POWER SYSTEM MODELING USING PETRI NETS

By

Lu, Ning

A Thesis Submitted to the Graduate
Faculty of Rensselaer Polytechnic Institute
in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Electric Power Engineering

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Rensselaer Polytechnic Institute
Troy, New York

September 20
(For Graduation Dec. 2002)
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# CONTENTS

LIST OF TABLES .................................................  v  
LIST OF FIGURES ................................................. vi  
ACKNOWLEDGMENT ................................................ ix  
Abstract ........................................................... x  

1. Introduction ..................................................... 1  
  1.1 Motivation .................................................. 1  
  1.2 Existing Modeling Methods ................................. 4  
  1.3 Petri Net Models ........................................... 7  
  1.4 Summary .................................................... 9  

2. Petri Net Models ............................................... 11  
  2.1 Basic Petri Net Notions ................................... 11  
  2.2 Matrix Analysis [18] ...................................... 14  
  2.3 Extensions, Abbreviations, and Particular Structures of PNs .... 17  
  2.4 Variable Arc Weighting Petri Nets ...................... 19  
  2.5 Colored Petri Nets ....................................... 29  
  2.6 Multi-layer Petri Nets ................................... 30  

3. Applications in Electric Power Systems ....................... 32  
  3.1 Physical Layer ............................................ 32  
  3.2 Information Layer ........................................ 38  
  3.3 Hybrid Models ............................................ 40  

4. A 3-zone Power System Dispatch Example ...................... 41  
  4.1 Problem Description ...................................... 41  
  4.2 Modeling Issues .......................................... 45  
  4.3 A Step-by-step Dispatch of a 3-Zone Power System .......... 47  
  4.4 Discussions ............................................... 62
5. Generator Bidding Strategies ............................................. 65
  5.1 Introduction ......................................................... 65
  5.2 Bidding Strategies for Steam Turbine Generators ................. 66
    5.2.1 Break-even Bid Curve ...................................... 68
    5.2.2 Maximum Profit Bid Curve ................................ 68
    5.2.3 High and Low Bid Curves .................................. 69
    5.2.4 Bid Curves Accounting for Generator Availability and Derating . 71
    5.2.5 Optimization of Block Bids ................................ 78
    5.2.6 Conclusions .................................................. 81
  5.3 Bidding Strategies for Pump-hydro Units .......................... 84
    5.3.1 Operational Constraints of a Pump-hydro Unit .............. 86
    5.3.2 Weekly MCP Variations ..................................... 87
    5.3.3 Optimal Bidding Strategies for Pump-hydro Units Based on Nominal Price Forecast ........................................ 87
    5.3.4 An Example for Unconstrained Scheduling of Pump-hydro Units . 91
    5.3.5 An Example for Constrained Scheduling of Pump-hydro Units . 94
    5.3.6 Optimization Under Uncertainties ........................... 98
    5.3.7 Comparison with a Basic Bidding Strategy .................. 99
    5.3.8 Conclusions .................................................. 102
  5.4 Summary .......................................................... 103
  5.5 Summary .......................................................... 103
6. A Price-feedback Market Simulator .................................... 105
  6.1 Motivations ....................................................... 105
  6.2 Model Structure .................................................. 106
  6.3 Modeling Issues .................................................. 111
  6.4 Example 1: Block Bids .......................................... 115
  6.5 Example 2: Bid-high Bids ....................................... 120
  6.6 Example 3: Insurance Bids ..................................... 123
  6.7 Summary .......................................................... 123
7. Conclusions and Future Work .......................................... 124
  7.1 Conclusions ....................................................... 124
  7.2 Future Work Recommendations ................................... 125
LITERATURE CITED ...................................................... 128
# LIST OF TABLES

2.1 Several interpretations of transitions and places [26] ............................................. 12
3.1 An example of the LMP scheme (1) ................................................................. 39
3.2 An example of the LMP scheme (2) ................................................................. 39
4.1 A load token vector list ......................................................................................... 42
4.2 A generator token vector list .................................................................................. 42
4.3 The nominal incidence matrix $C_o$ ........................................................................ 49
4.4 The diagonal elements of the adjustment matrix $D$ ............................................. 50
4.5 The firing vectors corresponding to different color pairs ....................................... 50
4.6 The marking evolution of the 3-Zone dispatch model ........................................ 64
5.1 $p_u$ with respect to $P_c$ ......................................................................................... 77
5.2 $k_L$ with respect to $P_c$ ......................................................................................... 77
5.3 $k$ with respect to $P_c$ ......................................................................................... 78
5.4 Results of the unconstrained case ......................................................................... 94
5.5 Results of the constrained case ............................................................................. 96
5.6 A basic bidding strategy ......................................................................................... 100
5.7 Profits of the optimal bidding strategy and the basic bidding strategy .................. 101
5.8 Profits obtained under incomplete information on MCP forecasts ...................... 102
6.1 The parameters of the generator bid curves .......................................................... 110
6.2 The implementation of optimal block bids ............................................................. 116
6.3 Bidding strategies for Generator 1 under different loads ...................................... 117
6.4 Simulation results for 3-equal block bids .............................................................. 117
6.5 Simulation results for optimal block bids ($\beta_1 = 0$) .......................................... 118
6.6 Simulation results under heavy load ($\beta_1 = 15$) ................................................ 118
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The cost curve of a steam turbine generator</td>
<td>4</td>
</tr>
<tr>
<td>1.2</td>
<td>The bid curve and the aggregated bid curves</td>
<td>6</td>
</tr>
<tr>
<td>1.3</td>
<td>The structure of a multi-layer PN model</td>
<td>7</td>
</tr>
<tr>
<td>1.4</td>
<td>The structure of a deregulated power market</td>
<td>8</td>
</tr>
<tr>
<td>2.1</td>
<td>A Petri net example</td>
<td>11</td>
</tr>
<tr>
<td>2.2</td>
<td>Firing a Petri net [25]</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>A Petri net example</td>
<td>15</td>
</tr>
<tr>
<td>2.4</td>
<td>A modification of the given Petri net example</td>
<td>18</td>
</tr>
<tr>
<td>2.5</td>
<td>A series network structure</td>
<td>23</td>
</tr>
<tr>
<td>2.6</td>
<td>A parallel network structure</td>
<td>25</td>
</tr>
<tr>
<td>2.7</td>
<td>A VAWPN model for a transmission line</td>
<td>27</td>
</tr>
<tr>
<td>2.8</td>
<td>A colored Petri net example</td>
<td>29</td>
</tr>
<tr>
<td>2.9</td>
<td>A multi-layer PN model for a 3-zone dispatch problem</td>
<td>31</td>
</tr>
<tr>
<td>3.1</td>
<td>The cost curve and bid curve for a generator unit</td>
<td>33</td>
</tr>
<tr>
<td>3.2</td>
<td>A Petri net representation of a generator</td>
<td>33</td>
</tr>
<tr>
<td>3.3</td>
<td>A subnet for the load aggregation process</td>
<td>35</td>
</tr>
<tr>
<td>3.4</td>
<td>The dispatching sequence of a load</td>
<td>36</td>
</tr>
<tr>
<td>3.5</td>
<td>A Petri net representation of an inter-zone transmission line</td>
<td>37</td>
</tr>
<tr>
<td>3.6</td>
<td>A combined Petri net model</td>
<td>38</td>
</tr>
<tr>
<td>3.7</td>
<td>A Petri net module for creating the priority list</td>
<td>40</td>
</tr>
<tr>
<td>4.1</td>
<td>A three-zone power dispatch example</td>
<td>41</td>
</tr>
<tr>
<td>4.2</td>
<td>The generation of the Load dispatch and generator dispatch tokens</td>
<td>43</td>
</tr>
<tr>
<td>4.3</td>
<td>The supply and demand curves</td>
<td>44</td>
</tr>
<tr>
<td>4.4</td>
<td>Distribution factors of each tie-line</td>
<td>45</td>
</tr>
</tbody>
</table>
4.5 The initial marking of the system ..................................................... 46
4.6 The first stage of the first-round dispatch ......................................... 48
4.7 The second stage of the first-round dispatch ...................................... 52
4.8 The first stage of round 2 dispatch ................................................... 55
4.9 The second stage of round 2 dispatch .............................................. 56
4.10 The third stage of round 2 dispatch ............................................... 57
4.11 The first stage of round 3 dispatch ............................................... 57
4.12 The second stage of round 3 dispatch ........................................... 59
4.13 The first stage of round 4 dispatch .............................................. 60
4.14 The first stage of round 5 dispatch ............................................... 61
4.15 The final marking of the system .................................................... 62
4.16 The final marking of the system - a MATLAB simulation result ........... 64

5.1 A typical cost curve of a steam generator ......................................... 67
5.2 Bid curves of a steam generator ....................................................... 68
5.3 Effects of “bid-high” and “bid-low” bid curves .................................. 70
5.4 The price margin for different slopes of the bid-high curve .................... 71
5.5 The unavailability curve of a steam unit .......................................... 74
5.6 $B_a(P)$ as a function of $k$ and $P$ ................................................... 74
5.7 (a) $B_d(P)$ as a function of $k$, (b) $B_d(P)$ as a function of $p_u$ ............. 78
5.8 A 3-segment bid curve of a steam generator ..................................... 79
5.9 (a) Normal distribution, (b) 3D plot of the expected profit vs $B_1$ and $B_2$, and (c) Equal profit contour versus $B_1$ and $B_2$ ......................... 82
5.10 (a) Uniform distribution, (b) 3D plot of the expected profit vs $B_1$ and $B_2$, and (c) Equal profit contour versus $B_1$ and $B_2$ ......................... 83
5.11 (a) A weekly MCP curve, (b) A weekly composite MCP curve ............. 87
5.12 An MCP curve for a time segment $T$ ............................................. 92
5.13 (a) Profit with respect to pumping time $t_p$, (b) Marginal profit with respect to pumping time $t_p$, (c) Energy storage, and (d) Generating/Pumping schedule 93
5.14 (a) $B_p$ and $B_g$ of iteration one, (b) Energy storage of iteration one, (c) $B_p$ and $B_g$ of iteration two, and (d) Energy storage of iteration two ........................................ 95

5.15 (a) $B_p$ and $B_g$ of iteration three, (b) Energy storage of iteration three, (c) $B_p$ and $B_g$ of iteration four, and (d) Energy storage of iteration four ........................................ 97

5.16 (a) Light load hours, (b) Heavy load hours ............................................................... 98

5.17 (a) Light-load hours, (b) Heavy-load hours ............................................................... 99

5.18 (a) Weekly MCP curves with uncertainties, (b) Composite MCP curves with uncertainties .............................................................. 99

5.19 Profits calculated based on probabilistic distributed MCPs ..................................... 102

6.1 The price-feedback market simulator .......................................................... 106

6.2 Generator bid curves .......................................................... 107

6.3 (a) Generator 1–1#, (b) Generator 1–2#, (c) Generator 1–3#, (d) Generator 1–4#, and (e) Generator 1–5# .......................................................... 108

6.4 (a) Generator 2–1#, (b) Generator 2–2#, (c) Generator 2–3#, (d) Generator 3–1#, and (e) Generator 3–2# .......................................................... 109

6.5 Aggregated supply curves .......................................................... 110

6.6 A 3-segment bid curve of a steam generator .......................................................... 114

6.7 Different loading conditions .......................................................... 115

6.8 (a) MCP curves, (b) Profit curves, and (c) Power output curves ........................................ 119

6.9 Fluctuations of the MCP .......................................................... 121

6.10 The strategy of bid-high .......................................................... 122

6.11 The aggregated supply curve considering derating ............................................. 123
ACKNOWLEDGMENT

I would like to express sincere thanks to my thesis committee members, Dr. George List, Dr. Robert C. Degeneff, and Dr. Hyde M. Merrill, for their help during different stages of my doctoral study.

I would like to take this chance to express my deepest gratitude to my thesis advisers, Dr. Alan A. Desrochers and Dr. Joe H. Chow, for their support and encouragement during the whole period of my doctoral study. They provided me with initiative, direction, and timely guidance while encouraging me to pursue my own thoughts. I always enjoy working with them as a student.

Words fail me to thank my parents for their encouragement and continuing support, without which, I would never have been able to make it this far. Finally, I would like to thank my friends for their friendship and their support, which made this long journey a joyful one.
Abstract

The Petri net model presented in this thesis focuses on the modeling and the simulation of the generator’s bidding behavior and the power system dispatch in a deregulated market, where Petri net models capture its stochastic and distributed characteristics. A hybrid multi-layer Petri net market simulator, which combines the modeling of physical flows and information flows, is proposed. The base layer is the physical layer, where the power transmission networks are modeled. On top of it are information layers modeling information flows which schedule the physical flow via discrete tokens. In between, there is an interface layer coded as programs and functioning as a control agent. Colored Petri Nets are used to simulate the information layer, in which the uncertainties presented in the decision making process during the bidding and the parallel behaviors of the bidders are modeled. An extension of continuous Petri nets, called a Variable Arc Weighting Petri net (VAWPN), is introduced to simulate the physical layer, in which vector tokens are used to match the information flows to the physical flows and distribution factors are used to obey the physical laws of the power flows.

The market simulator consists of three major modules: the ISO module, the Genco module and the Load module. The Independent System Operator (ISO) module simulates the priority-based dispatch process following a Locational-based Marginal Pricing (LMP) scheme. In addition to allowing the generator bids to be functions of price, the algorithm also accepts load bids as functions of price. Bidding strategies are developed for various types of generators, based on which a Generator Company (GenCo) module is developed to provide generator bids to the ISO module. With the price feedback from the ISO module, the GenCo module can allow the bidders to adjust their bids or switch bidding strategies. The Load Serving Entity (LSE) module generates load bids to the ISO module. LSEs are considered to be either a fixed aggregated demand or price sensitive block bids depending on the scenario studied. Impacts of generator bidding strategies on the market clearing prices (MCPs) are discussed based on the simulation results. Future research directions in this area are also addressed.
CHAPTER 1

Introduction

This chapter reviews the existing modeling methods for electric power systems and the topics relevant to the research. The motivation of the research is addressed in Section 1.1. Section 1.2 presents the existing modeling methods for the deregulated electric power market. Section 1.3 details the reason that Petri nets were selected as our approach to model the scheduling and dispatch problem of an open-access power market. The contributions of this thesis are summarized in Section 1.4.

1.1 Motivation

A major component of any service industry is a transport system that delivers services to the consumers. Examples of such systems include airlines, packages, freight, and electric energy. There are many commonalities between these systems such as pricing and congestion. It would be highly desirable that a common modeling approach can be applied to these systems serving diverse customers. This thesis focuses on the modeling of deregulated electric power markets and discusses a new modeling approach for power transport systems. Although the modeling techniques developed in this thesis are intended for power systems, with little modifications, they can be extended to many other transport systems as well, for example, air traffic systems [32].

The deregulated power market is more than ever determined by distributed decision-making and driven by discrete events, which are hard to simulate and model in traditional ways.

A regulated market has the following characteristics:

- A single optimization objective function
  An optimal economic generation schedule is established based on the assumption that a utility will serve the electric energy needs of its own customers at minimum cost. Interconnected operations and energy interchange agreements are the result of inter-utility arrangements with all of the parties sharing common interests [1].

- Centralized decision making
A centralized dispatch center controls the scheduling and dispatch of its own system. A carry-on-scheduling function is performed by each unit rather than self-scheduling. Therefore, all the generator behaviors are dependent and their functions are coordinated to meet a single economic objective for the utility that owns them.

- Deterministic
  The scheduling and dispatch problem is a deterministic optimization problem due to the fact that the cost curve of each generator unit is used to form an objective function, which is the total cost of the generation, together with constraints due to all the physical constraints. There are no human behaviors such as economic or physical withholding involved, which would disrupt market operation. Cost curves are known for each type of generator units with some uncertainties caused by the variations of water head for hydro units, the changes of the steam demand for cogeneration units, or the variations in fuel prices.

In a deregulated market, multiple parties in the bulk power systems engage in an open-access market competition with their own economic objectives to fulfill. The market is bid-based and three time-sequential energy markets are established: the bilateral trade market, the day-ahead market (DAM), and the real-time market (RT). The Independent System Operator (ISO) provides an equal access to transmission services for all qualified energy market participants. Generation companies (GenCos) compete in the DAM and RT markets to sell their energy. Load servicing entities (LSEs) buy forecasted load demands in the DAM and buy/sell the difference between the real demand and the committed DA power in the RT market.

The changes that deregulation has brought to the power industry are profound:

- Multiple optimization objective functions
  The economic objectives of the market participants are not identical. For GenCos, a minimization of the total energy production cost and a maximization of profit are the objectives. For LSEs, the goal is a payment minimization. Minimizing total generator cost will not, in general, result in minimum total load payment when the uniform price rule is used. For example, because the market clearing price is set by the most expensive generation dispatched, running an expensive local generator may indeed
lower the overall generation cost but it also results in a higher market clearing price which causes more load payment. For ISO, the market regulator who carries out a priority-based dispatch, where the generator bids are received and evaluated in a merit order based on both price and operational constraints, the goal is to find a near optimal solution within network physical constraints, while maintaining system reliability and security [2].

- Distributed decision making
  The ISO contributes the market dispatch, but it cannot dictate the supply and demand bids. Operations and planning of GenCos and LSEs are now decentralized and driven by market forces. However, their behaviors are not totally independent. In response to each other’s bidding strategy, market participants perform strategic bidding, gaming, and sometimes, tacit collusion, all of which further complicate the situation.

- Uncertainties
  In a deregulated market, the information available to GenCos and LSEs may be limited, regulated, or received with time delay [3]. In addition, a decision made by one participant may impact the overall system dispatch. These difficulties are compounded by the underlying uncertainty in fuel prices, unscheduled outages of generators and transmission lines, and tactics used by other market participants. Consequently, a market participant needs to hedge its supply and demand commitments to reduce potential risks arising from such uncertainties.

- Congestion Management
  Open access to the transmission system greatly encourages the interchange of power among different zones. Congestion caused by limited tie-line capacities can have a profound impact on energy prices in different zones because of the Locational-based Marginal Pricing (LMP) Scheme, which is introduced to differentiate the energy cost in congested zones with a shortage of inexpensive energy from uncongested zones that are well-supplied by inexpensive energy.

  These emerging changes call for new modeling methods and new dispatch algorithms to better represent and operate the restructured power system, as well as to provide more insights in its optimal operations.
1.2 Existing Modeling Methods

Traditionally, with only a single integrated electric utility operating both generators and transmission systems, the utility could establish dispatch schedules using unit commitment including hydrothermal scheduling [1] with an objective of minimizing its operating costs while taking into consideration all of the necessary physical, reliability, and economic constraints. A typical cost curve for a steam turbine unit is shown in Figure 1.1a. Usually, the cost can be represented by either a quadratic or a cubic function of the power output. An incremental cost curve \( \frac{dC}{dP} \) can be derived accordingly. For a quadratic cost curve, its incremental cost curve will be piece-wise linear as shown in Figure 1.1b. The beginning part and the ending part can be represented by different slopes to reflect the different rates of cost changes.

A simplified dispatch problem for \( T \) periods and \( N \) generators can thus be formulated as [1]:

\[
\min \sum_{t=1}^{T} \sum_{i=1}^{N} [F_i(P_t^i) + C_{st,i}]U_t^i = F(P_t^i, U_t^i) \tag{1.1}
\]

\[
U_t^i P_{t,\min}^i \leq P_t^i \leq U_t^i P_{t,\max}^i \quad \text{for } i = 1...N \quad \text{and } \quad t = 1...T
\]

\[
P_{t,\text{load}}^t - \sum_{i=1}^{N} P_t^i U_t^i = 0 \quad \text{for } \quad t = 1...T
\]

where

\( P_t^i = \) the power output of the unit \( i \) during period \( t \)

\( P_{t,\text{load}}^t = \) the total load during period \( t \)
\( C_{s_i,t} \) = the start up cost of unit \( i \) at time \( t \)

\( U^t_i = 0 \) if unit \( i \) is off-line during period \( t \)

\( U^t_i = 1 \) if unit \( i \) is on-line during period \( t \)

\( F(P^t_i, U^t_i) \) is the cost function of the generators

\( T \) is the total time period

\( N \) is the total number of generator units

A Lagrangian function can be formed as [1]

\[
\mathcal{L}(P, U, \lambda) = F(P^t_i, U^t_i) + \sum_{i=1}^{T} \lambda^t_i (P^t_{\text{load}} - \sum_{i=1}^{N} P^t_i U^t_i) \tag{1.2}
\]

Define

\[
q(\lambda) = \min_{P^t_i, U^t_i} \mathcal{L}(P, U, \lambda)
\]

the dual is

\[
q^*(\lambda) = \max_{\lambda^i} q(\lambda)
\]

where \( \mathcal{L}(P, U, \lambda) \) is the Lagrange function and \( \lambda \) is the Lagrange multiplier. The optimization is done in two steps [1]:

**Step 1** Find a value for each \( \lambda^i \) which moves \( q(\lambda) \) toward a larger value.

**Step 2** Assuming that the \( \lambda^i \) found in step 1 are now fixed, find the minimum of \( \mathcal{L} \) by adjusting the values of \( P^t \) and \( U^t \).

In a regulated system, a central dispatcher with all the cost curves \( F(P^t_i, U^t_i) \) available performs the optimization (1.2), the economic goal of which is to minimize the generation cost of the system as a whole. In a deregulated day-ahead market, an energy supplier submits to the ISO a set of piece-wise linear and monotonically increasing power-price supply bid curves (Figure 1.2a) for each generator or for a portfolio of generating units, one for each hour of the next day. These supply bid curves are aggregated by the ISO to create a single “supply bid curve”. On the other hand, an energy service company submits to the ISO an hourly power-price “demand bid curve” reflecting its forecasted demand. These demand bid curves are also aggregated by the ISO to create a single “demand bid curve”. Based on the demand and supply bid curves, the ISO determines a “Market Clearing Price” (MCP) for each hour as shown in Figure 1.2b.
The power to be scheduled for each bidder is then determined based on the individual bid curves and the MCP. All the power awards will be compensated at the MCP. Without knowing others’ bidding curves and with only an estimation of the MCP of the day ahead, there are many uncertainties in the process for the bidders to determine their bid prices. The goal of the bidder is to maximize profit rather than to minimize the generation cost of the system. They may evaluate historical MCPs, consider the impacts of their competitors’ strategies, and adjust their own bids accordingly. They may bid higher prices at peak-load hours when there is a shortage in generation and bid a lower price at light-load hours to remain dispatched in the system. The decision making process of each bidder maybe time varying, which introduces uncertainties into the price setting schemes and causes the volatility of the MCP in a deregulated market.

Game theories [4]-[12] have been used to investigate the possible bidding strategies as well as modified unit commitment methods [13] [14]. Discrete bidding strategies are described as “bid high”, “bid low”, or “bid medium” in matrix games and payoff matrices are constructed by enumerating all possible combinations of strategies. An “equilibrium” of the “bidding game” can be obtained. Modified unit commitment methods involve efforts to improve Lagrangian relaxation-based auction implementation and generation scheduling.

So far, dispatch models are based on mathematical forms. For a bidder to simulate the whole process, they need to have extensive knowledge of the power system generation, operation, and the network model.

The goal of this research is to develop a graphical representation of the dispatch process,
that can

- reflect market rules and reveal the operation mechanism of the power dispatch process,

- simulate the parallel, stochastic, and distributed bidding behaviors of the market participants,

- and provide the bidder a tool to develop better bidding strategies and evaluate them.

As a graphical and mathematical tool, Petri nets [15]-[23] have been successfully used in communication protocols and automated manufacturing systems, in which they offer a flexibility to simulate discrete systems. Therefore, we choose to use Petri nets as the modeling tool of the deregulated power market.

### 1.3 Petri Net Models

In general, a transport system can be divided into two layers (Figure 1.3): a physical layer and an information layer, which carry the physical flow and information flow, respectively. Information flow is discrete in nature and event-driven. Events are triggered either by the change in state of information flow or the change in state of the physical flow. A Petri net (PN) is a graphical and mathematical tool for describing and studying systems that are characterized as being concurrent, asynchronous, distributed, and stochastic. As a graphical tool, Petri nets are used as a visual communication aid similar to flow charts and networks. In addition, tokens are used in these nets to simulate the dynamic and concurrent activities
of systems. As a mathematical tool, it is possible to set up algebraic state equations with Petri nets, and other mathematical models governing the behavior of systems [24].

PN models can be readily used to describe a combined system model of several interacting subsystems. Each subsystem may interact with the other subsystems via token exchange. In the dispatch model that we will develop, we divide the model into three parts: the ISO, the GenCos and the LSEs, as shown in Figure 1.4. GenCos and LSEs submit generation bids and load bids to the ISO, which then dispatches the bids according to the merit lists set by the price priority of the bids. The ISO model is supposed to be a deterministic one, as all the information is available to the ISO, such as the amount of supply, and the amount of load and the transmission line capacities. The GenCo model and the LSE model are stochastic to simulate the uncertainties in their decision making process. The models interact with each other by exchanging tokens.

PNs are logical models derived from the knowledge of how the system works and allow the simulation of uncertainties via stochastic transitions [25]. The market rules can thus be incorporated into the net structure so that the net operates in the same way as the market operates. We will later show how the Locational-based Marginal Pricing (LMP) scheme is applied to operate the PN model.

PN models are able to describe “what-if” situations. For example, in strategic bidding, a generator owner may ask himself questions such as: “if others bid high (low) in a peak load time period, what are my bids?” or “what if the real time load is lower than the forecasted
Petri net models can simulate such kind of situations and provide an answer to these “what if” questions. Petri nets are widely used in the simulation of discrete-event systems. Because the decision-making system in a bid-based energy market is a discrete system that accommodates independent bidders and requires a bid-based priority dispatch, using Petri nets to describe the information layer is appropriate. It is the modeling of the physical layer, which contains continuous power flow, under the control of the information layer that poses the biggest challenge. In this thesis, Variable Arc Weighting Petri nets (VAWPN) are developed to model the physical layer. A third layer, the interface layer, is developed to control the operation of the VAWPN and provide communication links between the physical layer and the information layer. The structure of a multi-layer PN model is shown in Figures 1.3 and 1.4.

By extending ordinary Petri nets to Variable Arc Weighting Petri Nets (VAWPN), we would like the model to capture the discrete and distributed nature of the power market as well as reflect the distinct characteristics of the transportation of electric power energy.

1.4 Summary

This chapter has introduced the background of this research. Traditionally, with only a single integrated electric utility operating both generators and transmission systems, the utility could establish dispatch schedules that minimize its own operating costs while taking into consideration all of the necessary security and economic constraints.

A regulated market has the following characteristics

- A single optimization objective function
- Centralized decision making
- Deterministic

The changes deregulation has brought into the power industry are profound, including

- Multiple optimization objective functions
- Distributed decision making
- Stochastic
• Congestion management

A Petri net is a graphical tool as well as a mathematical tool to simulate and analyze systems that are concurrent, asynchronous, distributed, and stochastic. By extending PN models to the power system, we would like it to reflect the changes brought by the deregulation and provide an aid to market participants to understand the market rules and their operations. The model should serve as a simulator to evaluate the impacts of the various bidding strategies on the MCP. The interaction of the information flow and the physical flow is also addressed in building the model.

It is one of the goals of this research to develop a graphical representation of the dispatch process, which can

• reflect market rules and reveal the operation mechanism of the power dispatch process,

• simulate the parallel, stochastic and distributed bidding behaviors of the market participants,

• and provide the bidder with a tool to evaluate and develop better bidding strategies.

With little modification, the representation is expected to be extended to model other types of transportation systems sharing similar properties, for example, the air traffic systems.

Chapter 2 will introduce the basic notions of Petri nets and the extensions made to form VAWPN nets, as well as the operation mechanisms of the multi-layer model.

Chapter 3 develops A VAWPN ISO dispatch model.

Chapter 4 presents a 3-zone power system dispatch model to illustrate the multi-layer model.

Chapter 5 examines bidding strategies for different types of generators.

Chapter 6 presents the price-feedback market simulator.

Chapter 7 concludes the thesis and addresses the future research directions.
CHAPTER 2
Petri Net Models

This chapter introduces the basic Petri net notions in Section 2.1, the matrix analysis in Section 2.2, and the extensions, abbreviations and particular structures of Petri nets in Section 2.3. VAWPN nets are proposed and presented in Section 2.4. Colored Petri nets are introduced in Section 2.5. The multi-layer Petri net model for a 3-zone dispatch problem is addressed briefly in Section 2.6.

2.1 Basic Petri Net Notions

Mathematically, a Petri net (PN) (Figure 2.1) is defined as a 5-tuple [26], $PN = (P, T, A, W, M_0)$ where:

- $P = p_1, p_2, ..., p_m$ is a finite set of places,
- $T = t_1, t_2, ..., t_n$ is a finite set of transitions,
- $A \subseteq (P \times T) \cup (T \times P)$ is a set of arcs,
- $W : A \rightarrow 1, 2, 3...$ is a weight function,
- $M_0 : P \rightarrow 0, 1, 2, 3...$ is the initial marking,
- $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

![Figure 2.1: A Petri net example](image)

A PN consists of two types of nodes, called “places” ($P$) and “transitions” ($T$). Arcs ($A$) are either from a place to a transition ($P \times T$) or from a transition to a place ($T \times P$). Places are drawn as circles. Transitions are drawn as bars or boxes. Arcs are labelled with their weights ($W$), which take on positive integer values. The class of nets where we allow arc weightings greater than 1 are known as generalized Petri nets. When arc weightings are...
1, the class is known as ordinary PNs. The ordinary PN is considered to be the common language linking various versions of PNs. Table 2.1 gives a few possible interpretations of the places and transitions.

**Table 2.1: Several interpretations of transitions and places [26]**

<table>
<thead>
<tr>
<th>Input places</th>
<th>Transition</th>
<th>Output places</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resources needed</td>
<td>Task or job</td>
<td>Resources Released</td>
</tr>
<tr>
<td>Conditions</td>
<td>Clause in logic</td>
<td>Conclusion</td>
</tr>
<tr>
<td>Preconditions</td>
<td>Event</td>
<td>Postconditions</td>
</tr>
</tbody>
</table>

The marking at a certain time defines the state of the PN. The evolution of the state corresponds to an evolution of the marking, which is caused by the firing of transitions [24]. A marking is denoted by $M$, an $m \times 1$ vector, where $m$ is the total number of places. The $p^{th}$ component of $M$, denoted by $M(p)$, is the number of tokens in the $p^{th}$ place. The initial marking for the system represents the initial condition of the system and is denoted as $M_0$.

The state of the PN evolves from an initial marking according to two execution rules: enabling and firing. In an ordinary Petri net, if all the places that are inputs to a transition have at least one token, then the transition is said to be enabled and it may fire. When an enabled transition fires, a token is removed from each of the input places and a token is placed in each of the output places.

Figure 2.2 gives an example of firing a Petri net. The initial marking is $M_0 = (1 \ 1 \ 0 \ 1 \ 0)^T$ as shown in Figure 2.2a. With a default arc weighting of one, transition $t_1$ is enabled by the tokens in its upstream places $p_1$ and $p_2$. $t_1$ then fires, resulting one token removed from $p_1$ and $p_2$ and one token put into $p_3$ as shown in Figure 2.2b. The marking evolves to $M_1 = (0 \ 0 \ 1 \ 1 \ 0)^T$ after the firing of $t_1$. The tokens in $p_3$ and $p_4$ then enable transition $t_2$, the firing of which results in a marking of $M_2 = (0 \ 0 \ 0 \ 0 \ 1)^T$, as shown in Figure 2.2c. Note that the number of the tokens is not necessarily conserved in a PN model.

There are several behavioral properties of PNs [26]

- **Reachability**
  Reachability is a fundamental basis for studying the dynamic properties of any system. A marking $M_n$ is said to be reachable from a marking $M_0$ if there exists a sequence of
firings that transforms $M_0$ to $M_n$. The set of markings reachable from $M_0$ is denoted by $R(M_0)$.

- **Boundedness**
  A Petri net $(P, T, A, W, M_0)$ is said to be $k$-bounded or simply bounded if the number of tokens in each place does not exceed a finite number $k$ for any marking reachable from $M_0$, i.e. $k \geq M(p)$ for every place $p$ and every marking $M \in R(M_0)$. A Petri net $(P, T, A, W, M_0)$ is said to be safe if it is 1-bounded. By verifying that the net is bounded or safe, it is guaranteed that there will be no overflows in the buffers or registers, no matter what firing sequence is taken, and that the number of tokens in a place will not become unbounded.

- **Liveness**
  The concept of liveness is closely related to the complete absence of deadlocks in operating systems. A Petri net $(P, T, A, W, M_0)$ is said to be live if no matter what marking has been reached from $M_0$, it is possible to ultimately fire any transition in the net by progressing through some further firing sequence. This means that a live Petri net guarantees deadlock-free operation, no matter what firing sequence is chosen.

**Figure 2.2: Firing a Petri net [25]**

a) **Initial Marking**

b) **Marking after $t_1$ fires**

c) **Marking after $t_2$**
• Reversibility

A Petri net \((P, T, A, W, M_0)\) is said to be reversible if, for every possible marking reachable from \(M_0\), \(M_0\) is reachable from it. Thus, in a reversible net one can always get back to the initial marking or state.

2.2 Matrix Analysis [18]

The goal of any modeling methodology is to develop a mathematical description of a particular system in such a way that the model is accurate in its representation and also permits analysis of structural and/or performance properties. We hereby introduce the mathematical tools for analyzing Petri nets.

Arcs can be divided into two groups: input arcs and output arcs. Define:

1. \(I: P \times T \rightarrow 0, 1, 2, \ldots\) to be an input function that defines directed arcs from places to transitions, i.e., if \(I(p_i, t_j) > 0\), then we include an arc from \(p_i\) to \(t_j\). If \(I(p_i, t_j) = k\), \(k > 1\), then we label the arc \(k\).

2. \(O: P \times T \rightarrow 0, 1, 2, \ldots\) to be an output function that defines directed arcs from transitions to places, i.e., if \(O(p_i, t_j) > 0\), then we include an arc from \(t_j\) to \(p_i\). If \(O(p_i, t_j) = k\), \(k > 1\), then we label the arc \(k\).

**Enabling rule**: A transition, \(t_j\), of a PN is said to be enabled in a marking \(M\) if and only if \(M(p_i) \geq I(p_i, t_j)\) for all \(p_i\) which are members of the set of input places of \(t_j\).

**Firing rule**: An enabled transition can fire at any time and a new marking is reached according to the equation

\[
M_k(p_i) = M_{k-1}(p_i) + O(p_i, t_j) - I(p_i, t_j), \quad \forall p_i \in P
\]  

(2.1)

We next introduce the concept of an incidence matrix, which for a PN with \(m\) places and \(n\) transitions can be defined as an \(m \times n\) incidence matrix

\[
C(p_i, t_j) = O(p_i, t_j) - I(p_i, t_j)
\]  

(2.2)
Let $u_k$ represent the $k$th firing or control vector to indicate transition firing status then

$$M_k = M_{k-1} + C u_k$$  \hfill (2.3)

For the example shown in Figure 2.3, where $M_1 = M_0 + C \times u_1$, firing $t_1$ results in

$$M_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \left( \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Matrix analysis sometimes is called invariant analysis which may allow an evaluation of boundedness, liveness, and reversibility of the system model.

**P-invariant:**

For any marking $M_n$, we have

$$M_n = M_0 + C f_n$$  \hfill (2.4)

$$\sum_{k=1}^{n} u_k = f_n$$
where $f_n$ is the firing count vector at the $n^{th}$ firing. Premultiply the equation by $x^T$

$$x^T M_n = x^T M_0 + x^T C f_n \quad (2.5)$$

Notice that if $x^T C = 0$, then

$$x^T M_n = x^T M_0 \quad (2.6)$$

Invariants that satisfy the above conditions are called Place Invariants because for any invariant the weighted sum of tokens in its places is constant. We shall see that $x$ is actually the null space of $C^T$, which indicates that no matter what firing sequence is chosen, the sum of tokens in the places covered by the P-invariants will not change. The subnet formed by these places obeys the conservation of tokens, i.e., the number of tokens that come into the subnet shall be equal to the number of tokens that exit the subnet.

P-invariant analysis is very useful in checking whether or not a net structure is correct in reflecting built-in capacity constraints. For the example shown in Figure 2.3, the P-invariant is the null space of the incidence matrix $C^T$, which is

$$C = \begin{bmatrix} -1 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix}, \quad x^T C = 0 \quad \Rightarrow \quad x^T = [1 \ 0 \ 1]$$

The initial marking is

$$M_0 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \Rightarrow \quad x^T M = x^T M_0 \quad \Rightarrow \quad M(p_1) + M(p_3) = 1$$

If $p_3$ is considered to be a generator with a capacity of 1 MW and $p_1$ the capacity indicator, then the invariant analysis tells us that the structure satisfies the capacity constraints such that at any time, the token into $p_3$ will not exceed 1 MW. Because once $M(p_1)$ is 0, transition $t_1$ will be disabled. The analysis is as follows:

$$M(p_1) \geq 0 \quad \text{and} \quad M(p_1) + M(p_3) = 1 \quad \Rightarrow \quad M(p_3) \leq 1$$
In Section 2.4, another example of the use of P-invariants to check the structural integrity is given.

**T-invariant:**

For any marking $M$, we have

$$M = M_0 + Cf$$

Notice that if we have a firing sequence $f = y$ such that

$$Cy = 0 \implies M = M_0$$

Those values of $y$ are called T or transition invariants. The existence of T-invariants covering all transitions of the net is necessary but not sufficient to show reversibility. The reason is that we may find firing count vector solutions (T-invariants) that are not firable. It is easy to see that $y$ actually is the null space of the transition matrix $C$. For the example in Figure 2.3, because the $C$ matrix is full rank, there is no T-invariant possible, which means the net structure is not reversible. When the tokens in $p_2$ are depleted, the net reaches a deadlock, and marking $(1 0 0)^T$ will be the final marking of the net. In our Petri net models, a deadlock means the end of a state. For example, the marking of $(1 0 0)^T$ can be interpreted as the initial state for a generator, which is waiting for new load tokens to come. If a slight modification is made as shown in Figure 2.4, the T-invariant will be

$$C = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad Cy = 0 \implies y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

which means that after $t_1$ and $t_2$ all fire once, the net will reach its initial marking $(1 2 0)^T$.

### 2.3 Extensions, Abbreviations, and Particular Structures of PNs

Petri nets can be divided into three main classes in the literature: ordinary Petri nets, abbreviations, and extensions. In an ordinary Petri net, all the arcs have the same unity weight, there is only one kind of tokens, the place capacities are infinite (i.e., the number of tokens is not limited by place capacities), the firing of a transition can occur if and only if
every place preceding it contains at least one token, and no time is involved [24].

The abbreviations correspond to simplified representations, useful in order to lighten the graphical representation, to which an ordinary Petri net can always be made to correspond. Generalized PNs, finite capacity PNs, and colored PNs are abbreviations. They have the same power of description as the ordinary Petri nets [24].

The extensions correspond to models to which functioning rules have been added in order to enrich the initial model, enabling a great number of applications to be treated. Three main subclasses may be considered. The first subclass corresponds to models which have the descriptive power of Turing machines: an inhibitor arc PN and a priority PN. The second subclass corresponds to extensions allowing modeling of continuous and hybrid systems: continuous PNs and hybrid PNs. The third subclass corresponds to non-autonomous Petri nets, which describe the functioning of systems whose evolution is considered by external events and/or time: synchronized PNs, timed PNs, interpreted PNs, and stochastic PNs [24]. In [24], David and Alla have included a number of very good examples about these different types of PN models.

Recent years have seen a great deal of development in continuous PN models. Hybrid Petri nets containing both ordinary PNs and continuous PNs are especially valuable due to the fact that a hybrid PN can simulate systems where the physical flows are continuous or the amount of the particle flows is huge.

A Variable Arc Weighting Petri net (VAWPN) is a continuous Petri net. Several types of continuous Petri nets have been presented to model different types of continuous systems.
David and Alla [27] [28] used Instantaneous Firing Speeds (IFS) associated with transitions to describe the fluid nature of continuous systems. Balduzzi, Giua, and Menga [29] further developed this idea by introducing a linear algebraic formalism to analyze the first-order behavior of such nets. Trivedi [30] presented Fluid Petri nets where he described the system by a group of differential equations, the behavior of which is driven by a specific set of discrete Petri net markings. The VAWPN approach follows the idea proposed by Trivedi that continuous places hold continuous tokens and arcs carry token flows. Our contribution is the technique to calculate the arc weighting according to the characteristics of power systems and the use of an interface layer to convert vector tokens such that they can move across layers and carry information. As a result, the continuous physical layer model and the discrete information layer model can be analyzed separately while functioning as one interacting system. The transportation of power can be viewed as happening instantaneously. The IFS approach proposed by Balduzzi et al., which requires finite time transitions, is thus not applicable.

Our approach is to use the variable arc weighting (VAW) associated with each arc instead of the variable IFS associated with each transition, such that the VAWPN model preserves the same modeling flexibility as the IFS and fits into more scenarios involving immediate transitions. By using a total flow calculation and putting distribution factors in an adjustment matrix to account for the distributed flows in branches, we can use the incidence matrix to perform the marking calculation in the VAWPN models the same way as in discrete Petri net models. This approach does not change any conventions in discrete PN models except making the arc weights variables to reveal the nature of the continuous token flows in fluid PN models. The invariant analysis can still be performed to check the integrity of the network structure.

The VAWPN is primarily developed to model the unique characteristics of power flows, however, we believe that the modeling flexibility of variable arc weightings makes it possible to be applied to model many other systems containing continuous flow.

### 2.4 Variable Arc Weighting Petri Nets

A VAWPN is a 6-tuple net: \((P, T, TI, A, W, M_0)\). \(P\) is a set of places that can be divided into a set of physical layer places \(PF\) and a set of information layer places \(PI\) that
connect to the information layer transitions. These transitions are controlled by interface layer programs, which can convert the discrete tokens into continuous tokens or vice-versa. By doing so, all the places in the VAWPN nets are continuous places and all tokens are non-negative real number tokens representing the fluid level in each place. The transitions carry only time information. This greatly simplifies the problem and makes it possible to use traditional matrix methods to analyze the net structure and calculate the markings.

$TI$ is the Token Information vector. Similar to the tokens in Colored Petri nets [31], vector tokens allow information to be attached. Our innovation here is that we give the vector structure clear definition to meet our multi-layer concept.

- Initially, a token vector is a $k \times 1$ vector, such that there are $k$ pieces of information carried by the token. When a vector token moves through the VAWPN network, it encounters other vector tokens. The token vectors then add together to preserve the information carried by both, the process of which we call “token merging”. $n$ token vectors merging into one token vector results in a $k \times n$ vector.

- In our ISO model, a token vector is a $3 \times n$ vector. The first row is the identification number of the token, which indicates the origin of the token and corresponds to a color. The second row indicates the amount of power the token represents, which is the marking of the place. The third row carries the price information.

We will give an example later to show that this vector token concept is similar to a piece of mail getting stamped each time it passes a post office.

In power dispatch models, $TI(p_1) = (1 - 1\#, 200, 80)^T$ can be interpreted as follows. For this token in place $p_1$:

- The identification number is $1 - 1\#$, which represents Generator 1 in Zone 1 and corresponds to a token color of red.

- The amount of power (or the marking) is 200 MW.

- The bidding price of the 200 MW is $80/MW.

$W$ is the arc-weighting variable, which represents the physical flow in the path. Let $W_{in}$ represent input arc weightings and $W_{out}$ represent output arc weightings. Then we have
two sets of matrices

\[
I(p, t) = W_{\text{in}} = \begin{bmatrix}
    w_{\text{in}}(1, 1) & w_{\text{in}}(1, 2) & \ldots \\
    w_{\text{in}}(2, 1) & w_{\text{in}}(2, 2) & \ldots \\
    \cdots & \cdots & \cdots
\end{bmatrix}_{n \times m}
\]

\[
O(p, t) = W_{\text{out}} = \begin{bmatrix}
    w_{\text{out}}(1, 1) & w_{\text{out}}(1, 2) & \ldots \\
    w_{\text{out}}(2, 1) & w_{\text{out}}(2, 2) & \ldots \\
    \cdots & \cdots & \cdots
\end{bmatrix}_{n \times m}
\]

and

\[
C(p, t) = O(p, t) - I(p, t) = \begin{bmatrix}
    w_{\text{out}}(1, 1) - w_{\text{in}}(1, 1) & w_{\text{out}}(1, 2) - w_{\text{in}}(1, 2) & \ldots \\
    w_{\text{out}}(2, 1) - w_{\text{in}}(2, 1) & w_{\text{out}}(2, 2) - w_{\text{in}}(2, 2) & \ldots \\
    \cdots & \cdots & \cdots
\end{bmatrix}_{n \times m}
\]

where \(n\) is the number of places, \(m\) is the number of transitions, \(I(p, t)\) is the input matrix, \(O(p, t)\) is the output matrix, \(C(p, t)\) is the transition matrix, \(w_{\text{in}}(i, j)\) represents the weighting of the input arc from place \(i\) to transition \(j\) and \(w_{\text{out}}(i, j)\) is the weighting of the output arc from transition \(j\) to place \(i\).

As in power systems, assuming no transmission losses, the conservation of the power flow is held. Let \(C_0(p, t)\) represent the ordinary incidence matrix, where the arc weighting of each arc is set to one, \(F\) represent the total flow being transferred, and \(D\) be an adjustment matrix accounting for the distribution factors, we have

\[
C(p, t) = FC_0n(p, t)D
\]

(2.7)

By using a total flow calculation and putting distribution factors in an adjustment matrix to calculate the distributed flows in subbranches, we can use the incidence matrix to perform the marking calculation in the VAWPN models the same way as in ordinary Petri nets.

Once the arc weightings are set, the marking of the places can be calculated by

\[
M = M_0 + FC_0Df
\]

(2.8)
where $f$ is the firing count vector indicating which transitions fire.

Before we proceed to calculate the arc weightings, several assumptions need to be made. Because the VAWPN is specially designed to simulate and model the physical power flows (real power only), the following rules should be obeyed.

1. The places in the physical layer can be of the three types: resource places (loads), intermediate places (buses), and sink places (generators).

2. At any time the conservation of the physical flows should be obeyed.

3. Losses are ignored here for simplification purposes.

4. Arc weightings are calculated at each stage. The evolution of a stage is caused by either the marking of an activated information place goes to zero, or the marking of a VAWPN resource place goes to zero.

**Examples of Variable Arc Weighting Petri Nets**

To calculate arc weightings, we begin with two basic cases: series and parallel net structures.

**Case 1: Series net structure**

As shown in Figure 2.5a, $PF_1$ is a resource place and represents a load; $PF_2$ is an intermediate place (a bus); $PF_3$ is a sink place (a generator). $PI_1$ and $PI_2$ are information layer places. The interface layer control agent (not explicitly shown in the figures) will convert discrete tokens acquired from the information layer to vector tokens and then issue them to these information layer places. $PI_1$ represents the dispatch command coming from the dispatcher; $PI_2$ represents the capacity of the generator. Transition $t_1$ represents the dispatch of a load and $t_2$ represents the dispatch of a generator. The PN model in Figure 2.5a has an initial marking of $(5 \ 8 \ 100 \ 0 \ 0)^T$, which means that there are 100 MWs to be dispatched in total, the first load being dispatched is a 5 MW load, and the first generator being dispatched is an 8 MW generator. The incidence matrix is
where the incidence matrix $C$ can be decomposed as

$$C = F C_D = F \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

It is easy to see that, for a series net structure, a distribution factor of one is associated with each arc.

According to the above assumptions, the expression for the total flow $F$ in a series net structure is

$$F = \min(M(PI2), \min(M(PI1), M(PF1)))$$

For this example, we have

$$F = \min(8, \min(5, 100)) = 5$$

The control agent in the interface layer will perform the calculation and assign all the activated arcs with a weighting of 5. Once arc weightings are assigned, the transitions in Figure
2.5a can be enabled and then fired. In this case the marking evolves from \((5\ 8\ 100\ 0\ 0)^T\) to \((0\ 8\ 95\ 5\ 0)^T\) as shown in Figure 2.5b. Because \(M(PI1)\) reaches zero, no additional dispatch is being requested. The system moves to the next stage and waits for further command from the information layer. Analytical results are obtained by

\[
M_1 = M_0 + FC_o Df = \begin{bmatrix}
5 \\
8 \\
100 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
-5 & 0 \\
0 & -5 \\
-5 & 0 \\
5 & -5 \\
0 & 5 \\
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
8 \\
95 \\
5 \\
0 \\
\end{bmatrix}
\]

\[
M_2 = M_1 + FC_o Df = \begin{bmatrix}
0 \\
8 \\
95 \\
5 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
-5 & 0 \\
0 & -5 \\
-5 & 0 \\
5 & -5 \\
0 & 5 \\
\end{bmatrix} \begin{bmatrix}
0 \\
1 \\
0 \\
0 \\
0 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
3 \\
95 \\
0 \\
5 \\
\end{bmatrix}
\]

The token information vector (Figure 2.5a) in place \(PI1\) is represented as \(TI(PI1) = (1-1\#, 5, 80)^T\) (a 5 MW load in Zone 1 called 1-1# bids a price of $80/MW) and in place \(PI2\) as \(TI(PI2) = (1-1\#, 8, 50)^T\) (an 8 MW generator in Zone 1 called 1-1# bids a price of $50/MW). The token flow has been transported through the net during the firing of the transitions and the tokens reaching the sink place \(PF3\) (Figure 2.5b) are represented in the token information vector as

\[
TI(PF3) = \begin{bmatrix}
1-1\# & 1-1\# \\
8 & 5 \\
50 & 80 \\
\end{bmatrix}
\]

The information contained in the tokens reveals which generator is serving this load and the bid information of both sides. This is very useful if transmission cost is to be allocated among the loads and generators, because we can identify the suppliers for a given load.

**Case 2: Parallel net structure**

Figure 2.6 shows a parallel network structure, where \(PF1\) is a load, \(PF2\) is a bus, \(PF3\) is a generator, \(PI1\) is the load dispatcher, and \(PI2\) and \(PI3\) are the limits of the
transmission lines to the generator. The arcs $PF_2$ to $t_3$ and $PF_2$ to $t_2$ can be viewed as parallel transmission lines from the load to the generator. In this case, a decision needs to be made in $PF_2$, because $t_2$ and $t_3$ can both be enabled if there are tokens in $PF_2$. In power systems, power would flow on both paths with a ratio decided by the impedance of the paths. For a fixed power network, we can calculate a set of distribution factors [34]-[38], which indicate the flow distribution if we inject a certain amount of power in one node and extract the same amount of power from another node. If $d_f(i)$ represents the distribution
factors for the $i^{th}$ arc, we have

$$\sum_{i=1}^{n} d_f(i) = 1$$

The initial marking of the net is $(5 \ 3 \ 3 \ 100 \ 0 \ 0)^T$, which means that there are 100 MWs to be dispatched and the first load being dispatched is a 5 MW load. The capacities of the transmission lines connecting the load and the generator are 3 MW each.

The incidence matrix $C$ is

$$C = \begin{bmatrix}
  t1 & t2 & t3 \\
  PI1 & -F & 0 & 0 \\
  PI2 & 0 & -0.8F & 0 \\
  PI3 & 0 & 0 & -0.2F \\
  PF1 & -F & 0 & 0 \\
  PF2 & F & -0.8F & -0.2F \\
  PF3 & 0 & 0.8F & 0.2F \\
\end{bmatrix}$$

and again, $C$ can be decomposed as

$$C = FC_D = F$$

where the adjustment matrix $D$ is

$$D = \begin{bmatrix}
  1 & 1 & 1 & 1 & 1 & 1 \\
  1 & 0.8 & 1 & 1 & 0.8 & 0.8 \\
  1 & 1 & 0.2 & 1 & 0.2 & 0.2 \\
\end{bmatrix}$$

Note that $D$ now accounts for the effect of the distribution factors associated with the arcs $PF2$ to $t3$ to $PF3$ and $PF2$ to $t2$ to $PF3$. 
Given that $d_f(1) = 0.8$, $d_f(2) = 0.2$, the total flow is calculated as

$$F = \min(\min(M(PI1), M(PF1)), \min(M(PI2)/d_f(1), M(PI3)/d_f(2)))$$

$$= (\min(5, 100), \min(3/0.8, 3/0.2)) = 3.75$$

The $C$ matrix is readily obtained once the total flow $F$ is decided. Using (2.3), we can calculate the markings. The resultant arc weightings are shown in Figure 2.6b. After the marking of $PI2$ goes to zero, the system evolves to the next state. The calculation of the arc weightings for series and parallel structure nets is straightforward. For complicated net structures, where multiple resource places have interweaving pathes, network simplification and superimposing techniques have to be applied.

**Structure Analysis:**

There are many physical and operational constraints in electric power systems. PNs can readily incorporate these constraints into its structure. By using invariant analysis, we can verify whether or not these constraints have been satisfied by a specific network structure. Consider the example shown in Figure 2.7a, in which a transmission line allows bi-directional flow but is capacity limited. $P5$ and $P6$ are the terminal buses of the transmission line. $P2$ and $P4$ hold 1 token each, which indicates that the capacity of the transmission line is 1 MW. $P1$ and $P3$ are intermediate places and represent the token flows into the transmission line. As power cannot be stored in the transmission line, under further simplification these two places can be omitted as shown in Figure 2.7b. To make the example easier to understand we use the structure in Figure 2.7a to do the invariant analysis. The transition matrix is
Let $x^T C = 0$ and calculate vector $x$, we have

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Rightarrow C = O - I = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Then

$$x^T M = x^T M_0$$

For Figure 2.7a, where

$$M_0 = ( 0 \ 1 \ 0 \ 1 \ 1 \ 0 )^T$$

we have

$$x_1^T = ( 1 \ 1 \ 1 \ 1 \ 0 \ 0 )$$

$$(x_2 + x_3)^T = ( 1 \ 0 \ 1 \ 0 \ 1 \ 1 )$$

$$M(P1) + M(P2) + M(P3) + M(P4) = 2$$

$$M(P5) + M(P2) + M(P6) + M(P3) = 1$$

The invariant $M(P1) + M(P2) + M(P3) + M(P4) = 2$ means that at any time there are a total of 2 tokens in the four places. As the places $P1$ and $P3$ are intermediate places, the 2 tokens will be in either $P2$ or $P4$, which represent the transmission capacity in the direction
of $P6$ to $P5$ or $P5$ to $P6$. If 1 token is passed from $P5$ to $P6$, then $P4$ will be empty and $P2$ will have 2 tokens in it. In this situation, no token can be sent from $P5$ to $P6$, but 2 tokens can be sent in the opposite direction, which is from $P6$ to $P5$. However, the net flow, which is $|M(P2) - M(P4)|/2$, will never exceed 1. As negative tokens are not allowed, $x_2$ and $x_3$ are added up to obtain a new place invariant given by $M(P5) + M(P1) + M(P6) + M(P3) = 1$. It means that at any time there is only 1 token in one of these four places, which means, this network structure ensures that the flow in either direction at any time will not exceed 1 MW.

### 2.5 Colored Petri Nets

Colored Petri nets (CPNs) are used to simplify PNs in the cases when different kinds of resources are using the same resource allocation system. It is a more compact representation, which has been achieved by equipping each token with an attached data value - called the token color. The data value may be of arbitrarily complex type (e.g., a record where the first field is a real number, the second a text string, and the third is a list of integer pairs). For a given place all tokens must have token colors that belong to a specified type. This type is called the color set of the place [31]. Figure 2.8 shows a simply colored Petri net example,

![Figure 2.8: A colored Petri net example](image)

where the conflicts caused by enabling and firing $t1$, $t2$, or $t3$ are solved by assigning color to the tokens. A transition will be enabled only when each input place of the transition contains at least the number of tokens prescribed by the arc expression of the corresponding input arc. As shown in Figure 2.8a., when there are 50 red tokens in $PI1$ and the red arc
has a weighting of 50, \( t_1 \) will then fire and the resultant marking is shown in Figure 2.8b. Colored Petri nets are used in our information layer models. The colors are used to map the load control tokens and generator control tokens.

### 2.6 Multi-layer Petri Nets

So far, we are focusing on the VAWPN model of the physical layer, this is because information layer models can be constructed using conventional PNs, such as the CPN model we introduced in Section 2.5. Figure 2.9 shows a complete multi-layer model for the 3-zone dispatch model. Physical layers are power system networks. Information layers include bidding information and congestion information. Tokens in the information layer are integer tokens and tokens in the physical layer are continuous tokens. The control agent in the interface layer gathers and processes the information and then sends out control commands by issuing priority tokens and setting the transition firing sequences. It is coded by programs and functions through a communication link between the physical layer and the information layer.

Physical layer places can be viewed as resource locations that generate tokens, transfer stations that temporarily hold tokens, and consumer locations that consume tokens. Arcs are the paths where tokens can take to reach from the resource locations to the consumer locations. Each token can be viewed as a unity power to pass around to customers. It brings an information packet that is carried in the token vectors, each column of which contains the information needed to facilitate the control of physical flows. Rules can be interpreted either by transition firing sequences of the Petri net firing rules or by the priorities granted by the control agent in the interface layer. The tokens are flowing through the network the same way as they flow in the real power networks. Intermediate state information can be extracted from the system by tracking the marking of a certain place or the information contained in the vector tokens.
Figure 2.9: A multi-layer PN model for a 3-zone dispatch problem
CHAPTER 3  
Applications in Electric Power Systems

This chapter addresses the application issues. Basic Petri net modules for each power system component and the modeling issues of the physical layer network are discussed in Section 3.1. The bidding structure and market rules are introduced in Section 3.2. The multi-layer Petri net model is briefly discussed with an example 3-zone power system hybrid PN model in Section 3.3.

3.1 Physical Layer

The infrastructure of the power system can be divided into three parts: generation units, transmission networks, and distribution systems.

For a generator unit, the major factor to be considered during a dispatch is its Available Generation Capacity with respect to price. Transmission lines serve as a link between the generators and the loads. The most important physical constraint for a transmission line is the Available Transmission Capacity (ATC). The ATC is decided mostly by the network configuration, weather factors (such as temperature and wind conditions), and stability constraints. In the power grid, distribution systems can be treated as loads because the aggregation of loads mainly occurs in distribution systems. The behavior of individual small consumers is both parallel and highly stochastic in the load aggregation process. At the end of the transmission lines, a large amount of distributed loads become lumped loads, which can be described either by some stochastic distribution functions or by fairly accurate forecasted load curves.

Generators

Figure 3.1a. is a typical cost curve of a steam generator. Figure 3.1b shows a typical bid curve, \( B(P) \). The cost \( C \) of running a generator unit is usually a quadratic function or a cubic function with respect to its output \( P \). In some deregulated markets, the bid submitted by a generator unit is in blocks as described in Figure 3.1b. Two places and two transitions are used to represent a generator (Figure 3.2). The marking of \( PF1 \) indicates how many load tokens have been dispatched to this generator and the marking of \( PF2 \) indicates how many
more tokens the generator can take in. At the beginning of the dispatch, \( PF1 \) is empty and \( PF2 \) is full. The information layer control agent sends out dispatch commands by issuing tokens to \( PI2 \) according to the bids sent to the ISO. When \( PI2 \) receives a token, it enables transition \( t1 \), which then fires. Tokens will be removed from \( PF3, PI2, \) and \( PF2 \) and one token will be added into \( PF1 \).

When \( PF2 \) is empty, transition \( t1 \) will be disabled and the generator is then fully dispatched. Transition \( t2 \) is a deterministic timed transition, the duration of which depends on the bids interval. For example, if we use a step of 5 minutes, then every 5 minutes, the load bids refresh and so do the generator bids. The firing of \( t2 \) will reset the generator and make it ready for the next round of dispatch. Suppose that the generator has a capacity of 100 MW. Then this structure will guarantee that the total tokens in \( PF1 \) and \( PF2 \) will
never exceed 100. An invariant analysis will show that $PF_1$ and $PF_2$ are covered by a P-invariant of 100.

**Loads**

Loads can be divided into three categories: uninterruptible loads, interruptible loads, and loads with back-up generators (Figure 3.3a). Three kinds of load modules can be built accordingly. The load can switch into or out of the system after a time duration determined by certain probability distribution functions. If an exponential time duration is chosen, the random switching behavior can be simulated. Transitions $t_1$, $t_2$, and $t_3$ are deterministic transitions, which can drain load tokens at certain instances (like a sampling process) and aggregate them together. This load model can function as a load forecast model to provide the load information for the next dispatch interval to the bidder. To build such a load forecast model is one of our future research objectives. If the bid price and the amount of the load are assumed known, the load can be modeled as shown in Figure 3.3b.

The ISO receives the load bids and an aggregated demand curve can be created accordingly. The load tokens are to be dispatched in a sequence according to the bid prices such that the loads with higher bidding prices receive higher dispatch priorities during the dispatch. In Figure 3.3b, $PF_1$ holds all the load tokens while a control place $PI_1$ issues commands to determine how many tokens are to be dispatched at a time. The tokens in $PI_1$ will be given priorities merited by their bidding prices. Because priorities can be reflected into a different firing timing sequence, we can then use the controller place $PI_1$ to determine which load token is going to be passed to $PF_2$ first. $PF_2$ is an intersection among the inter-zone transmission lines, loads, and generators.

If a set of load bids are: 50 MW@$100; 30 MW@$60; 20 MW@$30, we have a dispatch sequence as shown in Figure 3.4.
Figure 3.3: A subnet for the load aggregation process
Figure 3.4: The dispatching sequence of a load
Transmission lines

There are two requirements for the proper representation of a transmission line. First, the model must be able to simulate bi-directional flows. Second, the capacity of a transmission line shall not be exceeded in either direction.

The resulting model is shown in Figure 3.5. PF5 and PF6 represent the two terminal nodes of a transmission line. If the power flows from PF5 to PF6, it will pass through PF1, and PF4 will be the transmission line capacity limiting place for the flow in this direction. The marking of PF2 represents the transmission line capacity in the direction of PF6 to PF3 and to PF5. Initially, when the net flow is zero, M(PF4) and M(PF2) both equal to $P_{tmax}$, where $P_{tmax}$ is the transmission line capacity in each direction. The power flow carried by the transmission line is calculated by

$$P_{tran} = \frac{(M(PF4) - M(PF2))}{2}$$

$M(PF(i))$ represents the number of tokens in $PF(i)$. In Petri net notation, it is called the marking of $PF(i)$. The power flow $P_{tran}$ may be negative or positive, which represents the direction of the power flow. An invariant analysis in Section 2.4 has shown that PF1, PF2, PF3, and PF4 are covered by a P-invariant. Note that the initial marking of PF2 and PF4 will be $P_{tmax}$, which will make sure that the markings of PF1 and PF3 will never exceed $P_{tmax}$. PF5 and PF6 will be intersections among inter-zone transmission lines, loads, and generators.

Figure 3.6 shows a combined Petri net model for a two-zone power network, which has loads in Zone 1 and generators in Zone 2. A transmission line connects the two areas together.
3.2 Information Layer

The information layers deal with the data collection, processing, storage, and distribution. The deregulated power market is bid-based. Therefore, for the dispatch model, there are two sets of data that are essential: the generator bids and the load bids. All bids are sent to the Independent System Operator (ISO). The ISO then dispatches the load according to their bidding prices. Bilateral transactions allow the load to hedge price volatilities. However, the load demand is normally hard to predict precisely. The surplus and the deficient power are all settled by the Locational-based Marginal Pricing (LMP) scheme.

In most deregulated power markets, generators bid in piece-wise continuous production curves. A generator priority list can be formed according to the bidding prices. This priority list can be used to determine the firing order of the generator enabling transitions. The cheapest generations are dispatched first. The LMP is the incremental cost to supply the next 1 MW of load at a specific location in the grid, if the load is not price sensitive.

In an open-access market, to make the market work in an interactive way, the loads also bid in the market for power supply. The load bidding price will be an upper limit price for the power they want to buy. The higher the bid is, the higher the priority of the load will be. According to the load bid price, we could obtain a load priority list, which can be used to determine the firing sequence of the load dispatch control tokens issued by $PI_1$ (Figure 3.3b). If we allow the load to bid in the market, the LMP is then defined as the minimum of the bid price of the last 1MW load dispatched and the bid price of the next 1MW generation available. As shown in Table 3.1, if a load of 100 MW bids $100$/MW and another load of 50 MW bids $90$/MW while two generators of 110MW and 100MW bid at $80$/MW and $95$/MW, then the 100 MW load will be dispatched first to the 110 MW generator. 10 MW
of the 50 MW load will be dispatched to the 110 MW generator as well. As the 100 MW generator bids a price of $95/MW, which is higher than the $90/MW load-bidding price, the 50 MW load is not fully served and the 100 MW generator is not committed. The LMP is then \(\min(90, 95) = 90\).

### Table 3.1: An example of the LMP scheme (1)

<table>
<thead>
<tr>
<th>Load bids</th>
<th>Generator bids</th>
<th>LMP ($/MWH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load 1: 100MWH@$100</td>
<td>Generator 1: 110MWH@$80</td>
<td>$90/MWH</td>
</tr>
<tr>
<td>Load 2: 50MWH@$90</td>
<td>Generator 2: 100MWH@$95</td>
<td>$90/MWH</td>
</tr>
</tbody>
</table>

If Load 2 had bid $96/MW instead of $90/MW, then Generator 2 will supply the rest of the 40 MW load of Load 2 and the LMP will be \(\min(96, 95) = 96\) (Table 3.2).

### Table 3.2: An example of the LMP scheme (2)

<table>
<thead>
<tr>
<th>Load bids</th>
<th>Generator bids</th>
<th>LMP ($/MWH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load 1: 100MWH@$100</td>
<td>Generator 1: 110MWH@$80</td>
<td>$95/MWH</td>
</tr>
<tr>
<td>Load 2: 50MWH@$96</td>
<td>Generator 2: 100MWH@$95</td>
<td>$95/MWH</td>
</tr>
</tbody>
</table>

The Colored Petri net structure shown in Figure 3.7 can serve as a priority list agent. Each time \(t_1\) fires, a color token is released to place \(P_1\). The color of the token represents the information of the location of this load. The price information is carried by the token as an attribute. Transitions \(t_2, t_3 \ldots t_n - 1\) then fire if the upstream place marking is greater than 1. If we want to create a load priority list, the firing of the transitions will move lower price tokens to the next place. If the bids are the same, remove the newly arrived token to the next place. Transitions \(t_1, t_2, \ldots, t_n\) are all deterministic time transitions. The time interval itself does not have a physical meaning but rather serves as a sequencing event trigger.

Transitions \(tt_1, tt_2, \ldots, ttn\) will fire in the next hierarchical level. This firing sequence will release the dispatch control token to \(P_{I1}\) which controls the dispatch of the load holder place \(P_{F1}\). The creation of the generator priority list is a similar process. The generator priority list, however, is in a price-ascending order instead of a price-descending order as the load priority list is.
3.3 Hybrid Models

If we link the physical layer and the information layer together with an interface layer, a hybrid Petri net model is then built.

This hybrid model is a demand side model, in contrast to the more common notation in power systems. In this model, tokens are moving from the loads to the generators instead of moving from the generators to the loads as in the traditional models. This is because the amount of the load actually determines the amount of power generated. A demand side model is a more natural way to commit generation. Otherwise, a feedback loop has to be provided to adjust the generations to meet the loads.

Load tokens either flow through the transmission lines to reach cheaper generators in other zones or are directly dispatched to generators in the same zone if cheaper remote energy sources are not accessible because of congestion. The tokens flow into the generator places just like water flowing into a container until the generators are fully loaded or the transmission line capacities have been depleted.

The information layers interface the physical layer by controlling the issue of tokens. In this model, we can view the generators and the loads as two groups of players. Each holds some tokens. The ISO, as a dispatcher, needs to pass these tokens to the right places while meeting the physical limits of the network.

In Chapter 4, a 3-Zone power dispatch model will be analyzed in detail to illustrate the operation of the hybrid PN model.
CHAPTER 4
A 3-zone Power System Dispatch Example

In this chapter, a dispatch problem of a 3-zone power system is presented in a step-by-step manner. The calculation of Variable Arc Weightings (VAW) is addressed. The advantages of using vector token concepts are discussed.

4.1 Problem Description

Figure 4.1: A three-zone power dispatch example

Figure 4.1 shows a 3-zone power system. The load and the generator bids from each zone are listed as well as the transmission line capacities. A load bid of 50 MW@$100 means that the amount of the load is 50 MW and the load will not accept any supply bids higher than $100. A generator bid of 30 MW@$20 means that the generator will provide a block of power of 30 MW at no less than $20.

Priority Lists and Dispatch Control Tokens:
Table 4.1: A load token vector list

<table>
<thead>
<tr>
<th>Load token</th>
<th>ID</th>
<th>Capacity (MW)</th>
<th>Bid price ($/MW)</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI1(1)</td>
<td>1-1#</td>
<td>50</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>TI1(2)</td>
<td>2-1#</td>
<td>100</td>
<td>90</td>
<td>2</td>
</tr>
<tr>
<td>TI1(3)</td>
<td>3-1#</td>
<td>120</td>
<td>70</td>
<td>3</td>
</tr>
<tr>
<td>TI1(4)</td>
<td>1-2#</td>
<td>30</td>
<td>60</td>
<td>4</td>
</tr>
<tr>
<td>TI1(5)</td>
<td>2-2#</td>
<td>40</td>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>TI1(6)</td>
<td>1-3#</td>
<td>20</td>
<td>30</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 4.2: A generator token vector list

<table>
<thead>
<tr>
<th>Generator token</th>
<th>ID</th>
<th>Capacity (MW)</th>
<th>Bid price ($/MW)</th>
<th>Priority</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI2(1)</td>
<td>1-1#</td>
<td>30</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>TI2(2)</td>
<td>1-2#</td>
<td>30</td>
<td>30</td>
<td>2</td>
</tr>
<tr>
<td>TI2(3)</td>
<td>3-1#</td>
<td>100</td>
<td>30</td>
<td>3</td>
</tr>
<tr>
<td>TI2(4)</td>
<td>2-1#</td>
<td>100</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>TI2(5)</td>
<td>1-3#</td>
<td>30</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>TI2(6)</td>
<td>2-2#</td>
<td>50</td>
<td>60</td>
<td>6</td>
</tr>
<tr>
<td>TI2(7)</td>
<td>3-2#</td>
<td>60</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>TI2(8)</td>
<td>1-4#</td>
<td>30</td>
<td>100</td>
<td>8</td>
</tr>
<tr>
<td>TI2(9)</td>
<td>2-3#</td>
<td>50</td>
<td>100</td>
<td>9</td>
</tr>
</tbody>
</table>

According to the bids, we can set up two priority lists for Figure 4.1: one for the load bids (Table 4.1) and the other for the generator bids (Table 4.2). 1-1# means Zone 1 - Load 1 and 2-1# means Zone 2 - Load 1. TI1(1) means information layer 1, token information vector 1. TI2(1) means information layer 2, token information vector 1.

According to the two priority lists, load dispatch control tokens and generator dispatch control tokens are generated as shown in Figure 4.2. The priorities given to the load dispatch control tokens are in price-descending order. The priorities given to the generator dispatch control tokens are in price-ascending order. The tokens are colored tokens (as shown shaded in Figure 4.2), the colors of which distinguish tokens from one another and match tokens to the right zone. This dispatch scheme follows the idea that an equilibrium price is obtained when the demand and the supply curve intersect as shown in Figure 4.3.

An equilibrium price of $60 is obtained by making the supply equal to the demand. This price would have been the market clearing price (MCP) if the power can be exchanged
Figure 4.2: The generation of the Load dispatch and generator dispatch tokens between zones without any limitations. However, as shown in Figure 4.1, the tie-line between Zone 2 and Zone 3 only has a capacity of 4 MW, which means, the amount of the power exchange among zones will be less than 4 MW. Therefore, for each zone, the localized supply curve may be well above the aggregated supply curve, because many less expensive generations outside the zone are not reachable. Thus, the MCP cannot simply be obtained by examining the aggregated supply curve and demand curve. This is why the Locational-based Marginal Pricing (LMP) scheme has been introduced to determine the MCP for each zone.
In the LMP scheme, the MCP is determined by the bid price of the next available 1 MW generation. If the demand bids are involved, then the LMP is determined by the minimum of the bid price of the next available 1 MW generation and the bid price of the last 1 MW load being dispatched. In our example, we apply the latter quantity to calculate the LMPs for each zone, because bids from both generators and loads are accepted and are used to set the priorities during the dispatch.

**Distribution factors:**

Distribution factors are used to calculate the power flow in each line. Distribution factors can be obtained by injecting 1 MW of power from Zone $i$ and withdrawing 1 MW power from Zone $j$, and then calculating the power flow in each tie-line. Let $d_f$ be the distribution factor matrix and $d_f(i, j, k)$ be the distribution factor from node $i$ to node $j$ to node $k$. We can calculate the line flow accordingly. For example, if 1 MW is injected into Zone 1, 0.9 MW directly reaches Zone 2 through tie-line 1-2 and 0.1 MW takes tie-line 1-3 and then 3-2 to Zone 2, then we have $d_f(1, 2, 2) = 0.9$ and $d_f(1, 3, 2) = 0.1$. In this given example, the distribution factors are assumed to be

- $d_f(1, 2, 2) = d_f(2, 1, 1) = 0.9$, $d_f(1, 3, 2) = d_f(2, 3, 1) = 0.1$
- $d_f(1, 3, 3) = d_f(3, 1, 1) = 0.8$, $d_f(1, 2, 3) = d_f(3, 2, 1) = 0.2$
- $d_f(2, 3, 3) = d_f(3, 2, 2) = 0.75$, $d_f(2, 1, 3) = d_f(3, 1, 2) = 0.25$

Figure 4.4 shows how the power flow will distribute along each tie-line.

**Transmission Line Capacities:**

![Figure 4.3: The supply and demand curves](image)
Let $P_{\text{tran}}(i,j)$ represent the tie-line capacity between node $i$ to node $j$. For this example, we use

$$P_{\text{tran}} = \begin{bmatrix} 0 & 140 & 130 \\ 140 & 0 & 4 \\ 130 & 4 & 0 \end{bmatrix}$$

The resulting system model is shown in Figure 4.5.

### 4.2 Modeling Issues

Before carrying out the dispatch, a few modeling issues need to be addressed first.

**Conflicts:**

We should note that there are conflicts in $PF1$, $PF3$, $PF6$, and $PF9$, where more than one arc come out of those places. In $PF1$, the conflicts are solved by the information carried by the dispatch control color tokens. For example, if the load dispatch control token is designated to Zone 1, then it enables the $t1$ leading to $PF3$; if it is designated to Zone 2, then $t2$ is enabled. The conflicts in $PF3$, $PF6$, and $PF9$ are solved by priorities. A generator transition $t4$ has a higher priority than a transmission line transition if the load token and the generator token are designated to the same zone, the information of which can be extracted from the identification number carried by each token.

**Rounds, stages, and congestions:**

A dispatch is carried out in such a way that the load token whose bid price is the highest in the priority list is picked to be dispatched first. A generator is then picked from the top of the generator priority list. The dispatch is carried out in rounds, and in each
round we dispatch a load bid. If the load is fully dispatched, (the marking of $PI_1$ will be zero), we can start another round of dispatch for the next load bid in the load priority list. Within each round, if the generator capacity is depleted (the marking of $PI_2$ is zero), we call it an end of a stage in this round of dispatch and move on to dispatch the next generator in the generator priority list. If the load and the generator are in the same zone, then it is called an internal dispatch. If the load and the generator are in different zones, then it is called a cross-zone dispatch. In a cross-zone dispatch, we need to transfer power through transmission lines. If the transmission line capacity is not exceeded during a dispatch, then the power exchange between the zones is valid. If the transmission line capacity has been exceeded, we consider it congested. Once a tie-line is congested, no more power transfer can proceed in that flow direction. In a congestion case, an internal dispatch is performed to dispatch the remaining load in the congested area.

The dispatch round, stage, and congestion correspond to the change of state in the PN

![Figure 4.5: The initial marking of the system](image-url)
model. The change of state is marked by depleting the resource of the system. Therefore, if the marking of any of the resource places becomes zero, the state of the PN model will change.

**Dispatch Patterns:**

Generally speaking, a dispatch can be reduced to three patterns: internal dispatch, cross-zone dispatch without congestion on the inter-zone tie-lines, and cross-zone dispatch with congestion in one or more of the inter-zone tie-lines. If the load and the generator tokens have the same color, that is, the load and the generator are from the same zone, the dispatch is called an internal dispatch. Otherwise, the dispatch is a cross-zone dispatch. If one of the transmission line capacity becomes zero during the cross-zone dispatch, then it is called a congestion case. Finding the dispatch patterns can simplify the calculation of the variable arc weightings (VAW). It also enables a network to be simplified, making it possible to analyze more complicated networks.

**“MW” and “MWH”:**

“MW” is the unit for power and “MWH” is the unit for electric energy. In this thesis, we expect the power consumption to be in a consecutive time frame and we use “$/MW” for energy prices which can be viewed as the energy price in a small time interval.

### 4.3 A Step-by-step Dispatch of a 3-Zone Power System

We hereby perform a step-by-step dispatch of the given system model and illustrate the calculation of the Variable Arc Weighting (VAW) according to different dispatch patterns.

**Internal dispatch:**

If the generator to be dispatched is in the same zone as the load is, then an internal dispatch will be carried out. That is, if the generator token and the load token have the same color, it reflects in a color pair of (1,1), (2,2) or (3,3) in this model. Let $P_d$ represent the demand bid power, $P_g$ represent the supply bid power, $P_t$ represent the transferring power, and $F$ represent the total flow, we have

$$P_t = \min(P_d, P_g)$$  \hspace{1cm} (4.1)$$

$$F = w_o(p_i, t_j) = w_i(p_i, t_j) = P_t$$  \hspace{1cm} (4.2)$$
From the Load Priority List, we find that the first load to be dispatched is Load 1 in Zone 1 with a bid of 50 MW at $100/MW. The first generator to be dispatched is Generator 1 in Zone 1, with a bid of 30 MW at $20/MW. The color pair is (1,1), which indicates that both the generator and the load are from Zone 1. This is an internal dispatch as shown in Figure 4.6a. Here $P_d$ is 50 MW, $P_g$ is 30 MW, and

$$F = P_t = \min(50, 300) = 30$$

where $F$ is the total amount of power being transferred through the network.

Figure 4.6: The first stage of the first-round dispatch
Arranging the places in Figure 4.5 in the order
\[ P = \{ PI1 \ PI2 \ PF1 \ PF2 \ PF3 \ PF4 \ PF5 \ PF6 \ PF7 \ PF8 \ PF9 \ PF10 \ PF11 \ PF12 \} \]

The initial marking is
\[ M_0 = \left( \begin{array}{ccccccccccccc}
50 & 30 & 360 & 0 & 0 & 140 & 140 & 0 & 4 & 4 & 0 & 130 & 130 & 0
\end{array} \right)^T \]

The ordinary incidence matrix \( C_o \) is shown in Table 4.3, where the blank entries denote zero.

**Table 4.3: The nominal incidence matrix \( C_o \)**

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The distribution factors will have an influence on the incidence matrix by putting a ratio on the arc weighting of transmission lines. Define \( D(i, j) \) as the adjustment matrix and let \((i, j)\) correspond to a color pair. Each \( D(i, j) \) is a 13 \( \times \) 13 diagonal matrix with the main entries shown in Table 4.4.

The firing vectors corresponding to different color pairs are shown in Table 4.5.

The color pair (1,1) corresponds to a firing vector of
\[ f = (1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T \]
Table 4.4: The diagonal elements of the adjustment matrix $D$

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Table 4.5: The firing vectors corresponding to different color pairs

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The marking will evolve from $M_0$ to $M_1$, according to

\[
M_1 = M_0 + F \times C_o D(1,1) f(1,1) = M_0 + 30 \times C_o D(1,1) f(1,1)
\]

\[
= ( 20 0 330 0 0 140 140 0 4 4 0 130 130 30 )^T
\]

Note that all the related arc weightings were set at $F = 30$. The transitions are enabled and then fired. The resulting marking is shown in Figure 4.6b. When $PI2$ reaches zero, $t_5$ is enabled and then fired. $TI3(1)$ moves to $PF12$ (Figure 4.6c), which serves as a collector of all the dispatched tokens. The marking of $PI2$ reaches 0 and calls an end to the first stage of the first-round dispatch. A generator dispatch control token is then fired to $PI2$ to
dispatch the next generator in the priority list. The generator is the Generator 2 in Zone 1 and the bid is 30 MW@$30. The color pair is still (1,1). Now $P_d$ is 20 MW, $P_g$ is 30 MW, and

\[
F = P_t = \min(20, 30) = 20
\]
\[
f(1, 1) = (1 0 0 1 0 0 1 0 0 0 0 0 0)^T
\]
\[
M_2 = M_1 + F \times C_o D(1, 1)f(1, 1) = M_1 + 20 \times C_o D(1, 1)f(1, 1)
\]
\[
= ( 0 10 310 0 0 140 140 0 4 4 0 130 130 50 )^T
\]

Thus, all the related arc weightings are set to 20. The transitions are enabled and then fired as shown in Figure 4.7. At this time, the $PI_1$ marking goes to 0 and calls an end to the first-round dispatch. Notice that $M(PF2) = 0$ and $M(PF12) = 50$, which are inconsistent with the markings shown in Figure 4.7b. This is because for illustration purposes, we do not consider the inhibitor arc effect on the markings. We can view $M(PF12) = 50$ as a sum of the markings of $PF2$ and $PF12$, which represents the total load token dispatched.

**Cross Zone Dispatch:**

In the second-round dispatch, a 100MW load in Zone 2 is to be dispatched to a generator in Zone 1 with 10MW capacity left as shown in Figure 4.8a. The color pair is (2,1), which is a cross-zone dispatch. To calculate the arc weightings, which are the power flows in each tie-line, we have to satisfy the following equation

\[
F = P_t = \min(P_d, P_g)
\]

with the constraints

\[
w_{i,j} = P_t \times d_f(i, j, j) \leq P_{\text{tran}}(i, j)
\]
\[
w_{i,k} = P_t \times d_f(i, k, j) \leq P_{\text{tran}}(i, k)
\]
\[
w_{k,j} = P_t \times d_f(i, k, j) \leq P_{\text{tran}}(k, j)
\]

where $P_{\text{tran}}(i, j)$ is the tie-line capacity from node $i$ to node $j$ and $w_{ij}$ is the arc weighting from node $i$ to node $j$.

If the constraints are satisfied, no congestion occurs. When including the above con-
Figure 4.7: The second stage of the first-round dispatch

straints, we have

\[
P_t(1) = \min(P_d, P_g)
\]

\[
w_{ij} = \min(P_t \times d_f(i, j), P_{\text{tran}}(i, j)) \Rightarrow P_t(2) = \frac{w_{ij}}{d_f(i, j)}
\]

\[
w_{ik} = \min(P_t \times d_f(i, k), P_{\text{tran}}(i, k)) \Rightarrow P_t(3) = \frac{w_{ik}}{d_f(i, k)}
\]

\[
w_{kj} = \min(P_t \times d_f(i, k), P_{\text{tran}}(k, j)) \Rightarrow P_t(4) = \frac{w_{kj}}{d_f(i, k, j)}
\]

\[
F = \min(P_t(1), P_t(2), P_t(3), P_t(4))
\]

\[
w_{ij} = F \times d_f(i, j)
\]

\[
w_{ik} = F \times d_f(i, k)
\]

\[
w_{kj} = F \times d_f(i, k)
\]
For the given example, with $P_d < P_g$,

\[
\begin{align*}
P_t(1) &= \min(10, 100) = 10 \\
w_{21} &= \min(10 \times 0.9, 140) = 9 \Rightarrow P_t(2) = \frac{9}{0.9} = 10 \\
w_{23} &= \min(10 \times 0.1, 4) = 1 \Rightarrow P_t(3) = \frac{1}{0.1} = 10 \\
w_{31} &= \min(10 \times 0.1, 130) = 1 \Rightarrow P_t(4) = \frac{1}{0.1} = 10 \\
F &= \min(10, 10, 10, 10) = 10
\end{align*}
\]

The color pair (2,1) corresponds to a distribution factor adjustment $D$ to the incidence matrix, where

\[
D = \text{diag}(1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0.9 \ 0.9 \ 0.1 \ 0.1 \ 0.1 \ 0.1)
\]

The marking is calculated as

\[
M_3 = M_2 + F \times C_o D(2,1)f(2,1) = M_2 + 10 \times C_o D(2,1)f(2,1)
\]

\[
= (90 \ 0 \ 300 \ 10 \ 0 \ 149 \ 131 \ 0 \ 3 \ 5 \ 0 \ 129 \ 131 \ 60)^T
\]

As the transmission line capacities are not exceeded, there is no congestion. The marking evolution is shown in Figure 4.8. TI2 again reaches zero and the first stage of the round 2 dispatch then ends. The next generator available is in Zone 3 with an amount of 100 MW as shown in Figure 4.9a. The color pair is (2,3). This is also a cross-zone dispatch and can be calculated as

\[
\begin{align*}
P_t(1) &= \min(90, 100) = 90 \\
w_{23} &= \min(90 \times 0.75, 3) = 3 \Rightarrow P_t(2) = \frac{3}{0.75} = 4 \\
w_{21} &= \min(90 \times 0.25, 131) = 22.5 \Rightarrow P_t(3) = \frac{22.5}{0.25} = 90 \\
w_{13} &= \min(90 \times 0.25, 131) = 22.5 \Rightarrow P_t(4) = \frac{22.5}{0.25} = 90 \\
F &= \min(90, 4, 90, 90) = 4
\end{align*}
\]
The marking thus is calculated by,

\[ M_4 = M_3 + F \times C_o D(2, 3) f(2, 3) = M_3 + 4 \times C_o D(2, 3) f(2, 3) \]
\[ = ( 86 \ 96 \ 296 \ 0 \ 0 \ 150 \ 130 \ 0 \ 8 \ 0 \ 0 \ 130 \ 130 \ 64 )^T \]

The transmission line capacity \( PF7 \) reaches 0, which requires a change of stage. As this is a direct congestion, where no countering flows can be provided, the remaining load must be dispatched inside its zone. As the load is in Zone 2, we then search the generator priority list, find a generator in Zone 2, and begin an internal dispatch as shown in Figure 4.10. When PI1 goes to 0, round 2 dispatch is ended. The next load is in Zone 3. We will resume dispatch load to Generator 3-1, which still has 96 MW capacity left. This again is an internal dispatch as shown in Figure 4.11.
Figure 4.8: The first stage of round 2 dispatch
Figure 4.9: The second stage of round 2 dispatch
Figure 4.10: The third stage of round 2 dispatch

Figure 4.11: The first stage of round 3 dispatch
After $TI2(3)$ reaches 0, we notice that $TI2(4)$ has a capacity of 14 MW left. We then dispatch it as shown in Figure 4.12 using

\[
P_t(1) = \min(24, 14) = 14
\]
\[
w_{32} = \min(14 \times 0.75, 8) \Rightarrow P_t(2) = \frac{8}{0.75} = 10.7
\]
\[
w_{31} = \min(14 \times 0.25, 130) = 3.5 \Rightarrow P_t(3) = \frac{3.5}{0.25} = 14
\]
\[
w_{12} = \min(14 \times 0.25, 150) = 3.5 \Rightarrow P_t(4) = \frac{3.5}{0.25} = 14
\]
\[
F = \min(14, 10.7, 14, 14) = 10.7
\]
\[
w_{23} = 10.7 \times 0.75 = 8
\]
\[
w_{21} = 10.7 \times 0.25 = 2.7
\]
\[
w_{13} = 10.7 \times 0.25 = 2.7
\]

Figures 4.13 and 4.14 show the last two rounds of dispatch and Figure 4.15 shows the final marking of the model. There are three remaining load tokens not dispatched in $PI1$ and four remaining generator tokens not dispatched in $PI2$. All the dispatched tokens are in $PF12$. 
Figure 4.12: The second stage of round 3 dispatch
Figure 4.13: The first stage of round 4 dispatch
Figure 4.14: The first stage of round 5 dispatch
As the $M(PF8)=0$, there is congestion in the dispatch process, so the LBMP price is calculated as:

\[ \text{LBMP} = \min(\text{the bids of the last 1MW load being dispatched, the bids of the next 1 MW generation available}) \]

Zone 1
\[ \text{LBMP} = \min(60, 100) = 60 \]
Zone 2
\[ \text{LBMP} = \min(50, 60) = 50 \]
Zone 3
\[ \text{LBMP} = \min(70, 80) = 70 \]

Figure 4.15: The final marking of the system

4.4 Discussions

We can see that the tokens in the final marking contain rich information. Take $TI3(4)$ as an example.

\[
TI3(4) = \begin{bmatrix}
2 - 1# & 100MW & $40 \\
2 - 1# & 86MW & $90 \\
3 - 1# & 10.7MW & $70 \\
2 - 2# & 3.3MW & $50
\end{bmatrix}
\]
The first row contains the generator information: the generator ID is Zone 2 No. 1 and the bids is 100 MW at a price of $40. The remaining rows contain the load dispatch information. The generator serves load bids 2-1#, 3-1# and 2-2# in the amount of 86 MW, 10.7 MW, and 3.3 MW, respectively.

There are a number of advantages to having the information:

- The information carried by the tokens enables us to readily calculate the LMPs. As shown in Figure 4.15, the LMPs are $60 for Zone 1, $50 for Zone 2, and $70 for Zone 3.

- This information is also helpful if we want to share transmission loss among loads. In this case, Load 3-1# should pay the transmission line tariff and pay for the losses during the transmission.

- We can also gain insight about the bidding process. As each step of the dispatch is clearly addressed by the model, we can see which bids affect the LMPs the most. By picking out those critical bids, Load serving entities and generation suppliers can analyze how to be more competitive and avoid being undispatched.

Table 4.6 shows the marking evolution in each round and stage. Those critical markings causing the change of state are shaded. Instead of calculating load flow each time, we use distribution factors to obtain an approximate flow. The distribution factor method is based on a sensitivity analysis of the physical network of the transmission system. It avoids solving differential equations, simplifies the power flow calculations, and therefore makes it possible to yield a reasonable result in less time. However, the distribution factor may result in a small error each time. If there are a large number of small bids involved, the accumulated error may become an issue. In practice, as the calculation of the arc weighting is an independent step, it is possible to have a power flow program as an auxiliary program in the interface layer to do the flow calculation every few steps to correct the error of using distribution factors.
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<th>PI2</th>
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<th>PF3</th>
<th>PF4</th>
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Table 4.6: The marking evolution of the 3-Zone dispatch model

Figure 4.16: The final marking of the system - a MATLAB simulation result
CHAPTER 5
Generator Bidding Strategies

This chapter develops optimal generator bidding strategies in a competitive deregulated power market. Section 5.1 introduces the existing research on generator bidding strategies. Section 5.2 [45] develops bidding strategies for steam turbine units. Section 5.3 discusses bidding strategies developed for pump-hydro units. Section 5.4 summarizes the chapter.

5.1 Introduction

In a deregulated power market, the day-ahead unit commitment is evaluated on a price-merit order based on the bid prices submitted by the participating generators. For each hour, a market clearing price (MCP) is obtained, which is equal to the incremental cost of supplying the next unit of power. Thus, the dispatch of and, hence, the profit received by a generator depend on its bidding strategies.

Many generator bidding strategies have been proposed and analyzed [39]-[42]. The purpose of this section is to develop optimal bidding strategies for individual generators in a competitive power market. By a competitive power market, we refer to the case where there is an ample energy supply allowing the MCP to be insensitive to the bid price variation of a single supplier, which is referred to as perfect competition in [40]. This assumption is appropriate in many deregulated power pools for most of the time, except possibly for peak load days when the demand approaches the available energy supply.

We build the optimal bidding strategies starting from the familiar generator cost curve, which we assume to be continuous and differentiable. Based on the perfect competition assumption, our analysis does not require the solution of a multi-hour unit commitment program. From the cost curve, we develop basic bidding concepts of the break-even bid curve and the maximum profit bid curve. The maximum profit bid curve can be further extended to account for generator availability. In this context, an estimate of the ratio of the day-ahead and real time energy prices is required. Analysis is also performed for generator bid curves consisting of multiple segments, which are permitted in most deregulated power pools. As a result, the continuous maximum profit bid curve needs to be approximated by
blocks. We show that the optimal blocks can be obtained from the probabilistic distribution of energy prices based on the load forecast.

In an attempt to keep the derivation simple, we only address the case of a single hour dispatch. For multiple-hour dispatches, the generator ramp rates have to be taken into account. We also have not taken congestion pricing into account, although the analysis can be extended to such cases. In addition, we have not included secondary effects such as losses into consideration.

5.2 Bidding Strategies for Steam Turbine Generators

In a vertically integrated power system, generators are dispatched according to their cost curves. The cost curves vary according to the types of generators, such as fossil, hydro, nuclear, and gas turbine, and are generally well understood by engineers. A commonly used cost curve for a fossil steam unit is shown in Figure 5.1 [1]. The cost \( C \) of running a steam unit consists of a start-up cost \( C_S \), a “min-gen” cost \( C_0 \) associated with the cost of operating the unit at its minimum generation \( P_{\text{min}} \), and the variable cost of operating the generator beyond its minimum loading, which is usually represented as a quadratic or cubic function, up to its maximum generation \( P_{\text{max}} \). Thus, the cost of operating a generator at a power level \( P \) is given by

\[
C(P) = C_S + C_0 + \beta_1(P - P_{\text{min}}) + \beta_2(P - P_{\text{min}})^2
\]

or

\[
C(P) = C_S + C_0 + \beta_1(P - P_{\text{min}}) + \beta_2(P - P_{\text{min}})^2 + \beta_3(P - P_{\text{min}})^3
\]

The costs \( C_S \) and \( C_0 \), and the coefficients \( \beta_1, \beta_2, \) and \( \beta_3 \) are functions of the fuel cost. More specifically, \( \beta_1 \) represents the fuel cost for generating beyond \( P_{\text{min}} \), and the quadratic and cubic terms model the decrease in the efficiency of the generator and the increased need of maintenance at high power levels. In the sequel, we will use the quadratic cost curve (5.1) to illustrate the bidding strategies. The results can readily be extended for (5.2). In a regulated market, the cost curves (5.1) and (5.2) are used in unit commitment programs to determine the most economic dispatch. In a system with no congestion, the optimal solution
occurs when the incremental cost is given by

$$\lambda_i = \frac{dC(P_i)}{dP_i}$$

where $\lambda_i$, the incremental cost for the $i$th unit, is the same for all units under dispatch.

In a deregulated power market using the uniform price scheme, the revenue for a generator is given by

$$R(P) = R_{\text{min}} + B(P)(P - P_{\text{min}})$$

where $R_{\text{min}}$ is the revenue received for its startup and minimum generation cost, and $B(P)$ is the bid curve as a function of the generation level $P$. Therefore, the generator is profitable if

$$R(P) > C(P)$$

Although a generator can submit any bid curve $B(P)$, in a competitive market, its objective is to ensure that not only (5.5) holds, but also its profit is maximized. Such bidding strategies should start from the cost curve (5.1). We will first examine two extreme cases: one to only recover the cost and the other to maximize profit.
5.2.1 Break-even Bid Curve

In the case where the generator is interested to only recover its cost, we have

\[ R(P) = C(P) \]  

(5.6)

Using the quadratic cost curve (5.1) and the revenue equation (5.4), (5.6) becomes

\[ R_{\text{min}} + B(P)(P - P_{\text{min}}) = C_S + C_0 + \beta_1(P - P_{\text{min}}) + \beta_2(P - P_{\text{min}})^2 \]  

(5.7)

In the following derivations, we will always assume that \( R_{\text{min}} = C_S + C_0 \). Denoting \( P_c = P - P_{\text{min}} \), (5.7) simplifies to the break-even strategy

\[ B_{\text{BE}}(P) = \beta_1 + \beta_2 P_c \]  

(5.8)

which has a linear slope (Figure 5.2). For a cubic cost curve (5.2), this analysis will result in a bid curve with a quadratic slope. The strategy (5.8) represents a competitive but unprofitable bidding strategy, whose parameters are determined from the traditional generator cost function. Furthermore, bidding below \( B_{\text{BE}}(P) \) would result in losing money.

![Bid curves of a steam generator](image)

Figure 5.2: Bid curves of a steam generator

5.2.2 Maximum Profit Bid Curve

In practice, a generator has to not only recover cost, but also make a profit. To maximize profit given a fixed MCP, a generator needs to reach a generation level such that
its incremental cost of generation would be equal to the MCP. Analytically, this maximum profit (MP) bid curve can be expressed as

$$B_{MP}(P) = \frac{dC(P)}{dP} = \beta_1 + 2\beta_2 P_c$$

(5.9)

This curve is also shown in Figure 5.2. Note that the slope of this curve is twice that of the break-even bid curve.

The maximum profit $\pi(P)$ as a function of the generation level $P$ is

$$\pi_{MP}(P) = B_{MP}P_c - (\beta_1 P_c + \beta_2 P_c^2) = \beta_2 P_c^2$$

(5.10)

Thus, when all generators bid into the market based on their maximum profit curves, the unit commitment dispatch would be identical to the dispatch based on the generator cost curves. However, instead of the generators recovering their individual cost plus a fixed regulated profit, under the uniform MCP scheme, the maximum profit bid curves would potentially produce a larger return to the generators.

We note that the maximum profit bid strategy has been derived in [43]. However, our derivation here does not require the use of a unit commitment optimization formulation.

### 5.2.3 High and Low Bid Curves

Once the maximum profit curve has been established, any bid curve that deviates from it is non-optimal unless the generator has some degree of market power. Two cases are possible: the “bid high” and “bid low” scenarios as shown in Figure 5.3. For simplicity, linear bid curves are used.

Assume that the unit is one of the units on the margin and submits the maximum profit bid curve $B_{MP}(P)$, resulting in a market clearing price (MCP) of $B_{MPo}$ (Figure 5.3). A bid-high curve $B_H(P)$ results in less power allocated to the unit with a potential increase in the MCP. A bid-low curve $B_L(P)$ results in more power allocated to the unit with a potential decrease in the MCP. If the unit receives a profit of $\pi_{MP}$ at a MCP of $B_{MPo}$, we can obtain an equal-profit curve as shown in Figure 5.3, from which we can observe the following.

- The “bid-low” curve is intended to depress market prices. It drives some competitors from the market and reduces incentives for new generation investment. It is a poor
short-term strategy because it reduces profitability. However, the “bid-low” curve can be used advantageously by units that must stay dispatched due to operational constraints.

- The “bid-high” curve may be more profitable than the curve $B_{MP}(P)$, depending on whether the increase of the MCP, $\Delta B = B_H - B_{MPo}$, will more than compensate for the reduction in the dispatched energy.

Pursuing the bid-high strategy further, let the bid-high curve be

$$B_H(P_H) = \beta_1 + b_2 P_{Hc}$$  \hspace{1cm} (5.11)

with a slope of $b_2 > 2\beta_2$ and $P_{Hc} = P_H - P_{\text{min}}$. Thus, the profit at a dispatch of $P_H$ is

$$\pi_H(P_H) = B_H(P_H)P_{Hc} - (\beta_1 P_{Hc} + \beta_2 P_{Hc}^2) = (b_2 - \beta_2)P_{Hc}^2$$  \hspace{1cm} (5.12)

To receive the same profit (5.10) as the maximum profit bid, we set $\pi_H = \pi_{MP}$ to obtain

$$(b_2 - \beta_2)P_{Hc}^2 = \beta_2 P_c^2$$  \hspace{1cm} (5.13)

The solution to (5.13) is given by

$$P_{Hc} = \sqrt{\frac{1}{b_2/\beta_2 - 1}} P_c$$  \hspace{1cm} (5.14)
and (5.11) becomes

\[ B_{H}^* = \beta_1 + b_2 \sqrt{\frac{1}{b_2/\beta_2 - 1}} P_c \]  \hspace{1cm} (5.15)

Thus, the MCP margin required to achieve equal profit is

\[ \Delta B_{H}^* = B_{H}^* - B_{MP} = \left( b_2 \sqrt{\frac{1}{b_2/\beta_2 - 1}} - 2\beta_2 \right) P_c \]  \hspace{1cm} (5.16)

For a generator using the bid-high curve, its profit will exceed the \( B_{MP} \) strategy if the MCP is higher than \( B_{H}^* \), that is, a unit becoming more profitable when it exercises economic withholding. This analysis can serve as an alternative measure of market power. As an illustration of this analysis, consider a unit with \( \beta_2 = 0.025 \), \( P_{\text{min}} = 100 \, \text{MW} \), and \( P_{\text{max}} = 800 \, \text{MW} \). A plot of \( \Delta B_{H}^* \) for a range of \( b_2 \) and \( P \) is shown in Figure 5.4. For the specific values of \( b_2 = 0.072 \) and \( P = 800 \, \text{MW} \), the margin \( \Delta B_{H}^* \) needs to be greater than $2. In a competitive market, we would expect this value to be a fraction of $2, thus deeming this bid-high strategy to be non-optimal.

### 5.2.4 Bid Curves Accounting for Generator Availability and Derating

Although we have illustrated that the “bid-high” strategy is not profit maximizing, such a strategy may be optimal if some other factors are taken into account. Here we consider...
the impact of generator availability on the bidding strategy. Most deregulated markets offer a multiple settlement system on the energy supply. Supply bids are submitted to the day ahead (DA) market (DAM) and the bids are evaluated using a bid-based unit commitment procedure. If a unit fails to deliver the amount of energy that it has been committed in the DAM, replacement energy needs to be purchased by the unit in real time (RT). Mostly likely, the RT energy supply tends to be more expensive than the DA prices, especially when the supply is tight on peak load days. Thus, the bidding strategy must also take unit availability and derating into account.

**Insurance Bid Curves on an ON/OFF base**

Let $p_a(P)$ be the availability of a unit for RT operation. The unit will have two modes of operation: “on” and “off”, with probabilities of $p_a(P)$ (availability) and $p_u(P)$ (unavailability), respectively. There are many ways to model $p_a(P)$. Here we assume that it is a continuous and differentiable function of the power level $P$. Furthermore, $p_a(P)$ is close to unity at low $P$ and decreases as $P$ increases. Thus, $1 \geq p_a(P) > 0$, and its slope versus $P$ is negative, that is, $dp_a(P)/dP < 0$. Furthermore, the unavailability $p_u(P)$ is given by

$$p_u(P) = 1 - p_a(P)$$ (5.17)

Taking unit availability into account, the expected profit of a unit given the DA price $B_a$, the RT price $B_{RT}$, and the committed power level $P$ is

$$\pi_a(P) = p_a(P)[B_aP_c - (\beta_1P_c + \beta_2P_c^2)] + p_u(P)(B_aP_c - B_{RT}P_c)$$ (5.18)

where $P_c = P - P_{min}$. The first term in (5.18) represents the expected profit from power delivery and the second term is the RT energy settlement in case the unit is unable to deliver any power. Without loss of generality, we assume that the RT price is proportional to the DA price by a factor $k$

$$B_{RT} = kB_a$$ (5.19)

where $k \geq 1$.

To maximize the profit expressed in (5.18), we solve for

$$\frac{d(\pi(P))}{dP} = 0$$ (5.20)
Applying (5.17) and (5.19), the maximization (5.20) yields

\[
\frac{dp_a}{dP}[B_a P_c - (\beta_1 P_c + \beta_2 P_c^2)] + p_a [B_a - (\beta_1 P_c + 2 \beta_2 P_c)]
\]

\[
- \frac{dp_a}{dP} (B_a P_c - kB_a P_c) + (1 - p_a)(B_a - kB_a) = 0
\]  

(5.21)

Rearranging the terms in (5.21) and solving for \( B_a(P) \), we can express the optimal bidding strategy as

\[
B_a(P) = \frac{B_{MP}(P)(1 + \alpha) - \alpha \beta_2 P_c}{1 + k \alpha + (1/p_a - 1)(1 - k)}
\]  

(5.22)

where

\[
\alpha = \frac{dp_a}{dP} \frac{P_c}{p_a}
\]  

(5.23)

Using \( dp_a/dP < 0 \), \( k \geq 1 \), and \( 0 < p_a \leq 1 \), the terms in (5.22) satisfy the inequalities

\[
1/p_a - 1 \geq 0, \quad 1 - k \leq 0, \quad \alpha < 0
\]

\[
1 + k \alpha + (1/p_a - 1)(1 - k) \leq 1 + \alpha
\]  

(5.24)

Here we have made an assumption that \( 0 > (k \alpha + (1/p_a - 1)(1 - k)) > -1 \), that is, the denominator of (5.22) is always positive. Hence from (5.22), the bid curve taking into account generator availability satisfies

\[
B_a(P) > B_{MP}(P)
\]  

(5.25)

implying that \( B_a(P) \) is a bid-high strategy. The condition (5.25) holds even for \( k = 1 \). The interpretation of the \( B_a(P) \) curve is that because the availability of the generator decreases as its power increases, a hedging mechanism is to increase the bid prices so that a somewhat smaller amount of power is committed in the DAM. Because the generator is now more likely to deliver the scheduled energy, the expected profit would be higher. In real time, the excess energy of the unit can then be bid into the RT market.

We would like to point out that the bidding strategy (5.22) would be a function of how the unit availability is represented. The derivation provided here is based on the unit either supplying the scheduled power or not supplying power at all. Cases involving unit de-rating may lead to different strategies, which we will analyze next.

As an illustration, consider a unit with \( P_{\text{min}} = 100 \text{ MW} \) and \( P_{\text{max}} = 800 \text{ MW} \). Its cost
coefficients are $\beta_1 = 20$ and $\beta_2 = 0.025$. Assuming its unavailability is given by

$$p_u = 0.01 + 0.00000018(P - 100)^2$$  \hspace{1cm} (5.26)

as shown in Figure 5.5. The maximum profit bids $B_a(P)$ for $1 \leq k \leq 1.5$ are shown in Figure 5.6.

By assuming a higher value of $k$, the corresponding $B_a(P)$ curve will also be higher. In practice, $k$ is not a constant. $k$ is smaller at low MCPs when the supply is ample and
becomes larger at higher MCPs when the supply is tight. Thus, a more realistic $B_a(P)$ strategy assumes an increasing $k$ as the MCP increases, as shown by the dashed curve in Figure 5.6.

**Insurance Bid Curves Considering Derating**

Derating refers to the case of a unit failing to deliver power at the committed level. An example of derating is that a unit operates at 600 MW in real time while its committed power output at the Day Ahead Market (DAM) is 800 MW. This unit then needs to buy 200 MW in the Real Time (RT) market to compensate for the discrepancy. Most likely, the RT energy supply tends to be more expensive than the DA prices, especially when the supply is tight on peak load days. Thus, a bidding strategy taking the probability of unit derating into account will provide some kind of insurance to the generator bidder for possible revenue losses.

An extreme case of derating is the loss of a unit, which is the case we studied above. Following similar analysis, let $p_a(P)$ be the probability of the unit generating the amount of power $P$ in RT after it has been committed in DAM and $p_u(i)$ the probability of the unit under-generating an amount of $P_L(i)$ at an power output of $P$, where

$$p_a(P) + \sum_{i=1}^{n} p_u(i) = 1$$

and $n$ represents the number of derating cases.

Taking derating into account, the expected profit of a unit given the DAM price $B_d$, the RT price $B_{RT}$, and the committed power level $P$ is

$$\pi_d(P) = p_a(P)(B_dP_c - (\beta_1 P_c + \beta_2 P_c^2)) + \sum_{i=1}^{n} [p_u(i)(B_dP_c - B_{RT}P_L(i) - (\beta_1(P_c - P_L(i)) + \beta_2(P_c - P_L(i))^2))]$$

(5.28)

where $P_c = P - P_{min}$ and $P_L(i)$ is the derated power.

The first term in (5.28) represents the expected profit from a full power delivery and the second term accounts for all the derating cases. Assume that the RT market price $B_{RT}$ is proportional to the DAM price $B_d$ by a factor $k(i)$

$$k(i) = \frac{B_{RT}(i)}{B_d(i)}$$

(5.29)
and

\[ P_L(i) = k_L(i)P_c \]  

(5.30)

where \( k_L(i) \) is the percent of power undergenerated. Applying (5.29) and (5.30), (5.28) becomes

\[
\pi_d(P) = p_u(P)(B_dP_c - (\beta_1 P_c + \beta_2 P_c^2)) + \sum_{i=1}^{n} [p_u(i)(B_dP_c - k(i)B_dk_L(i))P_c - (\beta_1(P_c - k_L(i))P_c + \beta_2(P_c - k_L(i))P_c^2)]
\]

(5.31)

To maximize the profit expressed in (5.31), we solve for

\[
\frac{d\pi_d(P)}{dP} = 0
\]

(5.32)

To obtain the maximum profit bid curve

\[
B_d = \frac{\beta_1 + 2\beta_2 P_c - \sum_{i=1}^{n} [p_u(i)(\beta_1 k_L(i) - 2\beta_2 k_L^2(i)P_c + 4\beta_2 P_c k_L(i))]}{1 - \sum_{i=1}^{n} p_u(i)k(i)k_L(i)}
\]

(5.33)

Letting

\[
B_{ins} = \beta_1 k_L(i) - 2\beta_2 k_L^2(i)P_c + 4\beta_2 P_c k_L(i)
\]

(5.34)

\[
k_{ins}(i) = k_L(i)k(i)
\]

(5.35)

(5.33) can then be simplified as

\[
B_d = \frac{B_{MP} - p_u^T B_{ins}}{1 - p_u^T k_{ins}}
\]

(5.36)

As

\[
\frac{B_d}{B_{MP}} = \frac{1 - p_u^T B_{ins} B_{MP}}{1 - p_u^T k_{ins}}
\]

(5.37)

if \( B_{ins}/B_{MP} < k_{ins} \), the \( B_d \) curve will be above the \( B_{MP} \) curve.

As an illustration, consider a unit with \( P_{min} = 100MW \) and \( P_{max} = 900MW \), its cost coefficients are \( \beta_1 = 5 \) and \( \beta_2 = 0.05 \). The unavailabilities are represented by a vector \( p_u \) shown in Table 5.1, where the column represents the probability of the loss of a certain amount of power at the given output level \( P_c \). For example, \( p_u(3,2) \) means that when
$P_c = 300$MW, the probability of derating by 200 MW generation is 0.01. The $k_L$ and $k$ are shown in Table 5.2 and Table 5.3.

Figure 5.7a illustrates the effect of different $k$ factors and Figure 5.7b illustrates the effect of different $p_u$ matrices. The uncertainties in determining the two matrices will cause the $B_d$ curves to shift up or down as shown in the figures. The knowledge of $p_u$ and $k$ is therefore essential to determine the $B_d$ curve. A risk averse bidder may tend to use the worst case scenario which results in a high bids at high power outputs. If most of the market participants are risk averse [51], then applying insurance bids generally result in a higher MCP price in heavy load area than not taking derating into account. This, however, should not be considered as an exercise of market power but a necessary way to hedge the risk.

**Table 5.1: $p_u$ with respect to $P_c$**

<table>
<thead>
<tr>
<th>$p_u$</th>
<th>100 MW</th>
<th>200 MW</th>
<th>300 MW</th>
<th>400 MW</th>
<th>500 MW</th>
<th>600 MW</th>
<th>700 MW</th>
<th>800 MW</th>
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<td>100 MW</td>
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<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
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<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
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<td>0</td>
<td>0.005</td>
<td>0.001</td>
<td>0.001</td>
<td>0.005</td>
<td>0.02</td>
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<td>0.001</td>
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<td>0.01</td>
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<td>0.005</td>
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<td>600 MW</td>
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<td>0</td>
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<td>0</td>
<td>0.03</td>
<td>0.01</td>
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<tr>
<td>800 MW</td>
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<td>0.05</td>
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**Table 5.2: $k_L$ with respect to $P_c$**

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<tr>
<th>$k_L$</th>
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<td>1/6</td>
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<td>3/8</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<td>5/8</td>
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<td>1</td>
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Table 5.3: $k$ with respect to $P_c$

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<th>$k$</th>
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<td>1.3</td>
<td>1.5</td>
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<td>1.4</td>
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<td>1.6</td>
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<td>2</td>
<td>1.6</td>
<td>2</td>
<td>2</td>
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<tr>
<td>500 MW</td>
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<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
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<td>600 MW</td>
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<td>2</td>
<td>3</td>
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</tr>
<tr>
<td>700 MW</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3.5</td>
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<td>800 MW</td>
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<td>0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Figure 5.7: (a) $B_d(P)$ as a function of $k$, (b) $B_d(P)$ as a function of $p_u$

5.2.5 Optimization of Block Bids

The generator bid curves derived from the continuous cost curves will also be continuous functions of $P$. However, many unit commitment programs require bid curves to be in the form of either discrete points with straight-line interpolation or multi-segment blocks. Maximum profit bid curves that are affine can be handled without approximation by discrete points with linear interpolation. However, multi-segment block bids require additional analysis including an assessment of the forecasted prices to ensure optimality.

A block bid consists of blocks of energy at some fixed prices. For example, a 3-segment
bid has, in addition to the min-gen bid, the form

\[
B_B(P) = \begin{cases} 
  P_1 & \text{if } B_{\text{MCP}} \geq B_1, \\
  P_2 & \text{if } B_{\text{MCP}} \geq B_2, \\
  P_3 & \text{if } B_{\text{MCP}} \geq B_3,
\end{cases} \quad B_1 < B_2 < B_3 \tag{5.38}
\]

where \( P_i \) is the amount in MW to be supplied when the MCP is at or above the bid price \( B_i \) (Figure 5.8). Note that to be consistent

\[
P_1 + P_2 + P_3 = P_{\text{max}} - P_{\text{min}} \tag{5.39}
\]

Figure 5.8: A 3-segment bid curve of a steam generator

From the discussion on the maximum profit bid, it follows that a necessary condition for a block bid curve \( B_B(P) \) to be optimal is that

\[
B_B(P) > B_{\text{MP}}(P) \tag{5.40}
\]

that is, \( B_B(P) \) is a bid-high strategy. If any part of \( B_B(P) \) is below \( B_{\text{MP}}(P) \), that part of \( B_B(P) \) would obviously not be profit maximizing. This strategy is shown in Figure 5.8. Thus, it follows

\[
B_1 = \beta_1 + 2\beta_2P_1 \tag{5.41}
\]

\[
B_2 = \beta_1 + 2\beta_2(P_1 + P_2) \tag{5.42}
\]
\[ B_3 = \beta_1 + 2\beta_2(P_1 + P_2 + P_3) = \beta_1 + 2\beta_2(P_{\text{max}} - P_{\text{min}}) \quad (5.43) \]

Figure 5.8 shows that the optimal strategy can be specified using only \( B_1 \) and \( B_2 \). From (5.41) and (5.42), we obtain the energy blocks as

\[
P_1 = \frac{B_1 - \beta_1}{2\beta_2}, \quad P_1 + P_2 = \frac{B_2 - \beta_1}{2\beta_2} \quad (5.44)
\]

The optimization of block bids with only a few segments can be challenging when the maximum profit bid curve has a steep slope. It has to rely on an estimate of the MCP, which is a function of demand (load) bids and supply (generation) bids. Furthermore, the demand bids may be price-responsive, that is, they are also functions of the MCPs. In a deregulated market, a generator owner does not have access to the demand and supply bids submitted by the other market participants. However, some aggregate information is available. For example, the MCP versus the forecasted load is available immediately after unit commitment has been performed by the system dispatcher (known as the independent system operator in many power markets). In addition, the MCP versus actual load is available with a time lag of several months. Based on the MCP over a period of time, one can establish the statistics of the MCP for a specific forecasted load at a specific hour and construct the MCP probability

\[
p_{\text{MCP}}(B), \quad \text{for} \quad B_{\text{min}} \leq B \leq B_{\text{max}} \quad (5.45)
\]

where \( B \) is the MCP, and

\[
\int_{B_{\text{min}}}^{B_{\text{max}}} p_{\text{MCP}}(B) \, dB = 1 \quad (5.46)
\]

Such a probability may be a Gaussian or uniform distribution.

The expected profit of a bidding strategy \( B_B(P) \) is obtained by integrating over the probability of the dispatch and the revenue minus the cost of each of the blocks:

\[
\pi_B(B_1, B_2) = \int_{B_1}^{B_2} p_{\text{MCP}}(B)[B \frac{B_1 - \beta_1}{2\beta_2} - \beta_1 \frac{B_1 - \beta_1}{2\beta_2} - \beta_2(B_1 - \beta_1)^2] \, dB
\]

\[
+ \int_{B_2}^{B_3} p_{\text{MCP}}(B)[B \frac{B_2 - \beta_1}{2\beta_2} - \beta_1 \frac{B_2 - \beta_1}{2\beta_2} - \beta_2(B_2 - \beta_1)^2] \, dB
\]

\[
+ \int_{B_3}^{B_{\text{max}}} p_{\text{MCP}}(B)[B(P_{\text{max}} - P_{\text{min}}) - \beta_1(P_{\text{max}} - P_{\text{min}}) - \beta_2(P_{\text{max}} - P_{\text{min}})^2] \, dB \quad (5.47)
\]
To solve for the optimal values of $B_1$ and $B_2$, we establish the first-order necessary conditions

$$\frac{d\pi_B(B)}{dB_1} = 0, \quad \frac{d\pi_B(B)}{dB_2} = 0$$

which are, in general, nonlinear if the probability function $p_{MCP}(B)$ is nonlinear. The equation (5.48) may admit many solutions. The optimal solution must satisfy $B_2 \geq B_1$.

As an illustration, we develop 3-segment block bids for a unit with $P_{\text{min}} = 20$ MW, $P_{\text{max}} = 120$ MW, $\beta_1 = 5$, and $\beta_2 = 0.25$ (such that $B_3 = 55$/MWH), using two different probability distributions $p_{MCP}(B)$. In Case 1, a normal distribution shown in Figure 5.9a is used, with the probability peaking at $30$/MWH and the prices mostly between $20$ and $40$ per MWH. The expected profit for $10 \leq B_1 \leq B_2 \leq 50$ are plotted in Figures 5.9b and 5.9c. The optimal values are $B_1 = 21.0$/MWH and $B_2 = 29.0$/MWH, with an expected profit of $\pi_B = 614$. The strategy is quite illuminating, namely, the lower two segments are bid in at below the expected MCP. This can be attributed to the fact that the unit is most profitable when the MCP is high and its cost of generation is low. Submitting a bid with low $B_1$ and $B_2$ will ensure that such opportunities are not lost.

In Case 2, a uniform distribution for prices ranging from $20$ to $40$/MWH is assumed for $p_{MCP}(B)$ (Figure 5.10a). In this case, the optimization yields $B_1 = 20$/MWH and $B_2 = 30$/MWH, with an expected profit of $\pi_B = 624$. The expected profits are plotted in Figures 5.10b and 5.10c. It is interesting to note that the optimal bids for Cases 1 and 2 are quite similar, even though the shapes of the two distributions are quite different.

### 5.2.6 Conclusions

We have investigated optimal bidding strategies for a generating unit in a competitive deregulated power market, in which the market clearing price is insensitive to the bid price of a single generator. Starting from the unit’s cost curve, the maximum profit bidding strategy has been developed, from which other optimal bidding strategies can be obtained. When unit derating is taken into account, the expected profit is optimized to develop a strategy which is a function of the estimated ratio of the real-time price versus the day-ahead price. Multiple-segment block bids have also been investigated using the probabilistic distribution of the market clearing price. The optimization strategy indicates that blocks are bid to avoid lost opportunities.
Several simplifying assumptions have been made in this analysis. The minimum generation is bid in at cost. Its bidding can perhaps be optimized. The bidding analysis here is based on a single hour. Extensions to 24-hour periods need to be considered, which will require the generator ramp rates. Reserve prices are not included in the consideration. Potentially they may impact on optimal bidding strategies.

Figure 5.9: (a) Normal distribution, (b) 3D plot of the expected profit vs \( B_1 \) and \( B_2 \), and (c) Equal profit contour versus \( B_1 \) and \( B_2 \).
Figure 5.10: (a) Uniform distribution, (b) 3D plot of the expected profit vs $B_1$ and $B_2$, and (c) Equal profit contour versus $B_1$ and $B_2$
5.3 Bidding Strategies for Pump-hydro Units

Pump-hydro units are the energy storage devices in a power network. Traditionally, they are built to help shave daily peak power needs by recycling water between two reservoirs: the upper reservoir and the lower reservoir.

In a vertically integrated market, hydro-thermal coordination \cite{1} \cite{46} \cite{47} is used to reduce the fuel cost by letting the pump-hydro units serve the peak load (a higher fuel-cost load) with hydro-energy and then pumping the water back into the upper reservoir at light-load periods (a lower fuel-cost load). Under a cost-based dispatch, it is not unusual for a pump-hydro unit to be always in either the generating or the pumping mode, except for the changeover periods.

In a competitive power market, the profits of low-cost generators are maximized if they are dispatched when the market clearing prices (MCPs) are high. Because price peaks and valleys do not necessarily coincide with load peaks and valleys, hydro-thermal coordination may not apply. The income of a pump-hydro unit includes the revenue received by selling energy when it is in the generating mode and by being accepted in the reserve market when not in the generating mode. The cost of operating a pump-hydro unit includes the operation and maintenance (O&M) cost and the payment for the energy needed to pump water into the upper reservoir when in the pumping mode. If the combined efficiency of the pumping and generating is $\eta (0 < \eta < 1)$, then after consuming 1 MWH of energy pumping water into the upper reservoir, the unit can only generate $\eta$ MWH of energy afterwards. Therefore, to be profitable, the MCP $B_g$, above which the pump-hydro unit starts to generate power and sell energy to the market must be at least $1/\eta$ times higher than the MCP $B_p$, below which the unit buys energy and pumps water for storage. When the MCP is between the two price thresholds, $B_g$ and $B_p$, the pump-hydro will stay off-line to be dispatched as non-synchronous 10 min or 30 min reserve. The pump-hydro unit can also be committed for synchronous reserve when it is in the pumping mode because it can readily reduce its pumping power and consequently reduce the overall system load. Thus, in a deregulated market, there are strong incentives for pump-hydro units to carefully schedule their pumping and generation so that generating occurs at price peaks and pumping occurs at price valleys with the price gaps greater than $1/\eta$.

The price dynamic is determined mainly by the system loads and to a smaller extent, by
the generation and transmission availabilities. The dynamic of the demand exhibits cyclical
daily, weekly, and seasonal patterns, and so does the market clearing price. The capacity
of a pump-hydro unit puts an upper limit on the continuous pumping or generating time,
which can take a value between zero and tens of hours depending on the amount of the
present water storage in the upper reservoir. Because the MCPs are generally lower on the
weekdays, allowing the upper-reservoir to be fully recharged, an optimization region of one
week is more efficient than on either a daily or a monthly base. A horizon of one day is too
short to consider the optimal utilization of the water storage capability of the pump-hydro
unit, while a horizon of one month is too long for the forecast of the MCPs. Therefore,
the algorithm developed in this section optimizes the pump-hydro units on a weekly base.
We feel that the algorithm can be extended to optimize the bidding strategies for other
generating units with fixed blocks of energy (such as the cases discussed in Alvarado’s paper
[48]).

In [49], Ni and Luh used an integrated generator bidding and scheduling with MCPs
being treated as Markov random variables for risk management and hydrothermal scheduling,
where they treated multi-unit scheduling. Our approach is different from theirs.

In this section, a simple yet effective looping optimization algorithm is developed to
schedule a pump-hydro unit. A weekly composite MCP curve, obtained from an estimated
weekly MCP curve, is first used to optimize the pumping and generating schedule of a
pump-hydro unit. A counterpart to the composite fuel cost curve, the composite MCP curve
indicates that in a bid-based market, price is the driving factor, while in a regulated market,
cost function is the driving factor in scheduling. The operational constraints and the capacity
constraints of the unit are accounted for by using a looping algorithm. The optimization is
a discrete process because prices are quantized on an hourly basis. The opportunities in the
reserve market will also be investigated and its impact on developing the optimal bidding
strategies considered.

This section is organized as follows. Section 5.3.1 discusses the operational constraints
for pump-hydro units. Weekly composite MCP curves are proposed in Section 5.3.2. Section
5.3.3 formulates the optimization problem. Sections 5.3.4 and 5.3.5 give two examples to
illustrate the algorithm developed in Section 5.3.3. Section 5.3.7 compares the optimal
bidding strategy with a basic bidding strategy. Section 5.3.8 concludes the section.
5.3.1 Operational Constraints of a Pump-hydro Unit

A pump-hydro power plant is usually built in such a way that the upper reservoir has little or no inflows other than rainfalls. Therefore, the power that it is able to generate depends on the power stored by pumping water up to the upper reservoir. Consider a unit having an efficiency of $\eta$ with an initial energy stored in the upper reservoir of $E_0$ and a maximum stored energy $E_{\text{max}}$. Assume that the unit pumps at $P_p$ and generates at $P_g$ within a time period $[0, T]$, where the total pumping time is $T_p$ and the total generating time is $T_g$. If the stored energy at $T$ is $E_T$, then

$$E_T = E_0 + E_{\text{in}} - E_{\text{out}}$$

where the inflow energy is

$$E_{\text{in}} = P_p t_p \eta$$

and the outflow energy is

$$E_{\text{out}} = P_g t_g$$

Therefore, $t_g$ and $t_p$ can be related by

$$t_g = \frac{P_p t_p \eta - E_T + E_0}{P_g}$$

Most of the optimization strategies would require $E_0 = E_T = E_{\text{max}}$ during a cycle and so (5.52) reduces to

$$t_g = \frac{P_p \eta t_p}{P_g}$$

The maximum time of pumping ($t_{\text{pmax}}$) within a fixed cycle of period $T = T_g + T_p$ can then be calculated from (5.53) as

$$t_{\text{pmax}} = \frac{T}{1 + \frac{P_p \eta}{P_g}}$$

Beyond $t_{\text{pmax}}$, the energy replacement requirement $E_0 = E_T$ cannot be met. Therefore, it can be a stopping criterion during the optimization.
5.3.2 Weekly MCP Variations

A weekly MCP curve $W$ for a typical system in the Northeast US usually takes a shape as shown in Figure 5.11a, where the daily peaks are due to industrial and commercial activities. MCP peaks are, in general, higher on weekdays than on weekends. DAM MCP is on an hourly base. Peak hours usually occur from noon to 7 pm and valley hours are from midnight to 6:00 am. Peaks and valleys may vary in magnitudes or shift in durations with respect to weather conditions, seasonal factors, and holiday schedules.

Sorted by ascending prices, a weekly composite MCP curve ($W_a$) shown in Figure 5.11b can be obtained. When the MCP is greater than $B_g$, the pump-hydro unit generates power to the grids. When the MCP is less than $B_p$, the unit pumps water for storage. The monotonicity of the curve allows us to optimize the profit of the unit by increasing the pumping time of the pump-hydro unit from zero to $t_{pmax}$ and checking for the maximum profits. The optimal hours of pumping and of generating during a weekly cycle are then found. This greatly simplifies the optimization process compared with solutions using Lagrange multipliers.

5.3.3 Optimal Bidding Strategies for Pump-hydro Units Based on Nominal Price Forecast

In a regulated power system, pump-hydro generators are dispatched to shave the peaks and fill the valleys of the demand curves. In a deregulated power market, a pump-hydro unit
will still conduct the same function if it is profitable to do so.

**Problem Formulation**

The cost of running a pump-hydro unit includes a fixed O&M cost $C_0$ and the payment $C_p$ for the power needed to pump water into the upper reservoir, which can be used to generate power later. Assume that the unit pumps at a fixed power level $P_p$ with a pumping time $t_p$ at a market clearing price $B_p$, the cost for a given time period $T$ is then given by

$$C = C_0 + C_p \quad (5.55)$$

where

$$C_p = P_p t_p B_p \quad (5.56)$$

In a deregulated power market using the *uniform* price scheme, assuming that a pump-hydro unit generates at a fixed power level $P_g$ for a price of $B_g$ in the generating mode and selling reserves with a fixed power level $P_r$ at a price of $B_r$ in the pumping mode or the off-line mode, the revenue $R$ received by the pump-hydro unit in any given hour $T$, during which the generating time is $t_g$, is given by

$$R(P) = R_r + R_g \quad (5.57)$$

where the revenue $R_r$ received from the ancillary service market is

$$R_r = P_r (T - t_g) B_r \quad (5.58)$$

and the revenue $R_g$ received from the DA market is

$$R_g = P_g t_g B_g \quad (5.59)$$

The profit is represented by $\pi = R_g + R_r - C_0 - C_p$. Thus, the profit maximization problem can be described as

$$\max(\pi) = \max(R_g + R_r - C_0 - C_p) \quad (5.60)$$

Based on the weekly composite MCP curve as shown in Figure 5.11, starting from
$t_p = 0$, increasing the $t_p$ to $t_{p_{\text{max}}}$ with a step size $T_s$ of one hour, and assuming the same reservoir level ($E_0 = E_T$) at the beginning and at the end of a time cycle $T$ ($T = 7 \times 24 = 168$ hr in our case), the problem is formulated as

$$\max \left( \sum_{i=1}^{N_g} P_g T_s B_g(i) + P_r (T - t_g)B_r - C_0 - \sum_{j=1}^{N_p} P_p T_s B_p(j) \right)$$  \hspace{0.5cm} (5.61)$$

$N_p$ is the number of hours pumping and $N_g$ is the number of hours generating.

**Optimality conditions**

Using (5.53), $t_g$ can be obtained in each step. We then find the corresponding $B_p = B(t_p)$ and $B_g = B(T - t_g)$ from the forecasted MCP curve. If $\pi(t)$ is a continuous function, then when the unit reaches its maximum profit, the marginal profit $d\pi(t_p)/dt$ is zero. As the MCP forecast is on an hourly basis, the profit calculated is a discretized function with a step of 1 hr. Therefore, the optimality condition is $\min(|\Delta \pi(t_p)|)$. We can then derive the relation between $B_p$ and $B_g$, which will help us to evaluate the price gap between selling and buying energy for a pump-hydro unit to be profitable.

As

$$\max \left( \sum_{i=1}^{T-t_g} P_g T_s B_g(i) + P_r (T - t_g)B_r - C_0 - \sum_{j=1}^{t_p} P_p T_s B_p(j) \right)$$  \hspace{0.5cm} (5.62)$$

$$\Delta \pi(t_p) = \Delta R(t_p) - \Delta C(t_p) = 0$$  \hspace{0.5cm} (5.63)$$

$$-P_r B_r \Delta t_g + P_g B_g \Delta t_g = P_p B_p \Delta t_p$$  \hspace{0.5cm} (5.64)$$

where

$$\Delta t_g = \frac{P_p \eta}{P_g} \Delta t_p$$  \hspace{0.5cm} (5.65)$$

we have

$$\frac{B_g}{B_p} = \frac{1}{\eta} + \frac{P_r B_r}{P_g B_p}$$  \hspace{0.5cm} (5.66)$$

Here we assume that $C_0$ is equal in all operating modes. If we also ignore the revenue received from reserve bids, the ratio of $B_g$ at which the pump-hydro unit sells power versus $B_p$ at which the unit buys power will simply be equal to the inverse of the efficiency of the unit (usually in the range of 1.5). If the effect of reserve payments is also included, the resulting ratio $B_p/B_g$ will be bigger, which means that the price gaps between the generating and pumping will have to be bigger for the unit to maximize its profit. This analysis suggests
that the unit is more profitable in days when the peak and valley prices vary significantly, such as on a summer day, than in days when the prices fluctuate within a narrow range, such as on a winter day.

**Capacity constraints**

The energy stored in the upper reservoir of the pump-hydro power plant has an upper limit and a lower limit, that is

\[ E_{\text{min}} < E(t) < E_{\text{max}} \]  

where the minimum energy \( E_{\text{min}} \) is usually taken to be zero. These limits impose constraints on the energy stored and the energy generated. The inequality should hold for an optimal solution.

**Algorithm**

To account for the constraint (5.67), a multi-stage looping optimization is carried out. In the first stage, an unconstrained optimization algorithm is applied, where the capacity constraint (5.67) is not enforced. The optimization process is as follows:

**Step A.1:** Obtain a weekly composite MCP curve \( W_u \), starting from \( t_p = 1 \).

**Step A.2:** Obtain \( t_g \) using (5.52) and find the corresponding \( B_p \) and \( B_g \) from \( W_u \) curve.

**Step A.3:** Check the optimality condition. If it is not satisfied, let \( t_p = t_p + 1 \). If the optimality condition is satisfied, calculate the profit \( \pi \) with (5.60) and stop.

**Step A.4:** If \( t_p \) is less then \( t_{p\text{max}} \), go to step 2. If \( t_p \) is equal to \( t_{p\text{max}} \), calculate the profit \( \pi \) with (5.60) and stop.

The unconstrained optimization yields an optimal pumping time \( t_p \) and an optimal generating time \( t_g \). From the \( W_u \) curve, a set of \( B_p \) and \( B_g \) is obtained. Using \((B_g, P_g)\) as the generation bid and \((B_p, P_p)\) as the energy purchase bid, a weekly generating and pumping schedule for the pump-hydro unit can be readily obtained. The energy stored in the upper reservoir with respect to each hour in the week, which reflects the pond level during each time interval, will also be obtained. The next stage of the optimization will then use the upper reservoir capacity limits to adjust the optimization time intervals.

The process is described as follows:

**Step B.1:** Solve the problem with the unconstrained optimization in a time interval \([t_0, T] \).
Step B.2: Check the Solution. If there are no violations of the capacity constraint, done. If either the maximum or the minimum capacity limit is violated, go to step B.3.

Step B.3: Subdivide the time interval into \([t_0, t_1]\), where \(t_1\) is the hour that the unit reaches its highest or lowest energy storage level. Perform step B.1.

Step B.4: Check the solution. If there are more violations, go back to B.3. If not, calculate the profit \(\pi\) with (5.60) for \([t_0, t_1]\). Set \(t_0 = t_1\) and go back to Step B.1.

The next two sections will illustrate how the algorithm works.

5.3.4 An Example for Unconstrained Scheduling of Pump-hydro Units

As an illustration of the unconstrained scheduling, consider a pump-hydro unit with \(P_p = 130 \text{ MW}, P_g = 100 \text{ MW}, P_r = 175 \text{ MW}, B_r = 1\text{/MWH}, \text{ and } \eta = 0.667^2\). Let the weekly cycle start from 8:00 a.m. on Monday and end at 8:00 a.m. the following Monday. This is because weekday peaks are generally higher than weekends, therefore, the time durations of load valleys are longer during weekends. As a result, at 8:00 a.m. on Monday, we can safely assume that the reservoir capacity is at its maximum. We then develop a bidding strategy based on the weekly MCP forecast curve with the following assumptions:

- There are three modes for the unit: generating, off line, and pumping.
- For simplification purposes, assume that the O&M cost is fixed regardless of the dispatch. Thus, this cost will not be included in the optimization.
- Assume a perfect market. Thus, the bids of the pump-hydro unit will not affect the hourly MCPs.

Obtain a weekly MCP data (such as those posted on the NYISO (New York Independent System Operator) website) and sort it by an ascending price order to form a weekly composite MCP curve as shown in Figure 5.12. The optimal \(P_p\) and \(P_g\) obtained from the optimization algorithm are also shown in Figure 5.12.

Figure 5.13a shows the profit with respect to \(t_p\) and Figure 5.13b shows the marginal profit with respect to \(t_p\). The profit reaches maximum when \(t_p = 40\), after which the incremental profit becomes negative such that any further increase of the pumping hours will result in a decrease of the total profit. Assume that the initial energy stored in the
upper reservoir is $E_0 = 1500$ MWH and the energy at the end of the week is $E_T = 1500$ MWH, the energy storage is shown in Figure 5.13c. Note that the capacity constraint is satisfied. The generating schedule is shown in Figure 5.13d. Note that the pump-hydro unit generates more on weekdays and pumps more on weekends, which coincide with the higher magnitudes and longer durations of price peaks on weekdays than those on weekends.

If the upper reservoir of the pump-hydro unit has a limited capacity, it is possible that the upper reservoir will be out of water during the weekdays, the case of which we will discuss in the next section.
Figure 5.13: (a) Profit with respect to pumping time $t_p$, (b) Marginal profit with respect to pumping time $t_p$, (c) Energy storage, and (d) Generating/Pumping schedule
The results are shown in Table 5.4. Notice that the ratio of $T_p/T_g$ is fixed at 1.155 in all three cases, because once this relation is satisfied, the energy balance requirement is satisfied. The ratio of $B_p/B_g$ is 1.5 when not bidding into the reserve market (set reserve market price $B_r$ to be zero). When bidding into the reserve market ($B_r = 1$ and $B_r = 2$), the ratios are 1.59 and 1.66. As predicted before, bidding into the reserve market will lower the pumping price, increase the generating price, and therefore, increase the price gap. Intuitively, if a pump-hydro unit is paid for the reserve when it stays off-line or pumping, it will then decrease some generating and pumping hours previously on the margin, where the profits earned by the price differences are not as attractive as the money collected by selling reserves when staying off line.

Table 5.4: Results of the unconstrained case

<table>
<thead>
<tr>
<th>$B_r$ ($/MWH$)</th>
<th>$B_p$ ($/MWH$)</th>
<th>$B_g$ ($/MWH$)</th>
<th>$T_p$ (hr)</th>
<th>$T_g$ (hr)</th>
<th>$R_r$ ($)</th>
<th>$R_g$ ($)</th>
<th>$\pi$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>22.68</td>
<td>34.05</td>
<td>40</td>
<td>34.7</td>
<td>0</td>
<td>145638</td>
<td>40872</td>
</tr>
<tr>
<td>1</td>
<td>21.87</td>
<td>34.80</td>
<td>34</td>
<td>29.5</td>
<td>24238</td>
<td>127249</td>
<td>64564</td>
</tr>
<tr>
<td>2</td>
<td>21.55</td>
<td>35.84</td>
<td>32</td>
<td>27.7</td>
<td>49105</td>
<td>121572</td>
<td>89008</td>
</tr>
</tbody>
</table>

5.3.5 An Example for Constrained Scheduling of Pump-hydro Units

As an illustration of the constrained case, consider the pump-hydro unit in the previous example having energy limits of $E_{\text{min}} = 0$ MWH and $E_{\text{max}} = 1500$ MWH. Using a weekly curve shown in Figure 5.14a, an unconstrained optimization yields an energy dynamic curve as in Figure 5.14b. Because the energy lower limit is violated, the whole time period [1,168] (the number of hours in a week) is then separated into [1, 63] with $E_0 = 1500$ MWH and $E_T = 0$ MWH, and [64, 168] with $E_0 = 0$ MWH and $E_T = 1500$ MWH. Unconstrained optimizations are carried out in each time segment and the upper and lower energy limits are checked again. As the lower energy limit is still violated (Figure 5.14d), [64, 168] is separated into [64, 86] with $E_0 = 0$ MWH and $E_T = 0$ MWH and [87, 168] with $E_0 = 0$ MWH and $E_T = 1500$ MWH. Figure 5.15 shows the rest of the optimization processes. The results are shown in Table 5.5.
Figure 5.14: (a) $B_p$ and $B_g$ of iteration one, (b) Energy storage of iteration one, (c) $B_p$ and $B_g$ of iteration two, and (d) Energy storage of iteration two
There are several observations based on the results:

- The total profit will decrease with the separation of the time periods and the setting of interior conditions because the intermediate optimization solutions are infeasible.

- Price thresholds in the sub-intervals will be different (Figure 5.15c). If the lower energy limit is violated, $B_p$ will decrease and $B_g$ will increase due to the reduced generating and pumping hours.

- If the prices in the beginning part of the week are higher than the later part of the week, the dispatch would result in an empty reservoir in the middle of the week (Figure 5.14b). The optimization then has to be done on a daily basis (Figures 5.14d, 5.15b and 5.15d). During weekends, the pump-hydro will be pumping most of the time and the upper reservoir will be refilled by Monday morning. In this period, the profits go negative (Table 5.5).

### Table 5.5: Results of the constrained case

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Time Seg.</th>
<th>$B_p$ ($/\text{MWH}$)</th>
<th>$B_g$ ($/\text{MWH}$)</th>
<th>$T_p$ (hour)</th>
<th>$T_g$ (hour)</th>
<th>Profit ($$$)</th>
<th>Y/N</th>
<th>$\sum$ Profit ($$$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-168</td>
<td>20.49</td>
<td>31.09</td>
<td>40</td>
<td>34.7</td>
<td>20867</td>
<td>Y</td>
<td>20867</td>
</tr>
<tr>
<td>2</td>
<td>1-63</td>
<td>19.91</td>
<td>31.2</td>
<td>8</td>
<td>22</td>
<td>54900</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>64-168</td>
<td>20.17</td>
<td>30.25</td>
<td>29</td>
<td>10.1</td>
<td>-34288</td>
<td>Y</td>
<td>20612</td>
</tr>
<tr>
<td>3</td>
<td>1-63</td>
<td>19.91</td>
<td>31.2</td>
<td>8</td>
<td>22</td>
<td>54900</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>64-86</td>
<td>20.31</td>
<td>30.53</td>
<td>6</td>
<td>5.2</td>
<td>3785</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>87-168</td>
<td>20.09</td>
<td>30.25</td>
<td>23</td>
<td>4.9</td>
<td>-38114</td>
<td>Y</td>
<td>20571</td>
</tr>
<tr>
<td>4</td>
<td>1-63</td>
<td>19.91</td>
<td>31.2</td>
<td>8</td>
<td>22</td>
<td>54900</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>64-86</td>
<td>20.31</td>
<td>30.53</td>
<td>6</td>
<td>5.2</td>
<td>3785</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>87-110</td>
<td>18.33</td>
<td>31.8</td>
<td>4</td>
<td>3.5</td>
<td>2656</td>
<td>N</td>
<td></td>
</tr>
<tr>
<td></td>
<td>111-168</td>
<td>19.97</td>
<td>30.25</td>
<td>18</td>
<td>0.6</td>
<td>-40863</td>
<td>N</td>
<td>20478</td>
</tr>
</tbody>
</table>

* Y: The capacity constraint is violated.
N: The capacity constraint is not violated.
Figure 5.15: (a) $B_p$ and $B_g$ of iteration three, (b) Energy storage of iteration three, (c) $\bar{B}_p$ and $\bar{B}_g$ of iteration four, and (d) Energy storage of iteration four
5.3.6 Optimization Under Uncertainties

The above analysis is performed assuming that an expected value MCP weekly curve is available, so the solution is an expected value solution. If we know the MCP probabilistic distribution around the expected value curve, we can perform a Monte Carlo simulation and obtain a solution set for $P_g$ and $P_p$.

![Graphs showing MCP price vs. load forecast for light and heavy load hours.](image)

**Figure 5.16: (a) Light load hours, (b) Heavy load hours**

From the NYISO released data, the MCPs versus the load forecast for the light-load hours is shown in Figure 5.16a, and the data points for the heavy-load hours are shown in Figure 5.16b. The scattering of the data can be used to derive a probability distribution of the MCP. Assume that during the light-load hours, the MCPs follow a uniform distribution as shown in Figure 5.17a, and during the heavy-load hours, the MCPs follow an exponential distribution as shown in Figure 5.17b.

If the uncertainties are analyzed in the time domain, the forecasted MCP curves may resemble a scattered plot in Figure 5.18a, with the corresponding composite weekly MCP curves shown in Figure 5.18b. The weekly $P_g$ and $P_p$ can be calculated and the results are also shown in Figure 5.18b. The $P_g$ and $P_p$ values will fall into a range of values. From the results, we noticed that as the optimization is done based on a weekly MCP, the composite MCP weekly curve will “absorb” some price volatilities. Therefore the $P_p$ and $P_g$ do not vary significantly.
5.3.7 Comparison with a Basic Bidding Strategy

A pump-hydro unit may choose to bid a fixed weekly schedule on a weekly basis. Table 5.6 shows an example of the basic bidding strategy, where “B” is the beginning point of the cycle, “E” is the end of the cycle, “P” denotes pumping, and “G” denotes generating. The schedule captures the general price peaks, which occur in the afternoon, and the valleys, which occur in the early morning. Beginning with a full reservoir at 8:00 a.m. on Monday and following the schedule of Table 5.6, we will get a full reservoir in the next Monday.
morning without violating any capacity constraints during the week.

Using the data of Jan. 2002 and July-Aug. 2001 obtained from the NYISO website [50], for the pump-hydro unit we studied above, we calculated the profits of the two bidding strategies: the optimal strategy we developed in Section 5.3.3, and the basic strategy as listed in Table 5.6. The results are listed in Table 5.7.

As the optimal bidding strategy can capture the weekly price peaks and valleys more accurately, the profits received are much higher than the basic bidding strategy. Especially in winter weeks, the profits received using optimal bids are significantly higher than using the basic bids, because the price dynamics are less significant and properly capturing the peak price period is therefore essential for the unit to make more profits. As the basic bidding strategy does not respond to possible shifts of the price peaks and valleys, the pump-hydro unit is going to lose revenue once the shifts occur. For example, in the week of Jan. 28 - Feb. 4, where the price peaks and valleys were shifted from normal hours, the profit is $-1948.

However, as the bidding strategy is based on probability density functions (PDFs) of the real MCP values, the optimality will depend on how well the forecasted value is. Consider the MCP values are varying from the forecasted value by 10% (uniformly distributed) in the price range of \( MCP < B_p \) and 50% (uniformly distributed) when the price is above \( B_p \). Another comparison is carried out between the optimal bidding strategy and the basic
Table 5.7: Profits of the optimal bidding strategy and the basic bidding strategy

<table>
<thead>
<tr>
<th>Time range</th>
<th>Profit of Opt. ($)</th>
<th>Profit of Basic ($)</th>
<th>Difference ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 7 - Jan. 14</td>
<td>33324</td>
<td>18063</td>
<td>15261</td>
</tr>
<tr>
<td>Jan. 14 - Jan. 21</td>
<td>40872</td>
<td>29012</td>
<td>11860</td>
</tr>
<tr>
<td>Jan. 21 - Jan. 28</td>
<td>20529</td>
<td>12202</td>
<td>8327</td>
</tr>
<tr>
<td>Jan. 28 - Feb. 4</td>
<td>11989</td>
<td>-1948</td>
<td>13937</td>
</tr>
<tr>
<td>July. 16 - July. 23</td>
<td>40442</td>
<td>22514</td>
<td>17929</td>
</tr>
<tr>
<td>July. 23 - July. 30</td>
<td>76910</td>
<td>66305</td>
<td>10605</td>
</tr>
<tr>
<td>July. 30 - Aug. 6</td>
<td>195490</td>
<td>132810</td>
<td>62680</td>
</tr>
<tr>
<td>Aug. 6 - Aug. 13</td>
<td>84700</td>
<td>76750</td>
<td>7950</td>
</tr>
</tbody>
</table>

bidding strategy. One resulting distribution of the profits is shown in Figure 5.19. The upper curve is the profit received by using optimal bidding strategy and the lower curve obtained by using basic bidding strategy. The results are shown in Table 5.8.

From the results, we noticed that

- When we have an accurate MCP curve, the profits obtained by using the optimal bidding strategy are much higher than using the basic strategy. When the MCP curve has a ±10% variation in the low MCP range \((MCP < B_p)\) and a ±50% variation in the high MCP range \((MCP \geq B_p)\), in most cases, the optimal bidding strategy is still superior than the basic bidding strategy. However, the advantages of using the optimal bidding strategy with inaccurate data are less prominent than with an accurate prediction of MCP prices, because the hours that are supposed to have a high or low MCP may not be so.

- Bidding in a fixed schedule ignores the daily variations caused by weather, special events, etc., therefore, this results in a loss of opportunity when price peaks occur outside the conventional price peak periods and an overpayment when the prices rise in conventional valley price time periods.

- The optimal bidding strategy developed above can tolerate the inaccuracy of the MCP weekly curve to some extent because the optimization is carried out on a weekly basis. As an accumulated weekly MCP curve is used, the inaccuracy of the data is compensated. However, when capacity limits are violated during the optimization, the profit
Table 5.8: Profits obtained under incomplete information on MCP forecasts

<table>
<thead>
<tr>
<th>Time range</th>
<th>$E[\pi_{\text{opt}}]$ ($)</th>
<th>$\sigma[\pi_{\text{opt}}]$ ($)</th>
<th>$E[\pi_{\text{basic}}]$ ($)</th>
<th>$\sigma[\pi_{\text{basic}}]$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 7 - Jan. 14</td>
<td>14465</td>
<td>10326</td>
<td>11872</td>
<td>9546</td>
</tr>
<tr>
<td>Jan. 14 - Jan. 21</td>
<td>12213</td>
<td>13265</td>
<td>11966</td>
<td>12906</td>
</tr>
<tr>
<td>Jan. 21 - Jan. 28</td>
<td>19165</td>
<td>11006</td>
<td>12236</td>
<td>9938</td>
</tr>
<tr>
<td>Jan. 28 - Feb. 4</td>
<td>32627</td>
<td>9346</td>
<td>12703</td>
<td>9056</td>
</tr>
<tr>
<td>July. 16 - July. 23</td>
<td>49918</td>
<td>12339</td>
<td>22721</td>
<td>9933</td>
</tr>
<tr>
<td>July. 23 - July. 30</td>
<td>28038</td>
<td>12679</td>
<td>22334</td>
<td>10714</td>
</tr>
<tr>
<td>July. 30 - Aug. 6</td>
<td>22460</td>
<td>12544</td>
<td>22596</td>
<td>10487</td>
</tr>
<tr>
<td>Aug. 6 - Aug. 13</td>
<td>22747</td>
<td>12715</td>
<td>22591</td>
<td>11113</td>
</tr>
</tbody>
</table>

obtained is more sensitive to the accuracy of the forecasted price. For example, during the two weeks from July 30 to Aug. 13 (Table 5.8), the profit decreased greatly and might even be lower than the profit of using the basic bidding strategy.

Figure 5.19: Profits calculated based on probabilistic distributed MCPs

5.3.8 Conclusions

We have investigated optimal bidding strategies for a pump-hydro generating unit in a competitive deregulated power market, in which the market clearing price is insensitive to the bid price of a single generator. Starting from an estimated MCP weekly curve, an unconstrained optimal bidding strategy has been developed. When reservoir capacity limits
are taken into account, a multi-stage-looping optimization has been carried out to meet the constraints within each time segment.

Several simplifying assumptions have been made. The O&M cost is considered to be the same for all operation modes: generating, pumping, and off-line. The pump-hydro unit is assumed to bid into the market with a fixed output of $P_g$ and a fixed reserve of $P_r$ and the pumping is also at a fixed power level $P_p$. The MCP curve used is obtained from the NYISO website. If the pump-hydro unit bids $P_g$ with respect to the MCPs in such a way that it tries to generate more power in the forecasted peak price hours, the developed optimal bidding strategy may change. Nevertheless, we can predict that the method will make the resulting bidding schedule more sensitive to the accuracy of the forecasted MCPs, because the pump-hydro unit will generate as much as it can in the forecasted peak price hours. $P_r$ has little influence on the resulting bidding strategy due to the fact that $B_r$ is usually in the range of a dollar or two and therefore, the revenue received has only a marginal influence on the overall profit.

5.4 Summary

This chapter focused on developing bidding strategies for various types of generators. Basic generator bidding strategies developed from steam generator cost curves generalized the bidding strategies that a steam unit can apply. The break-even bid curve provides a lower bound of a generator’s bid curve, below which the generator will not be able to recover its cost. The maximum profit (MP) bid curve provides an optimal bid curve in a perfectly competitive market scenario. Bid-low curves in any cases are not as profitable as the MP curve, but it will be applied by a generator unit for the purpose of staying dispatched during light load hours to avoid the shut-down and start-up cost. Bid-high curves are used to game during peak load hours to achieve more profits with certain risks. If a unit wants to hedge the risk of derating, it will then try to bid in an insurance bid curve in the day-ahead market. Under an expectation of a higher RT market price, the insurance bid curves are usually higher than the MP curve. If most bidders are risk averse and choose to bid in insurance bid curves, then the energy price at higher power outputs will be significantly higher than not taking derating into account. Pump-hydro unit bidding strategies are developed based on an optimization considering revenues received in both the ancillary service market and the day-
ahead market. The optimization is done in a weekly cycle to account for the limited reservoir capacity. We expect to extend the algorithm for developing optimal bidding strategies for other types of generators with limited generation capacity.

The bidding strategies developed so far can then be used to provide generator bids and those bids can serve as an input to the ISO model we developed in Chapter 3 and 4. In the next chapter, we are going to apply the bidding strategies to build a GenCo module and study the price feedback impact on generator bidding strategies.
CHAPTER 6
A Price-feedback Market Simulator

This chapter develops a price-feedback market simulator to apply optimal bidding strategies developed in Chapter 5 to provide generator tokens as an input to the ISO model proposed in Chapters 3 and 4. Section 6.2 introduces the structure of the simulator. Section 6.3 describes the inputs and outputs of the model as well as the assumptions made. Section 6.4 discusses the simulation results regarding block bids.

6.1 Motivations

To study the market participants’ bidding strategies, it would be illuminating to conduct the analysis in a market environment where interactions can be taken into account. For example, when developing the generator bidding strategies in Chapter 5, we assume a perfect market where individual bids do not affect MCPs. However, in practice, the MCPs may vary with respect to the changes of the individual bids. If so, then it is of the bidder’s interest to study that, to what extent, this MCP deviation will affect his profit, how he reacts to the new MCP, and whether a “dynamic equilibrium” can be reached if he adjusts his bids accordingly with the new MCP. To address these interactions, a straightforward way is to use the price as feedback information when developing bidding strategies. If such a feedback loop is available where the MCPs obtained using the previous bids can be fed back, then a new set of bids in response to the new MCP can be generated. This bidding adjustment process is analogous to an open-loop control versus a close-loop control.

Implementing the generator bidding strategies developed in Chapter 5, we can build a GenCo module with generator cost curves as inputs and generator bids as outputs. By cascading the GenCo model with the ISO model developed in Chapters 3 and 4, where the inputs are generator bids, load bids, and transmission line capacities, and the outputs are the MCPs and the unit commitments, we can build a market simulator as shown in Figure 6.1.
6.2 Model Structure

The simulator proposed (Figure 6.1) does not take time evolution into consideration. In a single hour, for a given load, the ISO receives bids from the Load Serving Entities (LSE) and Generator Companies (GenCos) and performs a dispatch as illustrated in Chapter 4. It yields a MCP and allocates generation to each generator. Generators will then calculate their profits and adjust their bidding strategies based on the MCP and loading commitments. A new set of generator bids will be submitted to the ISO and will result in a new MCP which is then fed back to the generators again. An equilibrium will be reached after several iterations.

The inputs of the simulator include generator bid curves, the load bids, and the inter-zone tie-line capacity data. Generator 1 is the study generator whose bidding strategy is optimized based on the feedback MCPs. Generators 2 through 10 apply designated bidding strategies according to the different scenarios studied. Unless specified, the MCP is not fed back to these generators.

As the simulation focuses on the effectiveness of the bidding strategies which we developed assuming no congestion, congestion issues are not considered in the analysis. In the three-zone ISO module, the capacities of the transmission line are set sufficiently high such
that no congestion will be presented during the simulation.

Load bids are inelastic, i.e., the load bid price is set sufficiently high such that all load will be satisfied. Three kinds of load levels are considered: light load (lower than 40% \( \sum P_i \)), medium load (40% \( \sum P_i \) - 60% \( \sum P_i \)), and heavy load (over 60% \( \sum P_i \)), where \( \sum P_i \) represents the total capacity of the ten generators.

The Genco model contains a Discretization module, which is used to discretize the generator bid curves based on the bidding strategies developed in Chapter 5. Assume that the cost curves are quadratic cost functions which lead to incremental bid curves \( B(P) = \beta_1 + \beta_2(P - P_{\min}) \). Because Generator 1 has incomplete information regarding the rest of the generators in the network, the slopes \( \beta_2 \) of the rest of the generators are assumed uniformly distributed in \([\beta_2^0, \beta_2^0 + \Delta \beta_2]\) (Figure 6.2), where \( \Delta \beta_2 \) can take a value between 0 - 50% \( \beta_2^0 \) depending on different modeling purposes. The start-up and no-load costs \( C_0 \) of generators 2 - 10 are assumed known to Generator 1. Also, assume that \( C_0 \) is covered by a fixed payment \( R_{\min} \).

Table 6.1 lists the parameters of the bid curves of all ten generators. Figures 6.3 and 6.4 show the bid curves of all the ten generators with their upper limits and lower limits. Use 3-equal block bids as an initial condition for each generator. The aggregated bid curves are shown in Figure 6.5, from which we can see that because of the uncertainties in each individual bidder’s bid curve, the aggregated curves are distributed over a certain range.
Figure 6.3: (a) Generator $1 - 1\#$, (b) Generator $1 - 2\#$, (c) Generator $1 - 3\#$, (d) Generator $1 - 4\#$, and (e) Generator $1 - 5\#$
Figure 6.4: (a) Generator 2 – 1#, (b) Generator 2 – 2#, (c) Generator 2 – 3#, (d) Generator 3 – 1#, and (e) Generator 3 – 2#
Table 6.1: The parameters of the generator bid curves

<table>
<thead>
<tr>
<th>Unit</th>
<th>$P_{\text{min}}$ (MW)</th>
<th>$P_{\text{max}}$ (MW)</th>
<th>$\beta_1$</th>
<th>$B_1^*$</th>
<th>$B_2^*$</th>
<th>$B_{\text{max}}$</th>
<th>$\beta_2^0$</th>
<th>$\frac{\Delta \beta_2}{\beta_2^0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1#</td>
<td>0</td>
<td>1020</td>
<td>15</td>
<td>6.80</td>
<td>13.60</td>
<td>20.40</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>1-2#</td>
<td>200</td>
<td>980</td>
<td>5</td>
<td>12.80</td>
<td>20.60</td>
<td>28.40</td>
<td>0.015</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>1-3#</td>
<td>500</td>
<td>1010</td>
<td>10</td>
<td>13.40</td>
<td>16.80</td>
<td>20.20</td>
<td>0.01</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>1-4#</td>
<td>1000</td>
<td>1600</td>
<td>5</td>
<td>5.40</td>
<td>5.80</td>
<td>6.20</td>
<td>0.001</td>
<td>0</td>
</tr>
<tr>
<td>1-5#</td>
<td>0</td>
<td>390</td>
<td>0</td>
<td>32.50</td>
<td>65.00</td>
<td>97.50</td>
<td>0.125</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>2-1#</td>
<td>0</td>
<td>2520</td>
<td>5</td>
<td>16.76</td>
<td>28.52</td>
<td>40.28</td>
<td>0.007</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>2-2#</td>
<td>200</td>
<td>980</td>
<td>0</td>
<td>26.00</td>
<td>52.00</td>
<td>78.00</td>
<td>0.05</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>2-3#</td>
<td>0</td>
<td>510</td>
<td>0</td>
<td>17.00</td>
<td>34.00</td>
<td>51.00</td>
<td>0.05</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>3-1#</td>
<td>0</td>
<td>600</td>
<td>5</td>
<td>11.60</td>
<td>18.20</td>
<td>24.80</td>
<td>0.0165</td>
<td>0 - 50%</td>
</tr>
<tr>
<td>3-2#</td>
<td>0</td>
<td>390</td>
<td>0</td>
<td>14.30</td>
<td>28.60</td>
<td>42.90</td>
<td>0.055</td>
<td>0 - 50%</td>
</tr>
</tbody>
</table>

Figure 6.5: Aggregated supply curves
6.3 Modeling Issues

In this section, modeling issues, such as assumptions made, simplifications applied, special considerations on the structure design, degree of uncertainties considered, convergence results, as well as the bidding strategies being modeled, are addressed in detail.

1. Assumptions

- GenCos bid in blocks of energy.
  The generator bid curves derived from the continuous cost curves will also be continuous functions of $P$. However, many unit commitment programs require bid curves to be in the form of either discrete points with straight-line interpolation or multi-segment blocks. The ISO module we developed using MATLAB accepts generator inputs as blocks of energy, therefore a discretization of the continuous cost curve needs to be done.

- The dispatch is based on a price-merit order and the Locational-based Marginal Pricing (LMP) scheme is used to set market clearing prices (MCPs).
  A generator priority list can be formed according to the bidding prices. This priority list can be used to determine the firing order of the generator enabling transitions. The cheapest generations are dispatched first. The LMP is the incremental cost to supply the next 1 MW of load at a specific location in the grid.

- Ramp rates are not considered during the dispatch.
  As ramp rates are not considered, the simulation will only be informing for a single hour dispatch instead of for a multiple-hour dispatch. Adding ramp rates into the GenCo model will be a future direction of this research, which will make the simulator a more powerful tool to perform market analysis in the time domain and optimize the generator bidding strategies to capture the price dynamic responses in a 24-hr period.

- Congestion issues are not addressed.
  In this thesis, an emphasis has been placed on the simulation of the generator bidding strategies developed in Chapter 5 and thus, price feedback influences on generator bidding strategies are our major interests. We set the transmission line
capacities in the ISO module to be sufficiently high such that no congestion will occur during the dispatch. Nevertheless, the congestion issues are important and will have influences on a generator’s bidding behavior if the generator is located in a critical area where congestions can make significant differences on MCPs. Studying generator bidding strategies under congestion can be a potential use of this simulator.

2. Generator Supply Curves

The capacity of each generator is chosen in such a way that the aggregated supply curve follows the general shape of a typical power system supply curve, which has three parts:

- a low cost region

The low cost region of a typical power system supply curve is rather flat, because nuclear power plants and large fossil steam power plants can be very efficient when generating at their rated output and tend to have a very flat cost curve in the low cost region. These generators supply base load. Generator 1-4# belongs to these type of generators. Because the start-up cost and the shut-down cost are usually prohibitively high for the nuclear plants and large fossil steam units, they tend to bid very low prices, sometime even negative prices, to stay dispatched. Therefore, the prices normally are not volatile in the low cost region.

- a medium cost region

There are a large number of smaller fossil steam power plants and other types of generations competing in the medium price range. Their individual capacities are small and the slopes of their cost curves can vary significantly with the change of fuel prices and weather conditions. Therefore, the supply curve becomes steeper in the medium cost region and the price volatilities are increasing as well.

- a high cost region

Gas turbines usually will supply energy in the peak price period. Based on [54], the entry price for a gas turbine in the New England area is $75/MW and will be even more expensive in New York City. The number of such peak-hour units are small and so the price can be significantly higher, showing a scarcity of supply.
The market price can be very volatile in this high price region when generators game for profits because of the scarcity in generation.

In our model consisting of only ten generators, each submitting a 3-block bid, we choose to select one base generator (1-4#) and one big supplier (2-1#). As shown in the aggregated supply curve (Figure 6.5), it follows the shape of a common supply curve. Different markets will have different combinations of generators and properly setting the generator cost curves will make the simulation reflect more realistically the characteristics of the market.

3. Degree of Uncertainties

As pointed out in Section 6.2, uncertainties in forecasting other suppliers’ bids are accounted for by setting randomized slopes using

$$\beta_2(i) = \beta_2^0(i) + \alpha_1\beta_2^0$$

(6.1)

where $\alpha_1$ is the percent of variations and is in a range of $0 - 50\%$.

The forecasted demands are also probabilistic values and can be generated by

$$P_D = P_D^0 + \alpha_2P_D^0$$

(6.2)

where $\alpha_2$ is the percent of variations and is in a range of $\pm 2\%$.

4. Bidding Strategies

- Block bids

As we have studied in Chapter 5, a block bid consists of blocks of energy at some fixed prices. For simplification purposes, 3-segment bids are used during the simulation. A 3-segment bid has, in addition to the min-gen bid, the form

$$B_B(P) = \begin{cases} P_1 & \text{if } B_{\text{MCP}} \geq B_1, \\ P_2 & \text{if } B_{\text{MCP}} \geq B_2, \quad B_1 < B_2 < B_3 \\ P_3 & \text{if } B_{\text{MCP}} \geq B_3, \end{cases}$$

(6.3)
where $P_i$ is the amount in MW to be supplied when the MCP is at or above the bid price $B_i$ (Figure 6.6). Note that to be consistent

$$P_1 + P_2 + P_3 = P_{\text{max}} - P_{\text{min}}$$

(6.4)

Figure 6.6: A 3-segment bid curve of a steam generator

- **High and low bids**
  When applying the bid-low strategy, we refer to the case that a generator is bidding in a cost curve between its equal-profit curve and maximum profit curve, i.e., $\beta_2^L$ is within $[\beta_2, 2\beta_2]$. When applying the bid-high strategy, we refer to the case that a generator is bidding in a cost curve with $\beta_2^H$ greater than $2\beta_2$ (refer to Section 5.2.3).

- **Insurance bids**
  In practice, a generator will bid its last block at a higher price due to the fact that the availability of the unit usually decreases significantly when fully loaded (refer to Section 5.2.4). We consider this case as bidding with insurance.

5. **Number of trials and convergence**

In each experiment, 100 trials are carried out. Due to the probabilistic nature of the problem, an equilibrium is considered to be reached when the resulting MCP falls into a certain bid tolerance.
6. Results

The results are presented with mean values and standard deviations. The MCP, the profit of each unit \( \pi(i) \), and the power output of each unit \( P_i \) are reported.

6.4 Example 1: Block Bids

This section presents simulation results for the optimal bidding strategy of block bids (refer to Section 5.2.5). The bidding strategy is tested under light, medium, and heavy load conditions. Figure 6.7 shows the upper limit and the lower limit of the supply curves, which are obtained by setting \( \Delta \beta_2/\beta_2^0 \) of each generator in Table 4.2 to be zero and 0.5, respectively.

![Figure 6.7: Different loading conditions](image)

In the light-load region and the lower part of the medium-load region, the supply greatly exceeds the demand. For example, from mid-night to early morning, the dispatch is mainly base loaded without much fluctuation. The supply mostly comes from the must-run generators such as nuclear power plants and low-cost large steam power plants. The demand curve intersects the supply curve at the low-cost generation region as shown in Figure 6.7, where the shape of the supply curve is practically flat. Because in this region, there are abundant generations available, the price sensitivity is low and the price volatility is only a few dollars. Therefore, the market can be assumed to be perfectly competitive. The MCP
can be well represented by a normal or uniform distribution within a price range of a few dollars.

For generator bidders, their major concern is to have as much energy dispatched as possible while receiving maximized profits. The optimization of block bids introduced in Section 5.2.5 aims at maximizing the bidder’s profit based on the MCP probability distribution function (PDF). The introduced method for the separation of the bidder’s bid curve is most suitable under these circumstances, because the assumption is that any change brought by an individual bidder’s bid will not affect the PDF of the MCP and it holds well when the supply greatly exceeds the demand.

Table 6.2 implements the bidding strategy. As an illustration, consider a unit with $\beta_1 = 0$, $\beta_2 = 0.05$, $P_{\min} = 0$ MW, and $P_{\max} = 900$ MW. If the forecast of the MCP follows a normal distribution with an expected value at $E[B_{MCP}] = $20 and a standard deviation of $\sigma[B_{MCP}] = $1, then the generator will bid in the first block at no more than $19 to guarantee selection and the second block at $20 to have as much power dispatched as possible. That results in a first block bid as (190 MW, $19) and a second block bid as (20 MW, $20).

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$[P_1, B_1]$</th>
<th>$[P_2, B_2]$</th>
<th>$[P_3, B_3]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Condition</td>
<td>$[\frac{P_{\max} - P_{\min}}{3}, \frac{B_{\max} - B_{\min}}{3}]$</td>
<td>$[\frac{P_{\max} - P_{\min}}{3}, \frac{B_{\max} - B_{\min}}{3}]$</td>
<td>$[\frac{P_{\max} - P_{\min}}{3}, \frac{B_{\max} - B_{\min}}{3}]$</td>
</tr>
<tr>
<td>$E[B_{MCP}] &lt; B_{\min}$</td>
<td>$[P_1, B_{\min} + 1]$</td>
<td>$[P_2, B_{\min} + 3]$</td>
<td>$[P_3, B_{\max}]$</td>
</tr>
<tr>
<td>$B_{\max} \leq E[B_{MCP}] &lt; B_{\max}$</td>
<td>$[P_1, E[B_{MCP}] - \sigma[B_{MCP}]]$</td>
<td>$[P_2, E[B_{MCP}]]$</td>
<td>$[P_3, B_{\max}]$</td>
</tr>
<tr>
<td>$E[B_{MCP}] \geq B_{\max}$</td>
<td>$[P_1, B_{\max} - 3]$</td>
<td>$[P_2, B_{\max} - 1]$</td>
<td>$[P_3, B_{\max}]$</td>
</tr>
</tbody>
</table>

Table 6.2: The implementation of optimal block bids

Using the data of the generator cost curves given in Table 6.1, choosing the 3-equal block bids as an initial condition for each generator (except that the $\beta_1$ of Generator 1 is set
to 0), assuming $\Delta \beta_2 = 0.05\beta_2$, and letting the demand vary from 1000 MWH to 5000 MWH with a variation of ±2%, we then developed the bidding strategies (Table 6.3) for Generator 1 for light load and medium load cases based on the price feedback. The results are shown in Tables 6.4 and 6.5.

Peak load cases are simulated by setting $\beta_1$ to $15$ for Generator 1 and letting the demand vary from 6000 MWH to 7500 MWH with a variation of ±2%. The results are shown in Table 6.6.

There are several insights from the simulation results:

- Modifications

If we can precisely forecast the MCP, implying that the deviation of the MCP is small, we can guarantee the selection of the first two blocks (as shown in Table 6.4 when the load is around 1000 MW). When there are deviations in the MCP forecasted value, it is then possible that we may lose the dispatch of one or even both of the two block

Table 6.3: Bidding strategies for Generator 1 under different loads

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$E[P_{load}]$</th>
<th>$[P_1, B_1]$</th>
<th>$[P_2, B_2]$</th>
<th>$[P_3, B_3]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-equal</td>
<td>1000</td>
<td>218, 6.80</td>
<td>340, 13.60</td>
<td>340, 20.40</td>
</tr>
<tr>
<td>Opt</td>
<td>1000</td>
<td>48, 5.30</td>
<td>755, 20.40</td>
<td></td>
</tr>
<tr>
<td>Opt</td>
<td>2500</td>
<td>14, 13.07</td>
<td>366, 20.40</td>
<td></td>
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<td>Opt</td>
<td>3000</td>
<td>8, 13.57</td>
<td>342, 20.40</td>
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<tr>
<td>Opt</td>
<td>4000</td>
<td>15, 17.06</td>
<td>167, 20.40</td>
<td></td>
</tr>
<tr>
<td>Opt</td>
<td>4500</td>
<td>30, 17.36</td>
<td>152, 20.40</td>
<td></td>
</tr>
<tr>
<td>Opt</td>
<td>5000</td>
<td>44, 17.89</td>
<td>126, 20.40</td>
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</tr>
</tbody>
</table>

Table 6.4: Simulation results for 3-equal block bids

<table>
<thead>
<tr>
<th>$E[P_{load}]$</th>
<th>$E[B_{MCP}]$</th>
<th>$\sigma[B_{MCP}]$</th>
<th>$E[\pi]$</th>
<th>$\sigma[\pi]$</th>
<th>$E[P]$</th>
<th>$\sigma[P]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2500</td>
<td>13.21</td>
<td>0.23</td>
<td>3311</td>
<td>75</td>
<td>340</td>
<td>0</td>
</tr>
<tr>
<td>3000</td>
<td>13.57</td>
<td>0.10</td>
<td>3894</td>
<td>602</td>
<td>466</td>
<td>165</td>
</tr>
<tr>
<td>4000</td>
<td>17.23</td>
<td>0.30</td>
<td>7089</td>
<td>203</td>
<td>680</td>
<td>7</td>
</tr>
<tr>
<td>4500</td>
<td>17.34</td>
<td>0.37</td>
<td>7166</td>
<td>249</td>
<td>680</td>
<td>0</td>
</tr>
<tr>
<td>5000</td>
<td>18.91</td>
<td>0.37</td>
<td>8234</td>
<td>252</td>
<td>680</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.5: Simulation results for optimal block bids ($\beta_1 = 0$)

<table>
<thead>
<tr>
<th>$E[P_{load}]$</th>
<th>$E[B_{MCP}]$</th>
<th>$\sigma[B_{MCP}]$</th>
<th>$E[\pi]$</th>
<th>$\sigma[\pi]$</th>
<th>$E[P]$</th>
<th>$\sigma[P]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>5.4</td>
<td>0</td>
<td>729</td>
<td>0</td>
<td>265</td>
<td>0</td>
</tr>
<tr>
<td>2500</td>
<td>13.16</td>
<td>0.23</td>
<td>4330</td>
<td>149</td>
<td>647</td>
<td>7</td>
</tr>
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<td>3000</td>
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<td>0.10</td>
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<td>66</td>
<td>674</td>
<td>4</td>
</tr>
<tr>
<td>4000</td>
<td>17.04</td>
<td>0.17</td>
<td>7255</td>
<td>148</td>
<td>844</td>
<td>7</td>
</tr>
<tr>
<td>4500</td>
<td>17.35</td>
<td>0.34</td>
<td>7524</td>
<td>291</td>
<td>851</td>
<td>15</td>
</tr>
<tr>
<td>5000</td>
<td>17.93</td>
<td>0.43</td>
<td>8030</td>
<td>381</td>
<td>870</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 6.6: Simulation results under heavy load ($\beta_1 = 15$)

<table>
<thead>
<tr>
<th></th>
<th>$E[\pi]$ (3-equal)</th>
<th>$E[\pi]$ (Opt.)</th>
<th>$E[B_{MCP}]$ (3-equal)</th>
<th>$E[B_{MCP}]$ (Opt.)</th>
<th>$E[P_{load}]$ (3-equal)</th>
<th>$E[P_{load}]$ (Opt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6000</td>
<td>4150</td>
<td>3792</td>
<td>28.59</td>
<td>27.32</td>
<td>467</td>
<td>581</td>
</tr>
<tr>
<td>7000</td>
<td>5592</td>
<td>5580</td>
<td>30.03</td>
<td>29.95</td>
<td>678</td>
<td>712</td>
</tr>
<tr>
<td>7500</td>
<td>9301</td>
<td>6899</td>
<td>35.14</td>
<td>31.64</td>
<td>719</td>
<td>782</td>
</tr>
</tbody>
</table>

bids. So it is more risk-averse to lower the bid price of the first block to guarantee the selection. For example, when the load is 2500 MW (Table 6.5), the first block is at a price of $12.80 instead of $(13.16-0.23). Therefore, the bidding strategy is modified so that when $B_{min} \leq E[B_{MCP}] < B_{max}$, the block bid is $[P_1, E[B_{low}]], [P_2, E[B_{MCP}]],$ and $[P_3, B_{max}]$. $B_{low}$ is set at a price that guarantees 95% chance of selection.

- **Limitations**

Based on the data in Tables 6.4 and 6.5, we can plot Figure 6.8, which shows the resulting MCPs, the profit, and the power output curves using the 3-equal block bids and the optimal bids, respectively. As expected, the bidding strategy works well in the light load region and the lower part of the medium load region. The power outputs (Figure 6.8c) of Generator 1 increased significantly with a slight drop of the market clearing price. As a result, the profits (Figure 6.8b) which are related with both the price and the power output increased significantly. However, towards the peak load, where the price sensitivity is higher, i.e., when cheaper power is available, the price (Figure 6.8a) will drop significantly at the same time (as the case when the load is
5000 MWH). The profits gain by having more power dispatched may be offset by the drop of the MCP.

Therefore, the optimal bidding strategy developed for block bid based on the PDFs of the MCPs is most suitable for light load and medium load situations. Steam generators usually fall into this load range. Gas turbines usually are more expensive and they join the competition when load is peaking. For them, the optimal block bids may not be a good bidding strategy. For example, if the $\beta_1$ is set to be $15$ for Generator 1 and the load is 7500 MWH, the MCP will drop from $35.14$/MWH to $31.04$/MWH,
which results in a profit drop from $9301 to $6899 as shown in Table 6.6.

- Stop criteria

To show the convergence of the program, 20 feedback iterations are performed for each load level. The MCPs after price feedback in each iteration when the load increases from 1000 MWH to 5000 MWH are shown in Figure 6.9. When $P_{\text{load}} = 1000$ MWH (Figure 6.9a), the MCPs will remain at $5.5/\text{MWH}$, because the bids of the base generator, Generator 1-4#, is fixed and will always be the price setter. However, in Figures 6.9b-6.9f, the MCPs fluctuate within a small range. These fluctuations are due to the uncertainties in the forecasted bid curves of the other bidders and the previous feedback price will slightly affect the MCPs as well. Therefore, the equilibrium will not be at a certain price, but rather fall into a small range. We will then have to set a stop criterion such that when the expected MCP value falls into this certain range, the optimization ends.

6.5 Example 2: Bid-high Bids

The above simulation shows that when bidding its high cost portion of generation, a generator should take a strategy that will not depress the MCPs. Therefore, bid-high is a proper strategy to apply. Using 3-equal block bids and with $k$ representing the ratio of the bid price to the price using maximum bids, we compare the MCP, the profit, and the output of Generator 1 under a load of 7500 MWH and 8000 MWH, respectively. The results are shown in Figure 6.10. There are several observations:

- Market prices will rise with the bid-high strategy (Figures 6.10a and 6.10b). Due to the increasing price sensitivities in the power market, we can expect the price increase to be significant when in a heavy load period.

- Profits will increase even though the power outputs decrease significantly (Figures 6.10c, 6.10d, 6.10e, and 6.10f). The relation of the MCP and power output to achieve maximum profit was analyzed in Section 6.4.

- To apply the bid-high strategy successfully, it is essential that the profit obtained by the price increase should exceed the profit loss of the unscheduled power.
Figure 6.9: Fluctuations of the MCP

(a) $P_{\text{load}} = 1000$ MWH

(b) $P_{\text{load}} = 2500$ MWH

(c) $P_{\text{load}} = 3000$ MWH

(d) $P_{\text{load}} = 4000$ MWH

(e) $P_{\text{load}} = 4500$ MWH

(f) $P_{\text{load}} = 5000$ MWH
Figure 6.10: The strategy of bid-high
6.6 Example 3: Insurance Bids

Figure 6.11 shows that the aggregated supply curve when all the generators are considering derating. Thus, they bid their last block at a higher price as an insurance to possible revenue losses. The dash line is the original upper and lower bound of the supply curve and the solid line is the upper and lower bound of the supply curve after considering the derating effect. As discussed in Section 5.2.4, this will result in a high energy price in a heavy load period.

![Figure 6.11: The aggregated supply curve considering derating](image)

6.7 Summary

This chapter presented a price feedback market simulator, which consists of an ISO module and a GenCo module with fixed inputs from the load. The simulator utilizes the bidding strategies developed in Chapter 5 to provide generator bids to the ISO dispatch model developed in Chapter 3 and 4 to study the interaction of bidders and the price feedback influences. Bidding strategies were tested and their limitations discussed.
CHAPTER 7
Conclusions and Future Work

7.1 Conclusions

The objective of this research was to build a multi-layer Petri net model to model and simulate the deregulated electricity market. A price feedback market simulator was built based on the concepts.

The model structure is as follows. A new type of fluid Petri net model: Variable Arc Weighting Petri net (VAWPN) model, is used to model the physical layer, which consists of the power systems physical infrastructure: generation, distribution and transmission networks. On top of it, Colored Petri net models were proposed to model the market participants: the Load Serving Entities (LSEs), the Generator Companies (GenCos), and the market regulator: the Independent System Operator (ISO).

The ISO model follows the Locational-based Marginal Pricing scheme and dispatches the generator bids and the load bids based on a price merit order. It also takes the transmission line capacity into consideration and can be used to study the congestion effects on the market clearing prices (MCPs).

Bidding strategies were implemented in the GenCo model which provides bids to the ISO model. Starting from generator cost curves, basic bidding strategies were derived. Risk hedging issues were addressed by accounting for the generator availability and derating, based on which insurance bid curves were developed. Bidding strategies for generators with limited capacities, such as pump-hydro units were also analyzed in detail. A novel algorithm was developed to schedule the generators with limited capacity within a certain time range. The optimization across the day-ahead market and the ancillary service market were briefly addressed. Price feedback influences on the bidder’s bidding strategies were studied based on the simulation.

In a market scenario, the power flows are driven by the information flows. Each market participant is driven by an incentive of achieving maximized profit for themselves. Based on the information they know towards each other, they will choose only those bidding strategies that will achieve maximum profit with their generator units operated within constraints
instead of considering minimizing the total generation cost as well as the safe operation of the system as a whole. It is then essential to study the influence of the information to guide market participants to do the right things. Therefore, we consider it is a major contribution of this research to try to set up such a bridge connecting the market mechanism with the operation of the physical networks of this complex large scale power network so that the interaction between the information flow and the physical flows can be modeled, simulated and studied.

To summarize, the contributions of this research include:

- A new type of fluid Petri net model, a Variable Arc Weighting Petri net (VAWPN), has been proposed. By taking distribution factors into account, the VAWPN models follow the physical laws of power flows and avoid solving differential equations.

- The interaction of the information flow and the physical flow has been modeled by a multi-layer Petri net structure, where the model contains both discrete Petri nets and fluid Petri nets to address the distinct characteristics of the different flows. The model combines pricing information with physical power flows including congestion and distribution.

- Vector tokens have been proposed to communicate between different layers.

- Generator bidding strategies have been studied in detail for steam units and pump-hydro units with emphasises on the risk hedging and multi-market optimizations.

- A price-feedback market simulator has been developed to address the interactive behaviors of the bidders in the energy market.

7.2 Future Work Recommendations

In the deregulated electricity market, different parties have different needs for information and they place different values on information as well. From a generator bidder’s perspective, answers to the following problems may be of great interest:

- Which piece of information is essential for a generator to make the best move when there are several pieces of information available at a price?
An example of this information are the MCP forecast, the load forecast, and congestions.

- How accurate should the information be?
  
  For some information, a mean value and a variation will be good enough, while for others more detailed probabilistic distributions may be needed.

- In what time frame is the information valuable?
  
  For example, for generators with limited capacities, such as a pump-hydro unit, a weekly MCP forecast may be of interest, while for a steam turbine day-ahead MCP forecast would be enough.

From the operator or the regulator’s perspective, different questions may be asked:

- Knowing what kind of information will encourage bidders to submit sensible bids but not to game for profit?

- To what extent the bidders should be informed?

- In what time scale should the information be released to help the bidders schedule the bids in a long run that can bring the cost down?

- What will be the response of a new policy? For example, they would definitely want to know what the consequences are by putting price caps on bids? Or, what will be the consequences of selling Transmission Congestion Contracts?

In the present power market, the data one can obtain is from the released bid information provided by the ISO’s open access same time information system (OASIS). The data can be half a year old or older, from which much data mining work needs to be done to extract the information one wants to have. The information then can only be available at a significant cost. Real time data such as load forecast and market clearing prices are also provided but are incomplete. From this incomplete information, are there ways to reconstruct or at least estimate what is the unknown? These questions remain open.

If such a market simulator is available, so that when feeding in the historical data and the real time data, it can estimate the state of the market which may well include market clearing prices for DAM, RT market, and the ancillary service market, tie-line loading
condition, and the aggregated supply and demand curves, then we will not have to learn from our mistakes in the real market scenario when implementing a new policy or when making decisions on buying or selling practices. This will be the ultimate goal of our research efforts.

Regarding the multi-layer Petri net model we have developed, there are several immediate developments that can be made:

1. Investigating load side bidding strategies.
   Load side bidding strategies can be developed to minimize total energy payment by submitting price sensitive bids and bidding across the day-ahead market and the real-time market. Based on these bidding strategies, a load serving entity (LSE) module can be built. The market simulator then will be complete to model and simulate the behavior of all the market participants.

2. Making extensions to a 24-hour period.
   Our model has not taken ramp rate into consideration and therefore is unable to account for a 24-hour scheduling period. Adding this feature would make it possible to model the market response in the time domain and perform stochastic simulations of the market.

3. And performing sensitivity analysis between the information flows and the power flows.
   For example, it will be of great interest to study how the price can drive the power exchanges between zones.

As engineers, we usually consider our inputs to our controller these physical variables such as voltage, current, and machine angles. That is very effective under traditional vertical utility structure. Now, in a market environment, where price is changing every 5 minutes and where power flows are driven by money flows, it is then time to build price information into our control model, so the generator and the load can respond to the market well and so the whole system can respond to the market well.
LITERATURE CITED


