Midterm Exam III

Last Name:______________
First Name:______________
ID:______________ Section:___

Math 2243,
April 28, 2004

There are 4 partial credit questions.
NO GRAPHIC CALCULATORS are permitted. GOOD LUCK!
1. Answer all the following questions:

a) Is there any injective linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$. If yes, give an example. [5 pts.]

b) Give an example of a matrix satisfying the equation $A^2 + 2A + I_2 = 0$. (Hint: remember that every matrix satisfies its own characteristic equation). [5 pts.]

c) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the reflection about the line of equation $x = y$. Find the eigenspaces of $T$. [15 pts.]
2.

a) Using the conservation of energy principle (kinetic + potential = constant) write down the equation of motion for a simple pendulum (interpreting the angle made by the pendulum with the vertical equilibrium position as a function of time). DO NOT ATTEMPT TO SOLVE THE EQUATION YOU GET! [10 pts.]

b) Is the equation you obtain in part a) a linear differential equation? [10 pts.]
3. Solve the Initial-Value-Problem

\[ y'' + y' = e^{-t}; \quad y(0) = 0; \quad y'(0) = 0 \]

(Hint: try a particular solution \( y_p(t) = te^{-t} \)) [35 pts.]
4. Let $v_0 = \langle 1, 0, -1 \rangle \in \mathbb{R}^3$ and $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as

$$T(v) = v \times v_0.$$ 

What is the kernel and the range of this linear transformation? [15 pts.] What is the matrix of the linear transformation? [10 pts.]