Distance and Accumulated Change

November 1, 2013
The rate of change of distance with respect to time is velocity.

Distance = Velocity × Time

Area = Distance traveled = 120 miles
How Do We Measure Distance Traveled

- The rate of change of distance with respect to time is velocity.
- If we given the velocity, can we find the distance traveled?

\[ \text{Distance} = \text{Velocity} \times \text{Time} \]

<table>
<thead>
<tr>
<th>velocity (mph)</th>
<th>time (h)</th>
</tr>
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<tbody>
<tr>
<td>40</td>
<td>3</td>
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Area = Distance traveled = 120 miles
The rate of change of distance with respect to time is velocity.
If we given the velocity, can we find the distance traveled?
Suppose velocity is constantly 40 mph, what is the distance traveled?

Distance = Velocity × Time

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The rate of change of distance with respect to time is velocity.
If we given the velocity, can we find the distance traveled?
Suppose velocity is constantly 40 mph, what is the distance traveled?
Distance = Velocity × Time

Area=Distance traveled=120 miles
Example 1

Suppose that you travel 30mph for 2 hours, then 40 mph for 1/2 hour, then 60 mph for 4 hours. What is the total distance you traveled?
We look at an example where the velocity is continually changing. Suppose an object is moving with increasing velocity and suppose we measure the object’s velocity every two seconds. How far has the object traveled?

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Velocity Data for Every 2 Seconds

- We don’t know how fast the object is moving at every moment

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Since the velocity is increasing, the object is going at least $2 \times 50 = 100$ feet in the first two seconds. Similarly, it goes at least $2 \times 86 = 172$ feet in the next two seconds, and so on.

During the ten-second period it goes at least $2 \times 50 + 2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146$ feet.
We don’t know how fast the object is moving at every moment.
We can’t calculate the distance exactly.

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### Time (sec) | Velocity (ft/sec)
---|---
0 | 50
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4 | 114
6 | 134
8 | 146
10 | 150
Velocity Data for Every 2 Seconds

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$$2 \times 50 + 2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 = 1060$$

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Velocity Data for Every 2 Seconds

Underestimate:

$$2 \times 50 + 2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 = 1060$$

Shaded are estimates distance traveled. Velocity measured every 2 seconds.
Since the velocity is increasing, the object is going at most $2 \times 86 = 172$ feet in the first two seconds.

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Similarly, it goes at most $2 \times 114 = 228$ feet in the next two seconds, and so on.

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\[2 \times 86 = 172\] feet in the first two seconds.
Similarly, it goes at most \[2 \times 114 = 228\] feet in the next two seconds, and so on.
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\[2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 + 2 \times 150 = 1260\] feet.

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During the ten-second period it goes at most

$$2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 + 2 \times 150 = 1260$$

$$1060 \text{ feet} \leq \text{Total distance traveled} \leq 1260 \text{ feet}$$
Velocity Data for Every 2 Seconds

Overestimate:

\[2 \times 86 + 2 \times 114 + 2 \times 134 + 2 \times 146 + 2 \times 150 = 1260\]

Shaded are estimates distance traveled. Velocity measured every 2 seconds.
Since the velocity is increasing, the object is going at least \( 1 \times 50 = 50 \text{ feet} \) in the first second.

Similarly, it goes at least \( 1 \times 69 = 69 \text{ feet} \) in the next second, and so on.

During the ten-second period it goes at least

\[
1 \times 50 + 1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 + 1 \times 141 + 1 \times 146 + 1 \times 149 = 1115
\]

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Velocity Data for Every Second

Underestimate:

\[ 1 \times 50 + 1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 + \\
+ 1 \times 141 + 1 \times 146 + 1 \times 149 = 1115 \]

Shaded are estimates distance traveled. Velocity measured every 1 second.
Since the velocity is increasing, the object is going at most \(1 \times 69 = 69\) feet in the first second.

Similarly, it goes at most \(1 \times 86 = 86\) feet in the next second, and so on

During the ten-second period it goes at most

\[
1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 + 1 \times 141 + \\
1 \times 146 + 1 \times 149 + 1 \times 150 = 1215
\]
Velocity Data for Every Second

Overestimate:

\[1 \times 69 + 1 \times 86 + 1 \times 101 + 1 \times 114 + 1 \times 125 + 1 \times 134 + 1 \times 141 + 1 \times 146 + 1 \times 149 + 1 \times 150 = 1215\]

Shaded are estimates distance traveled. Velocity measured every 1 second.
Velocity Data for Every Second

1115 feet \leq \text{Total distance traveled} \leq 1215 feet

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Velocity Data for Every 1/2 Second

Underestimate

Shaded are estimates distance traveled. Velocity measured every 1/2 second.
Velocity Data for Every 1/2 Second

Overestimate

Shaded are estimates distance traveled. Velocity measured every 1/2 second.
Velocity Data for Every 1/4 Second

Underestimate

Shaded are estimates distance traveled. Velocity measured every 1/4 second.
Velocity Data for Every 1/4 Second

Overestimate

Shaded are estimates distance traveled. Velocity measured every 1/4 second.
Velocity Data for Every 1/10 Second

Underestimate

Shaded are estimates distance traveled. Velocity measured every 1/10 second.

Distance and Accumulated Change
Velocity Data for Every 1/10 Second

Overestimate

Shaded area estimates distance traveled. Velocity measured every 1/10 second.
If the velocity is positive, the distance traveled is the area under the velocity curve.
With time $t$ in seconds, the velocity of a bicycle, in feet per second, is given by $v(t) = 4t$ for $0 \leq t \leq 3$, and by $v(t) = 15 - t$ for $t > 3$. How far does the bicycle travel in 7 seconds?
A city’s population grows at the rate of 500 people/year for 3 years and then grows at the rate of 300 people/year for 4 years. What is the total change in the population of the city during this 7-year period?
Example

The rate of sales (in games per week) of a new video game is shown in the table. Assuming that the rate of sales increased throughout the 20-week period, estimate the total number of games sold during this period.

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of sales</td>
<td>0</td>
<td>585</td>
<td>892</td>
<td>1875</td>
<td>2350</td>
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Example

Total sales = Rate of sales × Number of weeks.

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Distance and Accumulated Change
The value of a mutual fund increases at a rate of

\[ R = 500 \cdot 2^{0.04t}, \]

where \( t \) is in years since 2010.

1. Using \( t=0,2,4,6,8,10 \) makes a table of values for \( R \)
2. Use the table to estimate the total change in the value of the mutual fund between 2010 and 2020.