GENERALIZATION OF THE FOSCOLO-GIBILARO ANALYSIS OF DYNAMIC WAVES

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1. Introduction

This note reports an attempt to obtain the particle phase pressure in the Foscolo-Gibilaro [2] one-dimensional, particle in a fluidized bed model, from a constitutive hypothesis which appears to be implied by their work. Our hypothesis leads to their expression plus another term which is proportional to the space derivative of the particle velocity. We think of this term as representing a change in the microstructure, the positions of the particles relative to one another. This term is also missing from the one-dimensional equations which were recently derived by G.K. Batchelor [1]. When the new term is added we find that the state of uniform fluidization is always unstable.

2. Dynamical equations

Foscolo and Gibilaro [2] start with the coupled one-dimensional equation for the particles and fluid phase. The particle phase equations are

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi u_p}{\partial z} = 0, \tag{1}$$

$$\phi \rho_p \left[\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} \right] = - \phi \rho_p g + \mathcal{F} - \frac{\partial p_p}{\partial z}. \tag{2}$$

where u_p is the particle velocity, ϕ is the particle volume fraction, $\phi = 1 - \varepsilon$ where ε is the fluids fraction, ρ_p is the particle density, $\mathcal{F}$ is the interaction force, the force that the fluid exerts on the particle and p_p is the particle phase pressure. The fluid equations are of the same form except that the subscript p is replaced by f, ϕ is replaced by ε and $\mathcal{F}$ by minus $\mathcal{F}$.

Foscolo and Gibilaro modeled the interaction force $\mathcal{F}$ and the particle phase pressure in a manner that decouples the equations for the fluid and solid phases. This gives rise to a system of equations for the particles only, called the particle bed model.

It is convenient to introduce a dynamic pressure π_p into (2) by writing

$$p_p = P + \pi_p , \tag{3}$$

$$\phi\rho_p g + \frac{\partial P}{\partial z} = 0. \tag{4}$$

Then (2) reduces to

$$\phi\rho_p\left[\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial z}\right] = \mathcal{F} - \frac{\partial \pi_p}{\partial z}. \tag{5}$$

3. <u>Drag on a particle in a steady fluidized suspension</u>

To get their equations they first derived an interesting expression F_d (1) for the drag force exerted by the fluid on a single particle in a uniform fluidized suspension. This expression relies strongly on the well-known correlation of Richardson and Zaki for fluidized and sedimenting beds of monosized spherical particles

$$u_c = V\varepsilon^n \tag{6}$$

where

$$u_c = u_p\phi + u_f\varepsilon \tag{7}$$

is the composite velocity, the volume flux divided by total area and u_c is independent of z, $\frac{\partial u_c}{\partial z} = 0$. Of course $V = u_c$ when $\varepsilon = 1$, the steady terminal velocity of a freely falling single sphere in a sea of fluid. The exponent n depends on the Reynolds number $\mathrm{Re} = \frac{dV}{\mu}$, where d is the diameter

$$n = \begin{cases} 4.65 \text{ for } \mathrm{Re} < 0.2, \\ 4.4\,\mathrm{Re}^{-0.03} \text{ for } 0.2 < \mathrm{Re} < 1, \\ 4.4\,\mathrm{Re}^{-0.1} \text{ for } 1 < \mathrm{Re} < 500, \\ 2.4 \text{ for } \mathrm{Re} > 500. \end{cases} \tag{8}$$

Foscolo-Gibilaro replace 4.65 with 4.8=2(2.4) for reasons to be made clearer later.

There is a huge amount of fluid mechanics buried in the Richardson-Zaki correlation. This is hidden in the drag law for particles falling under gravity in steady flow. Let $F_d(\varepsilon)$ be the drag on a single particle in a freely falling suspension with a water fraction ε. When $\varepsilon = 1$ we get a drag law for the free fall of a single sphere which is Stokes drag when V is small enough; for larger V the drag is given by

$$F_d(1) = \frac{\rho V^2}{2} \frac{\pi d^2}{4} C_D \tag{9}$$

where C_D is given by an empirical correlation, for example,

$$C_D = \left(0.63 + \frac{4.90}{\sqrt{\mathrm{Re}}}\right)^2$$

said to be due to Dallavalle. Foscolo and Gibilaro produce the formula

$$F_d(\varepsilon) = \varepsilon F_d(1) \tag{10}$$

from an argument which says that in a fluidized bed in steady flow, the total force F on a sphere is the sum

$$F(\varepsilon, \mathrm{Re}) = F_d(\varepsilon) - F_b(\varepsilon) \tag{11}$$

where

$$F_b(\varepsilon) = \frac{\pi d^3}{6}(\rho_p - \rho_c)\, g \tag{12}$$

is the buoyant force using the effective density

$$\rho_c = \varepsilon\rho_f + \phi\rho_p \tag{13}$$

of the composite fluid. Since $\phi = 1 - \varepsilon$

$$F_b(\varepsilon) = \frac{\pi d^3}{\rho}(\rho_p - \rho_f)\, g\varepsilon = \varepsilon F_b(1). \tag{14}$$

In steady flow, $F = 0$ and

$$F_d(\varepsilon) = F_b(\varepsilon) = \varepsilon F_b(1) = \varepsilon F_d(1). \tag{15}$$

We never see steady flow in a fluidized bed, the particles always jiggle about; steady is in some statistical sense, whatever that may be. In any interpretation

$$u_p = 0 \text{ in steady flow}. \tag{16}$$

Equation (15) is all that is required to get the drag on a single particle in a fluidized suspension in steady flow. The hydrodynamic content is all buried in the drag correlation (9). We may write $F_d(\varepsilon) = \varepsilon F_1(V)$. To see how $F_d(\varepsilon)$ depends on the fluidizing velocity u_c, Foscolo and Gibilaro note that (9) implies that

$$F_d = \varepsilon \begin{cases} 3\pi\mu V & \text{(laminar)} \\ 0.055\pi\rho d^2 V^2 & \text{(turbulent).} \end{cases}$$

They next note that the Richardson and Zaki correlation (6) and (8), with 4.8 replacing 4.65, implies that

$$F_d = \varepsilon^{-3.8} \begin{cases} 3\pi\mu d u_c & \text{(laminar)} \\ 0.055\pi\rho d^2 u_c^2 & \text{(turbulent).} \end{cases} \tag{17}$$

This is good, we have $F_d(u_c, \varepsilon) = \varepsilon^{-3.8} F_d(u_c)$, independent of V for low and high Reynolds numbers. Now we look for an equivalent expression, valid for all Reynolds numbers in steady flow and

$$F_d(\varepsilon) = F_d(\varepsilon, u_c, V) = \varepsilon^{-3.8} g(u_c, V) \tag{18}$$

which will reduce to (17) at low and high Re. Clearly

$$g(u_c, v) = \varepsilon^{4.8} F_d(1) = \left(\frac{u_c}{V}\right)^{\frac{4.8}{n}} F_d(1).$$

Hence

$$F_d(\varepsilon, u_c, V) = \varepsilon^{-3.8} \left(\frac{u_c}{V}\right)^{\frac{4.8}{n}} F_d(1). \tag{19}$$

This is just another way of writing $F_d(\varepsilon) = \varepsilon F_d(1)$ when 4.65 is replaced with 4.8 which is useful in motivating the constitutive equation (21) below.

4. <u>The first constitutive hypothesis giving the force per unit volume on the spheres</u>

Foscolo and Gibilaro assume that in unsteady flow the force on a particle is given by the expression (19) with u_c replaced by the slip velocity

$$u_c - u_p = (1-\varepsilon)u_p + \varepsilon u_f - u_p = \varepsilon(u_p - u_f). \tag{20}$$

Then the unsteady drag force is

$$F_d(\varepsilon, u_c - u_p, V) = \varepsilon^{-3.8}\left(\frac{u_c - u_p}{V}\right)^{\frac{4.8}{n}} F_d(1). \tag{21}$$

In steady flow, $u_p = 0$, and (21) reduces to

$$F_d(\varepsilon) = \varepsilon F_d(1) \tag{22}$$

where balancing drag and buoyancy for a single sphere gives

$$F_d(1) = \frac{\pi d^3}{6}(\rho_p - \rho_f)g.$$

The total force on single particle in a fluidized suspension is given by

$$F = F_d - F_b = \frac{\pi d^3 g}{6}(\rho_p - \rho_f)\left\{\varepsilon - \left[\frac{u_c - u_p}{V}\right]^{\frac{4.8}{n}} \varepsilon^{-3.8}\right\}. \tag{23}$$

The force per unit volume due to all *n* spheres is

$$\mathcal{F} = NF \tag{24}$$

where

$$N = \frac{\phi}{\pi d^3/6} = \frac{n}{\text{volume}}. \tag{25}$$

Hence, the total force on the particles per unit volume is

$$\mathcal{F} = \phi(\rho_p - \rho_f)g\left\{\varepsilon - \left[\frac{u_c - u_p}{V}\right]^{\frac{4.8}{h}} \varepsilon^{-3.8}\right\}. \tag{26}$$

In steady flow, u_p and F = 0.

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