Please Answer The Following Three Questions, (Turn Over the Page.)

**Question 1.** [5 points] Find the volume of the solid of revolution generated when the area bounded by the given curves is revolved around the $x-$axis.

\[ y = 2x, \quad y = 2, \quad x = 0. \]

1. **Cylindrical Shell Method:** Radius of shell is $y$, thickness is $dy$ and height is $x$-coordinate on $y = 2x$. Therefore, $dV = 2\pi y \frac{2}{2} dy$

\[ V = \int_0^2 2\pi y^2 dy = \left[ \frac{y^3}{3} \right]_0^2 = \frac{8\pi}{3}. \]

2. **Annulus (Washer) Method:** Inner radius is $y$-coordinate on $y = 2x$, outer radius is $y$-coordinate on $y = 2$, and thickness is $dx$.

\[ V = \int_0^1 \pi (4 - (2x)^2)dx = \pi \left[ 4x - \frac{4}{3}x^3 \right]_0^1 = \frac{8\pi}{3}. \]

**Question 2.** [5 points] Sketch the curves and find the areas of the region they bound.

\[ y = x^2 + 1, \quad y = 3 - x^2, \quad x = -3, \quad x = 2. \]

\[ A = \int_{-3}^{-1} [(x^2 + 1) - (3 - x^2)] dx + \int_{1}^{2} [(x^2 + 1) - (3 - x^2)] dx + \int_{1}^{2} [(3 - x^2) - (1 + x^2)] dx = \frac{56}{3}. \]

**Question 3.** [2 points Bonus!] Prove the mean value theorem for integrals by using the mean value theorem for derivatives. That is, if $f(x)$ is a continuous function on the closed interval $[a, b]$, then there exists some $c$ in the open interval $(a, b)$ such that

\[ \int_a^b f(t)dt = f(c)(b - a). \]

Let $F(x) = \int_a^x f(t)dt$. $F(x)$ is continuous on $[a, b]$ and differentiable on $(a, b)$, therefore by Mean Value Theorem for derivatives, there exists some $c$ in $(a, b)$ such that
\[ F'(c) = \frac{F(b) - F(a)}{(b - a)}, \quad \text{where by FTC } F'(x) = f(x). \]

Thus,

\[ f(c) = \frac{\int_a^b f(t)dt - \int_a^a f(t)dt}{(b - a)} \Rightarrow \int_a^b f(t)dt = f(c)(b - a). \]