Theorem

If \( F'(t) \) is continuous for \( a \leq t \leq b \), then

\[
\int_a^b F'(t)\,dt = F(b) - F(a)
\]

In other words: The definite integral of the derivative of a function gives the total change in the function.
Marginal Cost and Change in Total Cost

Assume $C'(q)$ is the marginal cost and $C(0)$ is the fixed cost.

- Cost to increase production from $a$ units to $b$ units
  $$= C(b) - C(a) = \int_a^b C'(q) dq$$
Assume $C'(q)$ is the marginal cost and $C(0)$ is the fixed cost.

- Cost to increase production from $a$ units to $b$ units:
  \[ C(b) - C(a) = \int_a^b C'(q) \, dq \]

- Total variable cost to produce $b$ units:
  \[ \int_0^b C'(q) \, dq. \]
Marginal Cost and Change in Total Cost

Assume \( C'(q) \) is the marginal cost and \( C(0) \) is the fixed cost.

- Cost to increase production from \( a \) units to \( b \) units
  \[ = C(b) - C(a) = \int_a^b C'(q) \, dq \]

- Total variable cost to produce \( b \) units
  \[ = \int_0^b C'(q) \, dq. \]

- Total cost of producing \( b \) units
  \[ = C(0) + \int_0^b C'(q) \, dq. \]
The marginal cost function of producing \( q \) mountain bikes is

\[
C'(q) = \frac{600}{0.3q + 5}.
\]

If the fixed cost in producing the bicycles is $2000, find the total cost to produce 30 bicycles.
The marginal cost function of producing $q$ mountain bikes is

$$C'(q) = \frac{600}{0.3q + 5}.$$ 

- If the fixed cost in producing the bicycles is $2000, find the total cost to produce 30 bicycles.
- If the bikes are sold for $200 each, what is the profit (or loss) on the first 30 bicycles.
The marginal cost function of producing $q$ mountain bikes is

$$C'(q) = \frac{600}{0.3q + 5}.$$

- If the fixed cost in producing the bicycles is $2000, find the total cost to produce 30 bicycles.
- If the bikes are sold for $200 each, what is the profit (or loss) on the first 30 bicycles?
- Find the marginal profit on the 31$^{th}$ bicycle.
The marginal cost function of producing $q$ mountain bikes is

$$C'(q) = \frac{600}{0.3q + 5}.$$ 

- $2000 + 2059.23 = 4059.23$
- If the bikes are sold for $200 each, what is the profit (or loss) on the first 30 bicycles
- Find the marginal profit on the $31^{th}$ bicycle.
The marginal cost function of producing \( q \) mountain bikes is

\[
C'(q) = \frac{600}{0.3q + 5}.
\]

- \( 2000 + 2059.23 = 4059.23 \)
- \( 20030 - 4059.23 = 1940.77 \)
- Find the marginal profit on the 31\(^{th}\) bicycle.
Example

The marginal cost function of producing $q$ mountain bikes is

$$C'(q) = \frac{600}{0.3q + 5}.$$

- $2000 + 2059.23 = 4059.23$
- $20030 - 4059.23 = 1940.77$
- $\pi'(31) = R'(31) - C'(31)$
The marginal cost function of producing $q$ mountain bikes is

$$C'(q) = \frac{600}{0.3q + 5}.$$  

- $2000 + 2059.23 = 4059.23$
- $2000 - 4059.23 = 1940.77$
- $\pi'(31) = 200 - \frac{600}{0.3 \cdot 31 + 5} \approx 158.04$