EXISTENCE AND NON-EXISTENCE FOR QUASILINEAR ELLIPTIC EQUATIONS WITH THE P-LAPLACIAN INVOLVING CRITICAL SOBOLEV EXPONENTS

By

M.C. Knaap

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M.C. Knaap*

1. Introduction

In this paper we consider the following quasi-linear elliptic problem:

\[
\begin{align*}
-\text{div}(|\nabla u|^{p-2}\nabla u) &= f(u), & u > 0, & \text{in } \Omega \\
u &= 0, & & \text{on } \partial\Omega,
\end{align*}
\]

where \( \Omega \) is a smooth, bounded domain in \( \mathbb{R}^N, \) \( 1 < p < N \) and \( f(u) \) is a given function with

\[ f(0) = 0, \quad f(s) > 0 \text{ if } s > 0. \]

The differential operator \(-\text{div}(|\nabla u|^{p-2}\nabla u)\) is commonly referred to as the p-Laplacian and denoted by \(-\Delta_p u\). If \( p = 2 \) it reduces to the ordinary Laplacian. Problem (1) has been studied by many authors (see for instance [E], [GP], [GV], [PS] and references given there). It turns out that several phenomena occurring for Problem (1) when \( p = 2 \) and \( N > 2 \), also manifest themselves for general values of \( p, 1 < p < N \). Thus we observe that if \( f(u) \) is given by a power of \( u \):

\[ f(u) = u^q, \]

then there exists a critical Sobolev exponent

\[ q^* = \frac{(p-1)N+p}{N-p} \]

and the following results hold:

(a) if \( 1 < q < q^* \), then there always exists a solution of Problem (1) for any bounded domain \( \Omega \).
(b) if \( q \geq q^* \), then there does not exist a solution of Problem (1) for any star-shaped domain \( \Omega \).

* Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands
One proves Result (a) by means of variational methods as in [AR], using the fact that the embedding

$$I : W^{1,p}_0(\Omega) \to L^{q+1}(\Omega)$$

(1.3)

is compact if $1 < q < q^*$. However, if $q \geq q^*$ this is no longer true.

Result (b) follows from a generalization of the Pohozaev Identity ([E], [GV], [PS]). It reads:

If $u$ is a solution of Problem (I), then it must satisfy

$$N \int_\Omega F(u) - \frac{N-P}{p} \int_\Omega f(u)u = \frac{p-1}{p} \int_{\partial\Omega} |\nabla u|^p(x,n),$$

(1.4)

where $F(u)$ is $\int_0^u f(s) \, ds$ and $n$ the outward pointing normal on $\partial\Omega$.

From [V], [T] it follows that $\frac{\partial u}{\partial n} < 0$ on $\partial\Omega$. The requirement that $\Omega$ is star-shaped is equivalent to $(x,n) > 0$. Hence the right-hand-side of (1.4) is positive. Upon substitution of $f(u) = u^q$ Result (b) is deduced.

It is well known that if one adds a lower order term to the critical power, one can have solutions of Problem (I) even on star-shaped domains. The first result in this direction was discovered by Brezis and Nirenberg.

Let $\Omega$ be a ball in $\mathbb{R}^N$ ($N > 2$) and

$$p = 2 \quad \text{and} \quad f(u) = u^{(N+2)/(N-2)} + \lambda u,$$

(1.5)

where $(N + 2)/(N - 2)$ is the critical Sobolev exponent for $p = 2$.

**Theorem 1.1.** [BN] If $p = 2$, then

(a) if $N = 3$, Problem (I) has a solution if and only if $\mu_1/4 < \lambda < \mu_1$,

(b) if $N \geq 4$, Problem (I) has a solution if and only if $0 < \lambda < \mu_1$,

where $\mu_1$ is the first eigenvalue of $(-\Delta)$ with the Dirichlet boundary condition.

The dimensions $2 < N < 4$ are called the critical dimensions of Problem (I). In this paper we shall discuss the phenomenon of critical dimensions for general values of $p$, $1 < p < N$ and throughout we shall assume

$$\Omega = B_R = \{ x \in \mathbb{R}^N : |x| < R \}.$$
The function \( f(u) \) given in (1.5) is generalized in the following way:

\[
f(u) = u^{q^*} + \lambda u^{p-1}
\]  

(1.6)

We choose the lower order term to be \( u^{p-1} \), because it is of the same order as the differential operator. Connected to it is the 'half-linear' eigenvalue problem:

\[
\begin{aligned}
(E1) \quad & -\Delta_p \psi = \mu \psi^{p-1}, \quad \psi > 0 \quad \text{in } B_R \\
& \psi = 0 \quad \text{on } \partial B_R.
\end{aligned}
\]

It is called 'half-linear' because the governing equation is invariant under multiplication by a constant.

We denote the first eigenvalue of Problem (E1) by

\[
\mu = \mu_1.
\]  

(1.7)

The following theorems were proved in [GV] and respectively in [E] and [GV]:

**Theorem 1.2.** If \( 1 < p < N \), then Problem (I) does not admit any solution if \( \lambda \geq \mu_1 \).

**Theorem 1.3.** If \( N \geq p^2 \), then there exists a solution of Problem (I) for every \( \lambda \in (0, \mu_1) \).

Conversely, in [APS] it is shown that:

**Theorem 1.4.** If \( p < N < p^2 \), then there exists a number \( \bar{\mu} > 0 \) such that if \( \lambda \in [0, \bar{\mu}) \) Problem (I) does not admit any radially symmetric solution.

Thus for general values of \( p \) the critical dimensions become:

\[ p < N < p^2. \]

In this paper we shall give a precise characterization of the number \( \bar{\mu} \), when \( \Omega \) is a ball and the solutions of Problem (I) are required to be radially symmetric. To do this we introduce a second, degenerate half-linear eigenvalue problem, defined on \( B_R \setminus \{0\} \):

\[
\begin{aligned}
(E2) \quad & -\Delta_p \varphi = \mu \varphi^{p-1}, \quad \varphi > 0 \quad \text{in } B_R \setminus \{0\} \\
& \varphi = 0 \quad \text{on } \partial B_R \\
& \varphi(x) - \frac{\nu^\nu}{|x|^\nu} \to 0 \quad \text{as } x \to 0,
\end{aligned}
\]  

(1.8, 1.9, 1.10)
where \( \nu = (N - p)/(p - 1) \). Problem (E2) has a radial solution exactly when \( p < N < p^2 \) as is proved in [KP]. We denote the eigenvalue associated to the first radially symmetric eigenfunction by:

\[
\mu = \mu^*. \tag{1.11}
\]

We shall prove

**Theorem A.** If \( p < N < p^2 \) and \( \Omega \) is the ball \( B_R \), then there exists a radially symmetric solution of Problem (I) if and only if

\[
\lambda \in (\mu^*, \mu_1). 
\]

**Remark 1.5.** If \( p = 2 \) the number \( \mu^* \) is identical to \( \mu_1/4 \), the constant occurring in Theorem 1.1.

**Remark 1.6.** The result of Theorem A was predicted by numerical computations carried out by Budd and Egnell [BuE].

## 2. Proof of Theorem A

In this section we shall establish existence and non-existence results for the following problem:

\[
\begin{aligned}
  -\text{div}(|\nabla u|^{p-2} \nabla u) &= u^{q^*} + \lambda u^{p-1}, & u > 0, & \text{in } B_R \\
  u &= 0 & & \text{on } \partial B_R,
\end{aligned} \tag{2.1}
\]

when

\[
p < N < p^2.
\]

Since we require the solutions to be radially symmetric, we can rewrite (2.2) into an ODE. To this ODE we apply the following transformation:

\[
t = \left(\frac{\nu}{|x|}\right)^\nu, \quad y(t) = u(|x|), \tag{2.3}
\]

with

\[
\nu = \frac{N - p}{p - 1}. \tag{2.4}
\]
This leads to the following problem

\[(II) \begin{cases} 
(y'|^{p-2}y')' + t^{-k}(y^{p-1} + \lambda y^{p-1}) = 0, 
& y(t) > 0 \text{ on } (T, \infty), \\
\lim_{t \to \infty} y'(t) = 0, 
& y(T) = 0,
\end{cases} \quad (2.5)
\]

\[\lim_{t \to \infty} y'(t) = 0, 
& y(T) = 0, \quad (2.6)\]

where

\[k = \frac{N - 1}{N - p}, \quad l = \frac{k - 1}{p - 1}, \quad (2.7)\]

\[q^* = pl - 1 \quad \text{and} \quad T = \left(\frac{\nu}{R}\right)^{\nu}. \quad (2.8)\]

Since \(y\) is concave on \((T, \infty)\), the boundary condition (2.6) implies that \(y' > 0\) on \([T, \infty)\), whence we can leave out the modulus signs in \(|y'|^{p-2}y'|\).

Remark that in this setting the conditions \(N > p\) and \(p < N < p^2\) become respectively

\[k > p \quad \text{and} \quad k > p + 1.\]

We shall prove the following theorem:

**Theorem 2.1.** Suppose \(k > p + 1\). Then Problem (II) admits a solution if and only if

\[\mu^* < \lambda < \mu_1,\]

where \(\mu_1\) and \(\mu^*\) are given by (1.7) and (1.8).

The half-linear eigenvalue Problem (E2) plays a major rôle in the proof of Theorem 2.1. Using the transformation (2.3) we rewrite equation (1.8) of Problem (E2) and the boundary conditions (1.9):

\[\beta(t) = \varphi(x), \quad \beta(T) = \varphi(R) = 0,\]

where \(T\) is given by (2.8). The 'initial' condition of \(\varphi\) at the origin is weakened to the following condition at infinity: \(\lim_{t \to \infty} \beta'(t) = 1\) and we arrive at:

\[(III) \begin{cases} 
((\beta')^{p-1})' + \lambda t^{-k}\beta^{p-1} = 0, 
& \beta(t) > 0 \text{ on } (T, \infty), \\
\beta(T) = 0, \quad \lim_{t \to \infty} \beta'(t) = 1.\end{cases} \quad (2.9)
\]

(2.10)

Problem (III) has a solution for every \(\lambda \in [0, \mu_1)\) precisely when \(k > p + 1\). The asymptotic expansion of \(\beta(t)\) for \(t\) large is given by

\[\beta(t) = t + a(\lambda) + O(t^{-k+p+1}). \quad (2.11)\]
The proof of these results can be found in [KP, Section 5]. The function \( a(\lambda) \) has the following properties:

**Lemma 2.2.** We have

(i) \( a(\lambda) = -T + O(\lambda) \) as \( \lambda \to 0 \)
(ii) if \( \lambda_1 > \lambda_2 \), then \( a(\lambda_1) > a(\lambda_2) \)
(iii) \( a(\mu^*) = 0 \),

Observe that if \( \lambda = \mu^* \), then the corresponding solution \( \beta(t) \) is equal to the first radial eigenfunction \( \varphi_1(|x|) \) of Problem (E2).

We shall show that the sign of \( a(\lambda) \) determines whether or not Problem (II) admits a solution. If \( 0 \leq \lambda \leq \mu^* \), then \( a(\lambda) \leq 0 \) and there does not exist any solution whilst for \( \mu^* < \lambda < \mu_1 \), \( a(\lambda) > 0 \) and a solution does exist.

To prove this we introduce the functional \( H(v) \) defined for functions \( v \) which satisfy the following problem:

\[
\begin{cases}
((v')^p - 1)' + t^{-k} g(v) = 0, & v(t) > 0 \text{ on } (T, \infty), \\
v(T) = 0.
\end{cases}
\]

It is given by

\[
H(v) = t(v')^p - v(v')^{p-1} + \frac{p}{p-1} t^{1-k} G(v),
\]

where \( G(v) = \int_0^v g(s) \) ds and its derivative with respect to \( t \) is

\[
\frac{d}{dt} H(v) = -t^{-k} \{plG(v) - g(v)v\}.
\]

We are going to compare \( H(v) \) when \( v = y \) and when \( v \) is a multiple of \( \beta \). The first step is to establish the negative result stated in Theorem 2.1. By Theorem 1.2 [GV], we only have to prove that there can not be a solution when \( \lambda \) lies in the interval \( [0, \mu^*] \).

**Lemma 2.3.** If \( k > p + 1 \) and \( \lambda \in [0, \mu^*] \), then Problem (II) does not have a solution.
PROOF. Arguing by contradiction, we suppose that we do have a solution \( y(t) \) for a certain \( \lambda \) in the interval \([0, \mu^*]\). Then we multiply \( \beta(t) \) by a constant \( \theta_1 > 0 \) such that
\[
\theta_1 \beta'(T) = y'(T)
\]
and set \( \tilde{\beta}(t) = \theta_1 \beta(t) \).

PROPOSITION 2.4. The functions \( y(t) \) and \( \tilde{\beta}(t) \) intersect an even number of times on \((T, \infty)\).

PROOF. Since \( \lim_{t \to \infty} y(t) < \infty \) while \( \tilde{\beta}(t) \sim \theta_1 t \) for \( t \) large, \( y(t) < \tilde{\beta}(t) \) for \( t \) sufficiently large. Hence we need to show that \( y(t) < \tilde{\beta}(t) \) in a right neighbourhood \((T, T + \delta)\) of \( T \). Suppose to the contrary that \( y(t) \geq \tilde{\beta}(t) \) in \((T, T + \delta)\). Then if one integrates (2.5) and (2.9) over \((T, t)\) one finds for every \( t \) in this interval:
\[
y'(t)^{p-1} = y'(T)^{p-1} - \int_T^t s^{-k}(y^{pl} + \lambda y^{p-1}) \, ds
\]
\[
< \tilde{\beta}'(T)^{p-1} - \int_T^t s^{-k} \lambda \tilde{\beta}^{p-1} \, ds = \tilde{\beta}'(T)^{p-1},
\]
which, since \( y(T) = \tilde{\beta}(T) = 0 \) yields a contradiction.

Now we introduce the function \( \tilde{\beta} \), which is another multiple of \( \beta \): If \( \tilde{\beta} \) and \( y \) intersect we denote the last two points larger than \( T \) at which they do by \( t_1 \) and \( t_2 \). Let \( \theta_2 \) be
\[
\theta_2 = \operatorname{inf}\{ \theta > 1 : \theta \tilde{\beta}(t) > y(t) \text{ on } [t_1, \infty) \}
\]
and define
\[
\tilde{\beta}(t) = \theta_2 \tilde{\beta}(t).
\]
Hence there exists a point \( \tau \in (t_1, t_2) \) such that
\[
\tilde{\beta}(t) > y(t), \quad \text{for all } t > \tau \tag{2.12}
\]
\[
\tilde{\beta}(\tau) = y(\tau), \quad \tilde{\beta}'(\tau) = y'(\tau). \tag{2.13}
\]
If \( y(t) \) and \( \tilde{\beta}(t) \) do not intersect for any \( t > T \), we define \( \tilde{\beta}(t) = \tilde{\beta}(t), \theta_2 = 1 \) and \( \tau = T \). Because \( \tilde{\beta}(T) = y(T) = 0 \) and \( \tilde{\beta}'(T) = y'(T) \), (2.12) and (2.13) remain true. Hence the point \( \tau \) lies in the interval \([T, \infty)\).
The function \( \tilde{\beta}(t) \) thus constructed, satisfies the equation (2.9), while its expansion for \( t \) large is given by

\[
\tilde{\beta}(t) = \theta_1 \theta_2 t + \theta_1 \theta_2 a(\lambda) + O(t^{p-k+1}).
\]

The functional \( H(v) \) becomes for \( y \) and \( \tilde{\beta} \) respectively.

\[
H(y)(t) = t(y')^p - y(y')^{p-1} + \frac{t^{1-k}}{k-1} (y^{pl} + \lambda y^p),
\]

\[
\lim_{t \to \infty} H(y)(t) = 0, \quad \frac{d}{dt} H(y)(t) = -\lambda(l-1)t^{-k}y^p.
\]

For \( v = \tilde{\beta} \), we find

\[
H(\tilde{\beta})(t) = t(\tilde{\beta}')^p - \beta(\tilde{\beta}')^{p-1} + \frac{t^{1-k}}{k-1} \lambda l \tilde{\beta}^p
\]

\[
\lim_{t \to \infty} H(\tilde{\beta})(t) = -(\theta_1 \theta_2)^p a(\lambda), \quad \frac{d}{dt} H(\tilde{\beta})(t) = -\lambda(l-1)t^{-k} \tilde{\beta}^p.
\]

This is the place where \( a(\lambda) \) manifests itself. Since the derivatives \( \frac{d}{dt} H(y)(t) \) and \( \frac{d}{dt} H(\tilde{\beta})(t) \) are of the same form one can easily compare them. After integrating them over \((\tau, \infty)\) we find

\[
-\tau y'(\tau)^p + y(\tau)y'(\tau)^{p-1} - \frac{\tau^{1-k}}{k-1} (y(\tau)^{pl} + \lambda y(\tau)^p)
\]

\[
= -\lambda(l-1) \int_{\tau}^{\infty} s^{-k} y^p ds
\]

\[
> -\lambda(l-1) \int_{\tau}^{\infty} s^{-k} \tilde{\beta}^p ds
\]

\[
= -(\theta_1 \theta_2)^p a(\lambda) - \tau \tilde{\beta}'(\tau)^p + \tilde{\beta}(\tau)\tilde{\beta}'(\tau)^{p-1} - \lambda l \frac{\tau^{1-k}}{k-1} \tilde{\beta}(\tau)^p,
\]

where we used (2.12). Finally substituting (2.13) we arrive at the inequality

\[
\frac{\tau^{1-k}}{k-1} y(\tau)^{pl} < \theta_1 \theta_2 a(\lambda).
\]

However, since \( y(\tau) \geq 0 \) and \( a(\lambda) \leq 0 \) on \([0, \mu^*]\) this yields a contradiction and Lemma 2.3 is proved.
Lemma 2.5 If \( \lambda \in (\mu^*, \mu_1) \) and \( k > p + 1 \), then Problem (II) admits a solution.

Proof. The proof of this Lemma is essentially contained in [KP]. For completeness we sketch it here using the results deduced in [KP].

Consider the subcritical problem

\[
(\Pi_\varepsilon) \begin{cases} 
((y')^{p-1})' + t^{-k}(y^{pl-1-\varepsilon} + \lambda y^{p-1}) = 0, & y(t) > 0 \text{ on } (T, \infty), \\
y(T) = 0, & \lim_{t \to \infty} y'(t) = 0,
\end{cases}
\]

Provided that \( \varepsilon < p(l - 1) \), Problem \( (\Pi_\varepsilon) \) admits a solution \( y_\varepsilon(t) \) as long as \( 0 < \lambda < \mu_1 \). We shall show that if \( \lambda \in (\mu^*, \mu_1) \), then \( y_\varepsilon(t) \) converges to a solution of Problem (II) as \( \varepsilon \searrow 0 \).

Suppose to the contrary that Problem (II) does not admit a solution. In [KP; Section 5] it is proved that then necessarily \( \gamma_\varepsilon = \lim_{t \to -\infty} y_\varepsilon(t) \) blows up:

\[
\lim_{\varepsilon \searrow 0} \gamma_\varepsilon = \infty.
\]

We use this number \( \gamma_\varepsilon \) to rescale \( y_\varepsilon(t) \); Set

\[
\eta_\varepsilon(t) = \gamma_\varepsilon^\omega y_\varepsilon(t),
\]

where \( \omega = 1 - \theta/(p - 1) \) and \( \theta = \varepsilon/(l - 1) \).

The following lemma is small adaptation of Lemma 6.1 in [KP].

Lemma 2.6. If

\[
\lim_{\varepsilon \searrow 0} \gamma_\varepsilon = \infty.
\]

Then we have

\[
\gamma_\varepsilon^{-\theta} \int_T^\infty y_\varepsilon(t)^{pl-\varepsilon} \, dt \to K
\]

and

\[
\eta_\varepsilon(t) \to k_1^{1/(p-1)} \beta(t) \quad \text{as } \varepsilon \searrow 0,
\]

uniformly on compact sets in \([T, \infty)\), where

\[
K = k_1 \frac{k - 1}{k - p} \frac{\Gamma \left( \frac{l}{k - p} \right)}{\Gamma \left( \frac{pl}{k - p} \right)}
\]

and
$$k_1 = (k - 1)^{1/(l-1)}.$$

Thus, we see that if Problem (II) has no solution, then the rescaled function $\eta_\varepsilon$ converges to a solution of the degenerate, half-linear eigenvalue Problem (III). However, we shall see that if we compare the functionals $H(\eta_\varepsilon)$ and $H(\beta)$, this leads to a contradiction when $\lambda \in (\mu^*, \mu_1)$. It turns out that $a(\lambda)$ then has the wrong sign. The functional $H(\eta_\varepsilon)$ is given by

$$H(\eta_\varepsilon)(t) = \gamma_\varepsilon^{p\omega} H(y_\varepsilon)(t)$$
$$= t(\eta_\varepsilon')^p - \eta_\varepsilon(\eta_\varepsilon')^{p-1} + \frac{t_{l-k}^{l-k}}{k-1} (\gamma_\varepsilon^{p\omega} y_\varepsilon^{pl-\varepsilon} + \lambda l \eta_\varepsilon^p),$$

and

$$\frac{d}{dt} H(\eta_\varepsilon)(t) = -t^{-k} \left( \frac{\varepsilon \gamma_\varepsilon^{p\omega}}{pl-\varepsilon} y_\varepsilon^{pl-\varepsilon} + \lambda (l-1) \eta_\varepsilon^p \right)$$
$$\lim_{t \to \infty} H(\eta_\varepsilon)(t) = 0.$$  (2.14)

Integrating (2.14) over $(T, \infty)$ yields

$$T \eta_\varepsilon'(T)^p = \frac{\varepsilon \gamma_\varepsilon^{p\omega}}{pl-\varepsilon} \int_T^\infty s^{-k} y_\varepsilon^{pl-\varepsilon} ds + \lambda (l-1) \int_T^\infty s^{-k} \eta_\varepsilon^p ds.$$  (2.15)

Using (2.11), we evaluate $H(\beta)(t)$ at infinity and find

$$\lim_{t \to \infty} H(\beta)(t) = -a(\lambda).$$

Hence in the same way as above we find for $\beta(t)$

$$T \beta'(T)^p = -a(\lambda) + \lambda (l-1) \int_T^\infty s^{-k} \beta^p ds.$$  (2.16)

Taking the limit as $\varepsilon \searrow 0$ in (2.15) and then substituting (2.16), Lemma 2.6 induces that

$$\frac{\varepsilon}{pl-\varepsilon} \gamma_\varepsilon^{(p-\theta)/(p-1)} \to -k_1^{p/(p-1)} K^{-1} a(\lambda).$$

But this is impossible since $a(\lambda) > 0$ on $(\mu^*, \mu_1)$. Hence $\gamma_\varepsilon$ must remain bounded as $\varepsilon \searrow 0$. Therefore we conclude that our supposition that Problem (II) did not admit any solution was false and that a solution of Problem (II) does exist. Moreover this solution can be obtained as the limit of the
solutions \( y_\varepsilon(t) \) of the subcritical problem (II\(_\varepsilon\)) as \( \varepsilon \downarrow 0 \). Since \( y_\varepsilon(t) < \gamma_\varepsilon \) on \( [T, \infty) \), it follows that \( y_\varepsilon(t) \) is bounded uniformly on \( [T, \infty) \) uniformly in \( \varepsilon \), for \( \varepsilon \) sufficiently small. Thus we infer after applying a compactness argument that

\[
y_\varepsilon(t) \to y(t) \quad \text{as} \quad \varepsilon \downarrow 0 \quad \text{uniformly on} \quad [T, \infty).
\]

where \( y(t) \) is a solution of Problem (II).

References


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<table>
<thead>
<tr>
<th>#</th>
<th>Author/s</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>721</td>
<td>Ian M. Anderson, Niky Kamran and Peter J. Olver</td>
<td>Internal, external and generalized symmetries</td>
</tr>
<tr>
<td>722</td>
<td>C. Foias and J.C. Saut</td>
<td>Asymptotic integration of Navier–Stokes equations with potential forces I</td>
</tr>
<tr>
<td>723</td>
<td>Ling Ma</td>
<td>The convergence of semidiscrete methods for a system of reaction-diffusion equations</td>
</tr>
<tr>
<td>724</td>
<td>Adelina Georgescu</td>
<td>Models of asymptotic approximation</td>
</tr>
<tr>
<td>725</td>
<td>A. Makagon and H. Salehi</td>
<td>On bounded and harmonizable solutions on infinite order arma systems</td>
</tr>
<tr>
<td>726</td>
<td>San-Yeh Lin and Yan-Shin Chin</td>
<td>An upwind finite-volume scheme with a triangular mesh for conservation laws</td>
</tr>
<tr>
<td>727</td>
<td>J.M. Ball, P.J. Holmes, R.D. James, R.L. Pego &amp; P.J. Swart</td>
<td>On the dynamics of fine structure</td>
</tr>
<tr>
<td>728</td>
<td>KangPing Chen and Daniel D. Joseph</td>
<td>Lubrication theory and long waves</td>
</tr>
<tr>
<td>729</td>
<td>J.L. Ericksen</td>
<td>Local bifurcation theory for thermoelastic Bravais lattices</td>
</tr>
<tr>
<td>730</td>
<td>Mario Taboada and Yuncheng You</td>
<td>Stability results for perturbed semilinear parabolic equations</td>
</tr>
<tr>
<td>731</td>
<td>A.J. Lawrence</td>
<td>Local and deletion influence</td>
</tr>
<tr>
<td>732</td>
<td>Bogdan Vernescu</td>
<td>Convergence results for the homogenization of flow in fractured porous media</td>
</tr>
<tr>
<td>733</td>
<td>Xinfu Chen and Avner Friedman</td>
<td>Mathematical modeling of semiconductor lasers</td>
</tr>
<tr>
<td>734</td>
<td>Yongzhi Xu</td>
<td>Scattering of acoustic wave by obstacle in stratified medium</td>
</tr>
<tr>
<td>735</td>
<td>Songmu Zheng</td>
<td>Global existence for a thermodynamically consistent model of phase field type</td>
</tr>
<tr>
<td>736</td>
<td>Heinrich Freistühler and E. Bruce Pitman</td>
<td>A numerical study of a rotationally degenerate hyperbolic system part I: the Riemann problem</td>
</tr>
<tr>
<td>737</td>
<td>Epifanio G. Virga</td>
<td>New variational problems in the statics of liquid crystals</td>
</tr>
<tr>
<td>738</td>
<td>Yoshikazu Giga and Shun’ichi Goto</td>
<td>Geometric evolution of phase-boundaries</td>
</tr>
<tr>
<td>739</td>
<td>Ling Ma</td>
<td>Large time study of finite element methods for 2D Navier–Stokes equations</td>
</tr>
<tr>
<td>740</td>
<td>Mitchell Luskin and Ling Ma</td>
<td>Analysis of the finite element approximation of microstructure in micromagnetics</td>
</tr>
<tr>
<td>741</td>
<td>M. Chipot</td>
<td>Numerical analysis of oscillations in nonconvex problems</td>
</tr>
<tr>
<td>742</td>
<td>J. Carrillo and M. Chipot</td>
<td>The dam problem with leaky boundary conditions</td>
</tr>
<tr>
<td>743</td>
<td>Eduardo Harabetian and Robert Pego</td>
<td>Efficient hybrid shock capturing schemes</td>
</tr>
<tr>
<td>744</td>
<td>B.L.J. Braaksma</td>
<td>Multisummability and Stokes multipliers of linear meromorphic differential equations</td>
</tr>
<tr>
<td>745</td>
<td>Tae Il Jeon and Tze-Chien Sun</td>
<td>A central limit theorem for non-linear vector functionals of vector Gaussian processes</td>
</tr>
<tr>
<td>746</td>
<td>Chris Grant</td>
<td>Solutions to evolution equations with near-equilibrium initial values</td>
</tr>
<tr>
<td>747</td>
<td>Mario Taboada and Yuncheng You</td>
<td>Invariant manifolds for retarded semilinear wave equations</td>
</tr>
<tr>
<td>748</td>
<td>Peter Rejto and Mario Taboada</td>
<td>Unique solvability of nonlinear Volterra equations in weighted spaces</td>
</tr>
<tr>
<td>749</td>
<td>Hi Jun Choe</td>
<td>Holder regularity for the gradient of solutions of certain singular parabolic equations</td>
</tr>
<tr>
<td>750</td>
<td>Jack D. Dockery</td>
<td>Existence of standing pulse solutions for an excitable activator-inhibitory system</td>
</tr>
<tr>
<td>751</td>
<td>Jack D. Dockery and Roger Lui</td>
<td>Existence of travelling wave solutions for a bistable evolutionary ecology model</td>
</tr>
<tr>
<td>752</td>
<td>Giovanni Alberti, Luigi Ambrosio and Giuseppe Buttazzio</td>
<td>Singular perturbation problems with a compact support semilinear term</td>
</tr>
<tr>
<td>753</td>
<td>Emad A. Fatemi</td>
<td>Numerical schemes for constrained minimization problems</td>
</tr>
<tr>
<td>754</td>
<td>Y. Kuang and H.L. Smith</td>
<td>Slowly oscillating periodic solutions of autonomous state-dependent delay equations</td>
</tr>
<tr>
<td>755</td>
<td>Emad A. Fatemi</td>
<td>A new splitting method for scaler conservation laws with stiff source terms</td>
</tr>
<tr>
<td>756</td>
<td>Hi Jun Choe</td>
<td>A regularity theory for a more general class of quasilinear parabolic partial differential equations and variational inequalities</td>
</tr>
<tr>
<td>757</td>
<td>Haitao Fan</td>
<td>A vanishing viscosity approach on the dynamics of phase transitions in Van Der Waals fluids</td>
</tr>
<tr>
<td>758</td>
<td>T.A. Osborn and F.H. Molzahn</td>
<td>The Wigner–Weyl transform on tori and connected graph propagator representations</td>
</tr>
<tr>
<td>759</td>
<td>Avner Friedman and Bei Hu</td>
<td>A free boundary problem arising in superconductor modeling</td>
</tr>
<tr>
<td>760</td>
<td>Avner Friedman and Wenxiang Liu</td>
<td>An augmented drift-diffusion model in semiconductor device</td>
</tr>
<tr>
<td>761</td>
<td>Avner Friedman and Miguel A. Herrero</td>
<td>Extinction and positivity for a system of semilinear parabolic variational inequalities</td>
</tr>
<tr>
<td>762</td>
<td>David Dobson and Avner Friedman</td>
<td>The time-harmonic Maxwell equations in a doubly periodic structure</td>
</tr>
<tr>
<td>763</td>
<td>Hi Jun Choe</td>
<td>Interior behaviour of minimizers for certain functionals with nonstandard growth</td>
</tr>
<tr>
<td>764</td>
<td>Vincenzo M. Tortorelli and Epifanio G. Virga</td>
<td>Axis-symmetric boundary-value problems for nematic liquid crystals with variable degree of orientation</td>
</tr>
<tr>
<td>765</td>
<td>Nikan B. Firoozeye and Robert V. Kohn</td>
<td>Geometric parameters and the relaxation of multiwell energies</td>
</tr>
<tr>
<td>766</td>
<td>Haitao Fan and Marshall Slemrod</td>
<td>The Riemann problem for systems of conservation laws of mixed type</td>
</tr>
<tr>
<td>767</td>
<td>Joseph D. Fehrbach</td>
<td>Analysis and application of a continuation method for a self-similar coupled Stefan system</td>
</tr>
<tr>
<td>768</td>
<td>C. Foias, M.S. Jolly, I.G. Kevrekidis and E.S. Titi</td>
<td>Dissipativity of numerical schemes</td>
</tr>
<tr>
<td>769</td>
<td>D.D. Joseph, T.Y.J. Liao and J.-C. Saut</td>
<td>Kelvin–Helmholtz mechanism for side branching in the displacement of light with heavy fluid under gravity</td>
</tr>
</tbody>
</table>
Chris Grant, Solutions to evolution equations with near-equilibrium initial values
B. Cockburn, F. Coquel, Ph. LeFloch and C.W. Shu, Convergence of finite volume methods
N.G. Lloyd and J.M. Pearson, Computing centre conditions for certain cubic systems
João Palhoto Matos, Young measures and the absence of fine microstructures in the $\alpha - \beta$ quartz phase transition
L.A. Peletier & W.C. Troy, Self-similar solutions for infiltration of dopant into semiconductors
H. Scott Dumas and James A. Ellison, Nekhoroshev's theorem, ergodicity, and the motion of energetic charged particles in crystals
Stathis Filippas and Robert V. Kohn, Refined asymptotics for the blowup of $u_t - \Delta u = u^p$.
Patricia Bauman, Nicholas C. Owen and Daniel Phillips, Maximum principles and a priori estimates for an incompressible material in nonlinear elasticity
Patricia Bauman, Nicholas C. Owen and Daniel Phillips, Maximal smoothness of solutions to certain Euler–Lagrange equations from nonlinear elasticity
Jack Carr and Robert Pego, Self-similarity in a coarsening model in one dimension
J.M. Greenberg, The shock generation problem for a discrete gas with short range repulsive forces
George R. Sell and Mario Taboada, Local dissipativity and attractors for the Kuramoto–Sivashinsky equation in thin 2D domains
T. Subba Rao, Analysis of nonlinear time series (and chaos) by bispectral methods
Nicholas Baumann, Daniel D. Joseph, Paul Mohr and Yuriko Renardy, Vortex rings of one fluid in another free fall
Oscar Bruno, Avner Friedman and Fernando Reitich, Asymptotic behavior for a coalescence problem
Johannes C.C. Nitsche, Periodic surfaces which are extremal for energy functionals containing curvature functions
F. Abegg and J.L. Bona, A mathematical theory for viscous, free-surface flows over a perturbed plane
Gunduz Caginalp and Xinfu Chen, Phase field equations in the singular limit of sharp interface problems
Robert P. Gilbert and Yongzhi Xu, An inverse problem for harmonic acoustics in stratified oceans
Roger Fosdick and Eric Volkman, Normality and convexity of the yield surface in nonlinear plasticity
H.S. Brown, I.G. Kevrekidis and M.S. Jolly, A minimal model for spatio-temporal patterns in thin film flow
Chao–Nien Chen, On the uniqueness of solutions of some second order differential equations
Xinfu Chen and Avner Friedman, The thermitor problem for conductivity which vanishes at large temperature
Xinfu Chen and Avner Friedman, The thermitor problem with one-zero conductivity
E.G. Kalnins and W. Miller, Jr., Separation of variables for the Dirac equation in Kerr Newman space time
E. Knobloch, M.R.E. Proctor and N.O. Weiss, Finite-dimensional description of doubly diffusive convection
V.V. Pukhnachov, Mathematical model of natural convection under low gravity
M.C. Knaap, Existence and non-existence for quasi-linear elliptic equations with the p-laplacian involving critical Sobolev exponents
Stathis Filippas and Wenxiong Liu, On the blowup of multidimensional semilinear heat equations
A.M. Meirmanov, The Stefan problem with surface tension in the three dimensional case with spherical symmetry: non-existence of the classical solution
Bo Guan and Joel Spruck, Interior gradient estimates for solutions of prescribed curvature equations of of parabolic type
Hi Jun Choe, Regularity for solutions of nonlinear variational inequalities with gradient constraints
Peter Shi and Yongzhi Xu, Quasistatic linear thermoelasticity on the unit disk
Satyanad Kichenassamy and Peter J. Olver, Existence and non-existence of solitary wave solutions to higher order model evolution equations
Dening Li, Regularity of solutions for a two-phase degenerate Stefan Problem
Marek Fila, Bernard Kawohl and Howard A. Levine, Quenching for quasilinear equations
Yoshikazu Giga, Shun’ichi Goto and Hitoshi Ishii, Global existence of weak solutions for interface equations coupled with diffusion equations
Mark J. Friedman and Eusebius J. Doedel, Computational methods for global analysis of homoclinic and heteroclinic orbits: a case study
Mark J. Friedman, Numerical analysis and accurate computation of heteroclinic orbits in the case of center manifolds
Peter W. Bates and Songmu Zheng, Inertial manifolds and inertial sets for the phase-field equations
J. López Gómez, V. Marréquez and N. Wolanski, Global behavior of positive solutions to a semilinear equation with a nonlinear flux condition
Xinfu Chen and Fahuai Yi, Regularity of the free boundary of a continuous casting problem
Eden, A., Foias, C., Nicolaenko, B. and Temam, R., Inertial sets for dissipative evolution equations Part I: Construction and applications
Jose–Francisco Rodrigues and Boris Zaltzman, On classical solutions of the two-phase steady-state Stefan problem in strips
Viorel Barbu and Srdjan Stojanovic, Controlling the free boundary of elliptic variational inequalities on a variable domain