Exponential Functions

September 4, 2013
Population Growth

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
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<tbody>
<tr>
<td>Population</td>
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<td>2.093</td>
<td>2.168</td>
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<td>2.327</td>
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Table: The population (in millions) of Nevada 2000–2006.

Review questions:

- Is this population function increasing?
Population Growth

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- Is this population a concave up or concave down function? Why?
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**Table:** The population (in millions) of Nevada 2000–2006.

**Review questions:**

- Is this population function increasing?
- Is this population a linear function?
- Is this population a concave up or concave down function? Why?
- Find the relative change for each year between 2000 and 2003.
Population Growth

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- Let $t$ be the number of years since 2000, then the population is given by

$$ P = 2.020(1.036)^t. $$
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The population is an exponential function (with respect to \( t \)).

1.036 represents the factor by which the population grows each year. It is called the growth factor.
Assuming that the formula holds for 50 years (since 2000).
Problem 1. When a patient is given medication, the drug enters the bloodstream. The rate at which the drug is metabolized and eliminated depends on the particular drug. For the antibiotic ampicillin, approximately 40% of the drug eliminated every hour. A typical dose of ampicillin is 250 mg. Suppose $Q = f(t)$, where $Q$ is the quantity of ampicillin, in mg, in the bloodstream at time $t$ hours since the drug was given. Find several initial values of $f(t)$.

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- $f(5) = 19.4$
Elimination of Drug from the Body

Is this linear? Increasing? Decreasing? Concave up? Concave down?

Exponential Functions
Elimination of Drug from the Body

- Is this linear? Increasing? Decreasing? Concave up? Concave down?
- Find the formula of $Q = f(t)$. 

Exponential Functions
Elimination of Drug from the Body

- $Q = f(t) = 250(0.6)^t$

This function is called an exponential decay function.
Elimination of Drug from the Body

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Elimination of Drug from the Body

- \( Q = f(t) = 250(0.6)^t \)
- This function is called an **exponential decay function**.
The General Exponential Function

Definition

We say that \( P \) is an exponential function of \( t \) with base \( a \) if

\[
P = P_0 a^t.
\]

- \( P_0 \) is the initial quantity.
The General Exponential Function

Definition

We say that $P$ is an exponential function of $t$ with base $a$ if

$$P = P_0 a^t.$$ 

- $P_0$ is the initial quantity.
- $a$ is the factor by which $P$ changes when $t$ increase by 1.
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- If \( a > 1 \), we have an exponential growth.
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The General Exponential Function

**Definition**

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- $P_0$ is the **initial quantity**.
- $a$ is the factor by which $P$ changes when $t$ increase by 1.
- If $a > 1$, we have an **exponential growth**.
- If $0 < a < 1$, we have an **exponential decay**.
- $a = 1 + r$, where $r$ is the decimal representation of the percent rate of change.
Comparison between Linear and Exponential Functions

Definition

- A linear function has a constant rate of change.
Comparison between Linear and Exponential Functions

**Definition**

- A linear function has a constant rate of change.
- An exponential function has a constant percent rate of change (relative rate of change).
**Problem 2.** A quantity can change rapidly. Suppose the initial value is 100. Find the formula for the quantity $Q$ at the time $t$ minutes later if $Q$ is:

- Increasing by 3 per minute.
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- Increasing by 3 per minute.
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- Increasing by 3 per minute.
- Decreasing by 7 per minute.
- Increasing by 4% per minute.
Example

**Problem 2.** A quantity can change rapidly. Suppose the initial value is 100. Find the formula for the quantity \( Q \) at the time \( t \) minutes later if \( Q \) is:

- Increasing by 3 per minute.
- Decreasing by 7 per minute.
- Increasing by 4% per minute.
- Decreasing by 6% per minute.
Problem 3. Sales at the stores of company A increase from $2503 millions in 1990 to $3699 millions in 1996. Assuming the sales have been increasing exponentially, find the equation of the sale function $P$ with respect to $t :=$ the number of years since 1990.

$P = P_0 a^t$
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\[ P = P_0 a^t \]

\[ P_0 = 2503 \]
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$$P = P_0 a^t$$

$$P_0 = 2503$$

$$a^6 = 1.478$$
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$P = P_0 a^t$

$P_0 = 2503$

$a^6 = 1.478$

$a = 1.07$
Definition

The values of $t$ and $P$ in a table could form an exponential function $P = P_0 a^t$ if ratios of $P$ values are constant for equally spaced $t$ values.
### Example

#### Exponential Functions

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>1</td>
<td>24</td>
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<td>2</td>
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<tr>
<td>3</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>81</td>
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<tr>
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<td>35</td>
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<th>$h(x)$</th>
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Families of exponential functions
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