Second derivative

September 26, 2013
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What does the second derivative tell us?

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- If \( y = f(x) \), then the second derivative can also be written by \( \frac{d^2y}{dx^2} \).
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- If $y = f(x)$, then the second derivative can also be written by $\frac{d^2y}{dx^2}$.
- which means $\frac{d}{dx} \left( \frac{dy}{dx} \right)$, i.e. the derivative of $\frac{dy}{dx}$. 
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Theorem

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- If $f'' < 0$ on an interval, then $f'$ is decreasing over that interval, so the graph of $f$ is concave down there.
We think that the derivative as a rate of change, then the second derivative as the rate of change of rate of change.
We think that the derivative as a rate of change, then the second derivative as the *rate of change of rate of change*. If the second derivative is positive, the rate of change is increasing; if the second derivative is negative, the rate of change is decreasing.
A population, $P$, growing in a confined environment often follow a *logistic* growth curve.

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What is the sign of the second derivative $\frac{d^2 P}{dx^2}$?
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What is the practical interpretation of $t^*$ and $L$?
Initially, the population is increasing, and at an increasing rate.
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At $t^*$ the population is increasing fastest.
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At \( t^* \) the population is increasing fastest. 
\( L \) is the limiting value of the population.
Example

Table show the number of abortions per year.

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<tbody>
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<td>1359</td>
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(a) Calculate the average rate of change for the time interval shown between 1972 and 2005.

(b) What can you say about the sign of \( \frac{d^2 A}{dt^2} \) during the period 1972-1995?
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