INSTRUCTIONS: Books and notes are not allowed. Calculators are allowed. Problems 1-3 are in “multiple choice” format. For these problems circle the answer you believe to be correct (only one of the answers listed for each problem is correct). Write complete solutions to problems 4-6 for full credit. You have 50 minutes to work on the problems.

Name: ___________________________ TA Section: ___________________________

1) (10 pts) Let \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) be a linear transformation with \( T((1, 0, 1)) = (3, 2, 1) \), \( T((0, 1, 0)) = (2, 2, 0) \), \( T((0, 0, 1)) = (-1, -1, -1) \). Then \( T((3, 3, 5)) \) is equal to

(A) (9, 6, 3)  
(B) (13, 10, 1)  
(C) (15, 12, 3)  
(D) (-2, -2, 1)  
(E) (9, 6, -2)

(Hint: Try first to write \((3, 3, 5)\) as a linear combination of vectors whose \(T\)-values you know.)
2) (10 pts) Let \( W(t) \) be the Wronskian of the vector-functions \( \mathbf{x}_1(t) = \begin{bmatrix} e^{-2t} \\ \cos 3t \\ -\sin 3t \end{bmatrix} \), 
\( \mathbf{x}_2(t) = \begin{bmatrix} 0 \\ \sin 3t \\ \cos 3t \end{bmatrix} \), \( \mathbf{x}_3(t) = \begin{bmatrix} e^{-2t} \\ -\cos 3t \\ \sin 3t \end{bmatrix} \). Then

(A) \( W(0) = 2 \) and the vectors are linearly independent.
(B) \( W(0) = 0 \) and the vectors are linearly dependent.
(C) \( W(0) = 0 \) and the vectors are linearly independent.
(D) \( W(0) = 2 \) and the vectors are linearly dependent.
(E) \( W(0) = 2 \) and the vectors may be either linearly dependent, or linearly independent.

3) (10 pts) Consider the differential system \( \mathbf{x}' = A\mathbf{x} + \mathbf{b}(t) \), with \( A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} \).
Assume you are given a solution \( \mathbf{x}_p(t) \). Then the general solution has the form

(A) \( c_1 e^{-t} \mathbf{v}_1 + c_2 e^{-t} \mathbf{v}_2 \)
(B) \( c_1 e^{-t} \mathbf{v}_1 + c_2 e^{-t} \mathbf{v}_2 + \mathbf{x}_p \)
(C) \( c_1 e^{-t} \mathbf{v}_1 + c_2 e^{-t} (\mathbf{v}_2 + t\mathbf{v}_3) + \mathbf{x}_p \)
(D) \( c_1 e^{t} \mathbf{v}_1 + c_2 e^{t} \mathbf{v}_2 + \mathbf{x}_p \)
(E) \( c_1 e^{-t} (\mathbf{v}_1 + t\mathbf{v}_2) + c_2 e^{-t} (\mathbf{v}_3 + t\mathbf{v}_4) + \mathbf{x}_p \)

End of multiple choice problems
4) (30 points) (a) Consider the matrix $A = \begin{bmatrix} 3 & -2 & -2 \\ 1 & 0 & -2 \\ 0 & 0 & 3 \end{bmatrix}$. Determine if $A$ is diagonalizable. In case it is, find a matrix $B$ such that $B^{-1}AB$ is a diagonal matrix.

(b) If $A$ is the matrix in part (a), find the solution to the initial value problem $x' = Ax$, $x(0) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. 
5) (25 pts) Find the general solution to the first-order linear differential system

\[\begin{align*}
  x_1' &= 5x_1 - 5x_2 \\
  x_2' &= 2x_1 - x_2
\end{align*}\]
6) (25 pts) Find two linearly independent solutions for the first-order linear differential system \( x' = Ax \), where \( A = \begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix} \). (Note that the matrix is the same as in problem 3, so you may use any of the calculations you did for that problem - however, you need to write them down here.)