August 29, 2013
Average rate of change

|------|------|------|------|------|------|------|------|------|------|------|------|------|------|

Table: Men 100m World Records (Running)

- **Review**: Is Men 100m Word Record a decreasing function (of year)? a linear function?
Average rate of change

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Table: Men 100m World Records (Running)

- **Review:** Is Men 100m Word Record a decreasing function (of year)? a linear function?
- From 1964-1994, the time decreased 0.21 s.
Average rate of change

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**Table:** Men 100m World Records (Running)

- **Review:** Is Men 100m World Record a decreasing function (of year)? a linear function?
- From 1964-1994, the time decreased 0.21 s.
- From 1999-2009, the time decreased 0.21 s.
Review: Is Men 100m Word Record a decreasing function (of year)? a linear function?

From 1964-1994, the time decreased 0.21 s.

From 1999-2009, the time decreased 0.21 s.

What is average rate of change of the time from 1964 to 1994?
Table: Men 100m World Records (Running)

|------|------|------|------|------|------|------|------|------|------|------|------|------|------|

- **Review:** Is Men 100m Word Record a decreasing function (of year)? a linear function?
- From 1964-1994, the time decreased 0.21 s.
- From 1999-2009, the time decreased 0.21 s.
- What is average rate of change of the time from 1964 to 1994?
- What is average rate of change of the time from 1999 to 2009?
Definition

If $y$ is a function of $t$, so $y = f(t)$, then the **average rate of change** of $y$ between $t = a$ and $t = b$ is

$$\frac{\Delta y}{\Delta t} = \frac{f(b) - f(a)}{b - a}.$$
The line passing through $A$ and $C$ is called the **secant line** between $x = a$ and $x = c$. 
The line passing through A and C is called the secant line between $x = a$ and $x = c$.

The average rate of change is represented by the slope of the secant line.
Concavity

Definition

The graph of a function is **concave up** if it bends upward as we move from left to right; the graph is **concave down** if it bends downward.
Problem 1: A ball is thrown up in the air. The height of the ball above the ground is represented by the table. Find the rate of change and average rate of change in the first 3 seconds.

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y (ft)</td>
<td>6</td>
<td>95</td>
<td>150</td>
<td>160</td>
<td>150</td>
<td>100</td>
<td>40</td>
</tr>
</tbody>
</table>
The average rate of change of the height with respect to time is **velocity**.
Distance, Velocity, and Speed

- The average rate of change of the height with respect to time is velocity.
- Velocity ≠ Speed.
The average rate of change of the height with respect to time is **velocity**.

- Velocity $\neq$ Speed.
- Speed is the magnitude of velocity.
**Problem 2:** Find the average velocity of the ball over the interval \( t = 2 \) and \( t = 3 \). Compare to the similar value over the interval \( t = 3 \) and \( t = 4 \). Explain the difference.

<table>
<thead>
<tr>
<th>( t ) (sec)</th>
<th>0</th>
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</tr>
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</table>
Distance, Velocity, and Speed

Definition

Average velocity = \frac{\text{Change in distance}}{\text{Change in time}}
Is a population increase of 30 a significant change? If the number of graduate students in the math department, there are total 100 students, increases by 30 people, the graduate students would definitely notice. On the other hand, if the number of IU Bloomington students, there are total about 43,000 students, increases by 30 people, almost no one will notice.
Is a population increase of 30 a significant change? If the number of graduate students in the math department, there are total 100 students, increases by 30 people, the graduate students would definitely notice. On the other hand, if the number of IU Bloomington students, there are total about 43,000 students, increases by 30 people, almost no one will notice.

To visualize the impact of the increase on the two different communities, we look at the change, 30, as a fraction of the initial population. This change is called the relative change.
Relative change

Definition

When a quantity $P$ changes from $P_0$ to $P_1$, we define

$$\text{Relative change in } P = \frac{\text{Change in } P}{P_0} = \frac{P_1 - P_0}{P_0}$$

The relative change is a number, without unit. It is often expressed as a percentage.