Math 1571H. Practice Midterm Exam III November 29, 2006

There are a total of 100 points on this exam, plus one 5-point extra credit problem that you should only work if you complete the rest of the exam. To get full credit for a problem you must show the details of your work. Answers unsupported by by an argument will get little credit.

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Total: ________
Problem 1 (15 points) Find the derivative $f'(x)$ and simplify.

1. $f(x) = \ln(\sec x + \tan x)$

2. $f(x) = \int_{\sin x}^{6} (\cos t)^t \, dt$

3. $f(x) = (\sin x)^{2\tan x}$ Don’t simplify in this case.
Problem 2 (15 points) Find the volume swept out when the area under the top half of the ellipse

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

is rotated about the x-axis.

Problem 3 (15 points) Find the volume swept out when the area under the curve

\[ y = \sin(x^2), \quad 0 \leq x \leq \sqrt{\pi} \]

is rotated about the y-axis.
**Problem 4** (20 points) Find the length of the curve with vector equation

\[ R(t) = e^t \sin t \mathbf{i} + e^t \cos t \mathbf{j} \]

between \( t_0 = 0 \) and \( t_1 = \frac{1}{2} \ln 2 \).
Problem 5  (15 points)  A spring has a natural length of 15 in. and a 10 lb. weight stretches it 2 in. How much work is done in stretching the spring from -2 in. to +3 in.?

Problem 6  (20 points)  A dam has a vertical side in the shape of a right triangle with vertex at the bottom, height of 10 ft. and width of 12 ft. at the top. The water (density 62.5 lbs./ft.\(^3\)) behind the dam is 8 feet deep. Compute the total force of the water against the dam.
Problem 7 (EXTRA CREDIT, 5 points) The function

\[ f(x) = |(x + 2)(x - 1)| \]

is continuous everywhere, so it has an antiderivative. Compute the antiderivative \( F(x) \) on the domain \( x > 0 \) such that \( F(1) = 0 \). Simplify your answer!
Brief solutions.

1. 1) \( \sec x \), 2) \(- (\cos[\sin x])^{\sin x} \cos x \), 3) \\
\((\sin x)^2 \tan x \left[ 2 \sec^2 x \ln(\sin x) + 2 \right]

2. \( \frac{4\pi ab^2}{3} \)

3. \( 2\pi \)

4. \( 2 - \sqrt{2} \)

5. 12.5 in-lbs.

6. 6400 lbs.

7. 
\( f(x) = \begin{cases} 
\frac{1}{3}(x - 1)^3 + \frac{3}{2}(x - 1)^2, & x \geq 1 \\
-\frac{1}{3}(x - 1)^3 - \frac{3}{2}(x - 1)^2, & 0 < x < 1 
\end{cases} \)