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- For $y = f(x)$

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\[ \frac{d}{dx} \text{ stands for “the derivative with respect to } x \text{ of ...”}. \text{ Thus } \frac{dy}{dx} \text{ could be viewed as} \]

\[ \frac{d}{dx}(y) \]
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Advantage and disadvantage

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Interpretations of the derivative
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- How to represent \( f'(2) \) by our new notation?
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$dy$, over horizontal run, $dx$.

How to represent $f'(2)$ by our new notation?

$$\left. \frac{dy}{dx} \right|_{x=2}$$
The units of the derivative of a function are the units of the dependent variable divided by the units of the independent variable.
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The units of $dA/dB$ are the unit of $A$ divided by the units of $B$.

If the derivative of a function is not changing rapidly near a point, then the derivative is approximately equal to the change in the function when the independent variable increases by 1 unit.
The cost $C$ (in dollars) of building a house $A$ square feet in area is given by the function $C = f(A)$. What are the unit and the practical interpretation of the function $f'(A)$?

We can think $dC$ as the extra cost of building an extra $dA$ square feet. If we are planning to build a house with area $A$ square feet, $f'(A)$ is approximately the cost per square foot of the extra area involved in building a slightly larger house, and it is called marginal cost.
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Example

The cost of extracting $T$ tons of ore from a cropper mine is $C = f(T)$ dollars. What does it mean to say that $f'(2000) = 100$?

We can think of $dC/dT$ dollars per square ton as the extra cost of extracting an extra $dT$ tons of ore. So $dC/dT\bigg|_{T=2000} = 100$ says that when 2000 tons of ore have already been extracted from the mine, the cost of extracting the next ton, the $2000^{1}$st ton, is about 100$. 

Interpretations of the derivative
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Interpretations of the derivative
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If \( q = f(p) \) gives the number of thousands of tons of zinc produced when the price is \( p \) dollars per ton, then what are the units and the meaning of

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- *thousand of tons per dollar*

- When the price is $900, the quantity produced increase by 0.2 thousand tons for one-dollar increase in price.
The time $L$ (in hours), that a drug stays in a person’s system is a function of the quantity administered, $q$, in mg, so $L = f(q)$. 

- Interpret the statement $f(20) = 5$.
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Interpretations of the derivative
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(a) Interpret the statement $f(20) = 5$.

(b) Interpret the statement $f'(20) = 0.2$. 
Example

If the velocity of a car at time $t$ seconds is measured in meters/sec, what is the units of the acceleration?
Fertilizers can improve agriculture. A research of corn in Kenya found that the average value, \( y = f(x) \), in Kenyan shillings of the yearly corn production from an average plot of land is a function of quantity, \( x \), of fertilizer used in kg.
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(a) Interpret the statement \( f(20) = 11,500 \) and \( f'(20) = 350 \).

(b) Use part (a) to estimate \( f(21) \) and \( f(30) \).

(c) Which estimation in part (b) is more reliable?
Using the derivative to estimate values of a function

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If \( y = f(x) \) and \( \Delta x \) is near 0, then \( \Delta y \approx f'(x)\Delta x \). Then for \( x \) near \( a \) and \( \Delta x = x - a \),

\[
f(x) \approx f(a) + f'(a)\Delta x.
\]

This is called the \textit{Tangent Line Approximation}. 

The **(instantaneous) relative rate of change** of $y = f(t)$ at $t = a$ is defined to be

$$\frac{dy}{dt} = \frac{f'(a)}{f(a)}.$$
Annual world soybean production, \( W = f(t) \), in million tons, is a function of \( t \) years since the start of 2000.
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(a) Interpret the statement \( f(8) = 253 \) and \( f'(8) = 17 \).
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(b) Calculate the relative rate of change of $W$ at $t = 8$, interpret it in terms of soybean production.
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Annual world soybean production, $W = f(t)$, in million tons, is a function of $t$ years since the start of 2000.

(a) Interpret the statement $f(8) = 253$ and $f'(8) = 17$.

(b) Calculate the relative rate of change of $W$ at $t = 8$, interpret it in terms of soybean production.
Solar photovoltaic (PV) cells are the world’s fastest growing energy source. Annual production of PV cells, $S$, in megawatts, is approximately $S = 277 \exp 0.368t$, where $t$ is in years since 2000. Estimate the relative rate of change of PV cell production in 2020 using

(a) $\Delta t = 1$,
(b) $\Delta t = 0.1$,
(c) $\Delta t = 0.01$,