Global Maxima and Minima

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Local maxima and minima occur where a function takes larger or smaller values than at nearby points. However, we are often interested in where a function is larger or smaller than at all other points.

**Definition**

For any function $f$:
- $f$ has a global minimum at $p$ if $f(p)$ is less than or equal to all values of $f$.
- $f$ has a global maximum at $p$ if $f(p)$ is greater than or equal to all values of $f$. 
Example

Global Maxima and Minima

\[ y = x^2 + 4x + 14 \]
Example

Global Maxima and Minima

\[ x^3 + x + 5 \]
Example

Global Maxima and Minima

\[ y = \sin(x) + 1 \]
How to find global maxima and minima?

To find the global maximum and minimum of a continuous function on an interval including end points: Compare values of the function at all critical points in the interval and at the endpoints.

![Diagram showing how to find global maxima and minima](image)
How to find global maxima and minima?

To find the global maximum and minimum of a continuous function on an interval excluding end points or on the entire real line: Find the values of the function at all the critical points and sketch a graph.

\[ y = x^4 - 8x^3 + x^2 + 9x + 5 \]
Example

\[ f(x) = x^3 - 9x^2 - 48x + 52. \]
Example

For time, \( t \geq 0 \), in days, the rate at which photosynthesis takes place in the leaf of a plant, represented by the rate at which oxygen is produced, is approximated by

\[ p(t) = 100(e^{-0.02t} - e^{-0.1t}). \]

When is photosynthesis fastest? What is that rate?
Example

\[ p(t) = 100(e^{-0.02t} - e^{-0.1t}). \]
Example: Minimizing Gas Consumption

We investigate how to set driving speeds to maximize fuel efficiency. We assume that gas consumption, $g$ (in gallons/hour), as a function of velocity, $v$ (in mph) is as shown in figure. We want to minimize the gas consumption per mile, not gas consumption per hour. Let $G = g/v$ represent the average gas consumption per mile (The units of $G$ are gallons/mile).
Example: Minimizing Gas Consumption

Minimize $G = \frac{g}{v}$.