OPTIMUM FILTER FOR DC/AC–CONVERTER IN ELECTRONICS

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Abstract. An optimalization of a filter in the sense of reducing its size as well as in realizing a small loss of power can lead to a high-dimensional nonlinear system of equations. We will give a fast and surprisingly simple algorithm to solve the problem for higher-dimensional cases.

1. The Classical Approach. We dealt with a DC/AC-converter as shown in Fig. 1, provided to feed small consumers such as measuring instruments, computers, ...

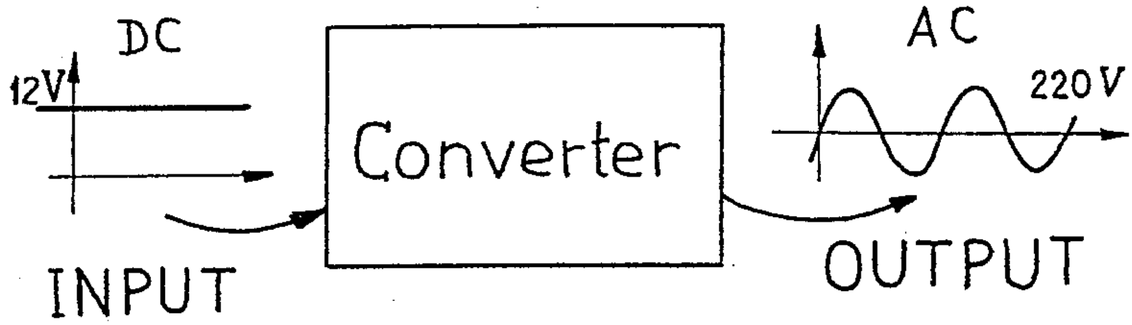

Fig. 1

An optimalization of a filter in the sense of reducing its size as well as in realizing a small loss of power can lead the following mathematical question (see [1]):

Which 2π-periodic signal $E(t)$ with values ± 1 or 0 only satisfies the conditions:

(1) $E(-t) = -E(-t)$ (odd function)

(2) $E(t) = E(\pi - t)$

(3) Vanishing 2^{nd}, 3^{rd}, ..., n^{th} harmonics with n large.

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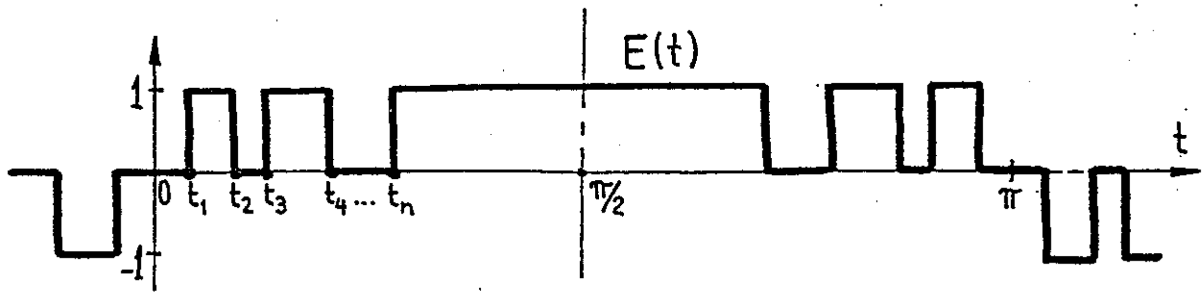

Fig. 2

Therefore, we have to computer n angles t_i for commutating the signal (see Fig. 2), where

$$0 < t_1 < t_2 < \cdots < t_n < \pi/2 \tag{1.1}$$

For the simple case

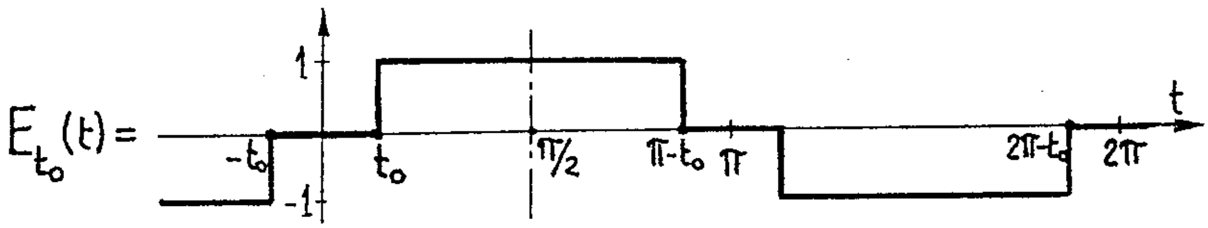

Fig. 3

we got the Fourier series (see Fig. 3)

$$E_{t_0}(t) = \frac{4}{\pi} \cdot \sum_{k=1,3,5\ldots}^{\infty} \frac{1}{k} \cos(kt_0) \cdot \sin(kt) \tag{1.2}$$

Superpositions lead to

$$E(t) = E_{t_1}(t) - E_{t_2}(t) + E_{t_3}(t) - \cdots \pm E_{t_n}(t) \tag{1.3}$$

and therefore we get

$$E(t) = \sum_{k=1,3,5\ldots}^{\infty} B_k \cdot \sin(kt) \qquad \text{with} \tag{1.4}$$

$$B_k = \cos(kt_1) - \cos(kt_2) + \cos(kt_3) - \cdots \pm \cos(kt_n) \tag{1.5}$$

Condition (3) for vanishing harmonics leads to

$$B_3 = B_5 = B_7 = \cdots = B_{2n+1} = 0 \tag{1.6}$$

and therefore to the following $n \times n$ - *nonlinear* system of equations for the angles t_i:

$$(1.7) \qquad \begin{cases} \cos 3t_1 & -\cos 3t_2 & +\cos 3t_3 & -\ldots \pm \cos 3t_n & = 0 \\ \cos 5t_1 & -\cos 5t_2 & +\cos 5t_3 & -\ldots \pm \cos 5t_n & = 0 \\ \ldots & & & \ldots & \\ \cos{(2n{+}1)}t_1 & -\cos{(2n{+}1)}t_2 & +\cos{(2n{+}1)}t_3 & -\ldots \pm \cos{(2n{+}1)}t_n & = 0 \end{cases}$$

This problem could be solved numerically up to $n = 17$ unknowns by *Newtons method*:

For n odd we got a *unique* solution, for n even we got *no* solution with the computer.

Table 1 gives the result for $n = 17$ in the center column after 10 iterations. The column to the right describes the right hand sides of (1.7)

Starting values	Solutions t_i(angles)	errors
sta 1= 0.1300	t_1= 0.14343461	f 1= - 0.00000034
sta 2= 0.1500	t_2= 0.16744260	f 2= 0.00000011
sta 3= 0.2700	t_3= 0.28724365	f 3= 0.00000024
sta 4= 0.3200	t_4= 0.33479213	f 4= 0.00000018
sta 5= 0.4200	t_5= 0.43179664	f 5= - 0.00000010
sta 6= 0.4900	t_6= 0.50194244	f 6= 0.00000014
sta 7= 0.5400	t_7= 0.57745609	f 7= 0.00000010
sta 8= 0.6400	t_8= 0.66876878	f 8= 0.00000020
sta 9= 0.7100	t_9= 0.72456684	f 9= 0.00000050
sta10= 0.8100	t_{10}= 0.83511161	f10= 0.00000042
sta11= 0.8400	t_{11}= 0.87344646	f11= 0.00000072
sta12= 0.9700	t_{12}= 1.00076675	f12= - 0.00000035
sta13= 1.0000	t_{13}= 1.02436447	f13= 0.00000069
sta14= 1.1500	t_{14}= 1.16547775	f14= 0.00000020
sta15= 1.1700	t_{15}= 1.17751884	f15= 0.00000028
sta16= 1.3400	t_{16}= 1.32892848	f16= 0.00000058
sta17= 1.3500	t_{17}= 1.33300971	f17= 0.00000085

Table 1
$n = 17$

The computation of the spectra gives Fig. 4

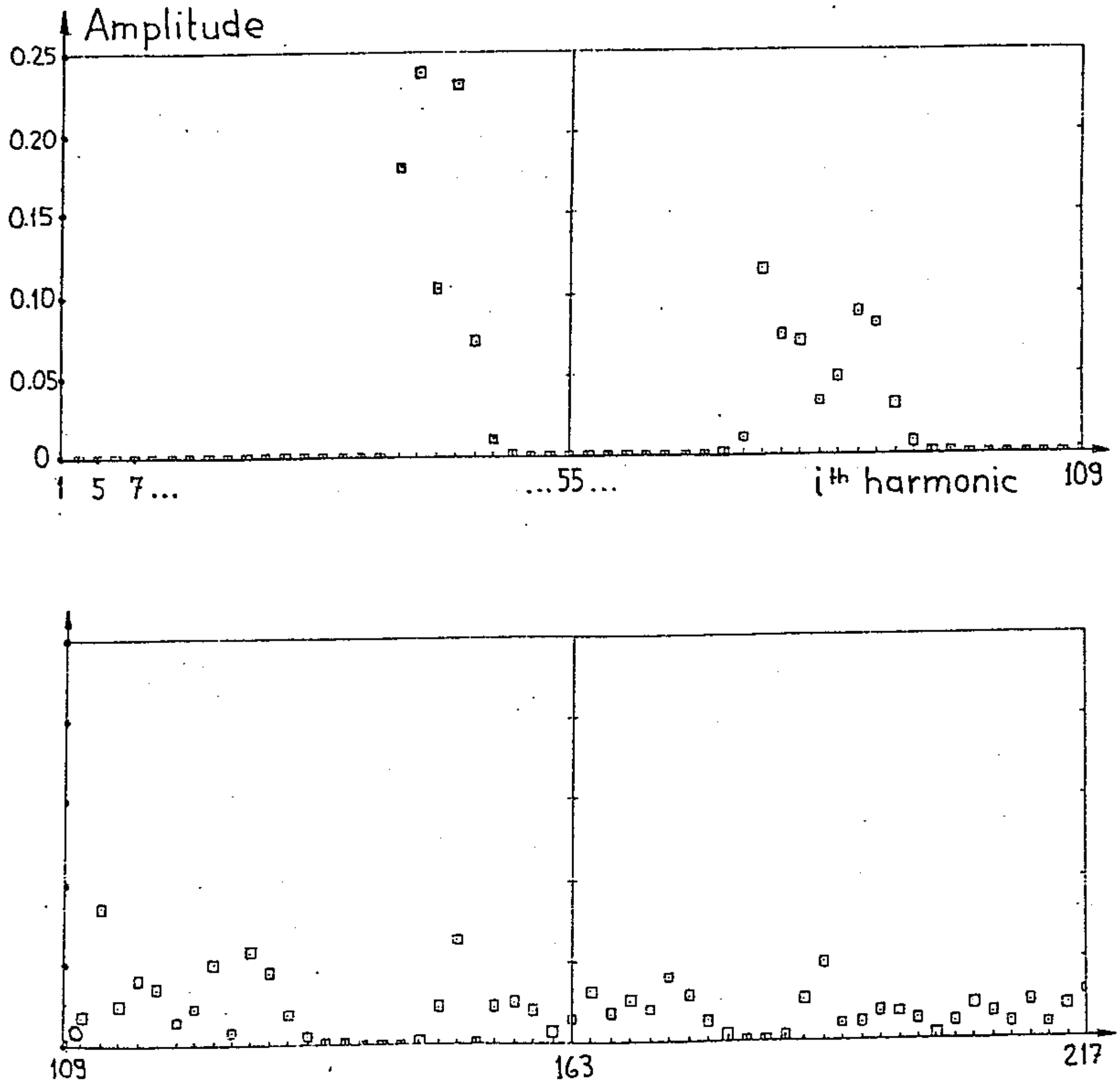

Fig. 4: Spectra with Newton's Method $n = 17$

2. A fast, simple and efficient algorithm.

Idea: To compute the angles t_i for commutating the signal, we start with the sine curve

$$(2.1) \qquad \sin t \qquad \text{with } 0 \leqq t \leqq \pi$$

using the following steps (see Fig. 5):

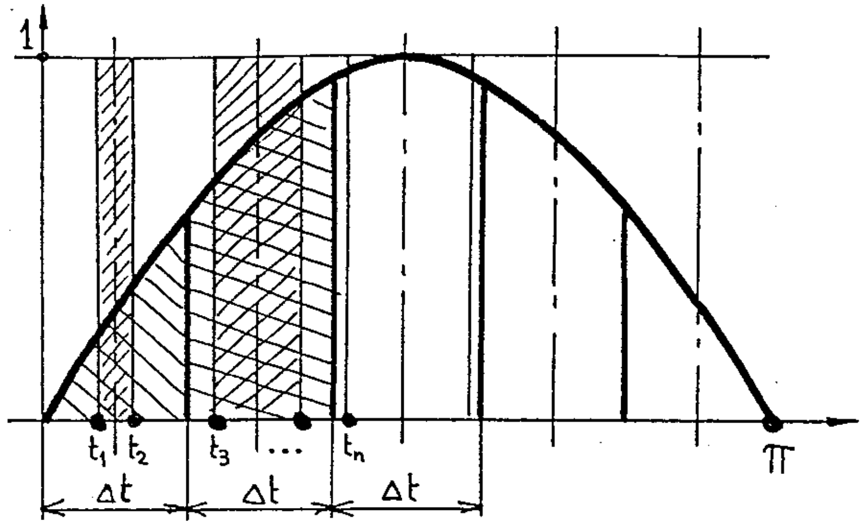

Fig. 5

I. Partition of the sine-area into n aequidistant stripes ($\Delta t = \frac{\pi}{n}$, n odd)

II. Replace stripe-areas by centered rectangles of the same area with amplitude = 1.

III. Take the points • as angles $t_1, t_2, \ldots, t_n$

The computation is simple (see Fig 6:)

$$
(2.1) \qquad \begin{cases} \Delta T = \int_\alpha^\beta \sin t \cdot dt = \cos\alpha - \cos\beta \\ \text{where } \Delta t = \frac{\pi}{n}, \beta = \alpha + \Delta t \end{cases}
$$

This implies

$$
(2.2) \qquad \begin{aligned} t_k &= \frac{\alpha+\beta}{2} - \frac{\Delta T}{2} \\ t_{k+1} &= \frac{\alpha+\beta}{2} + \frac{\Delta T}{2} \end{aligned}
$$

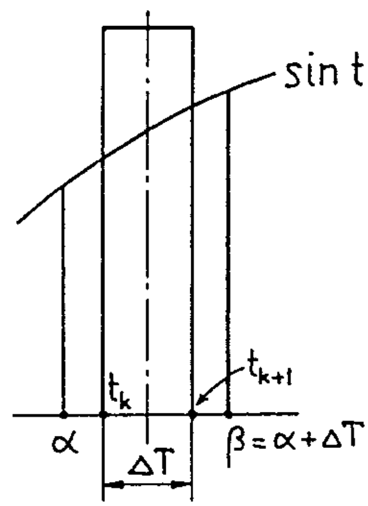

Fig. 6

For $h = 17$, we get now the spectra of Fig. 7 with "only" 13 practically vanishing first harmonics:

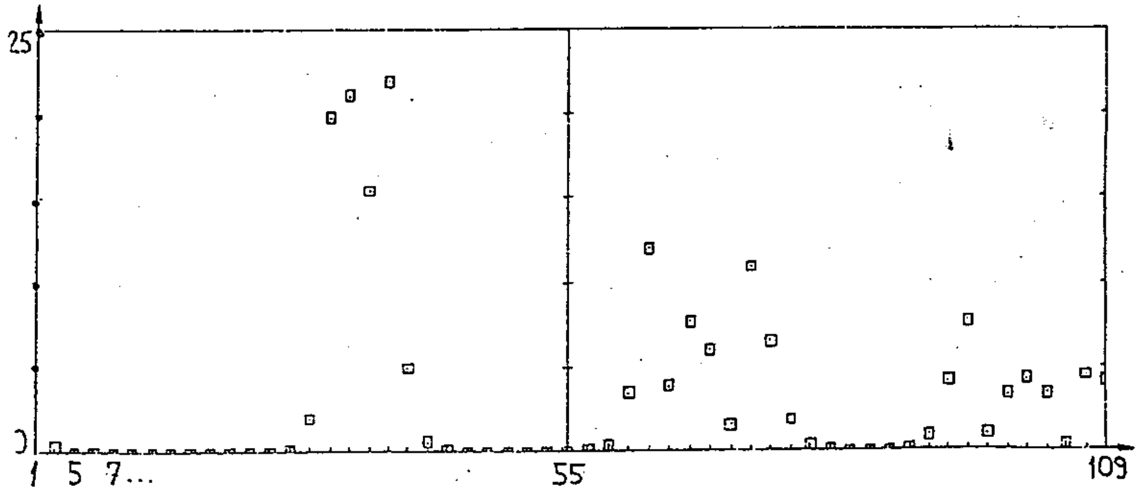

Fig. 7

However, the algorithm for $n = 71$ lead to the *excellent* spectra of Fig. 8:

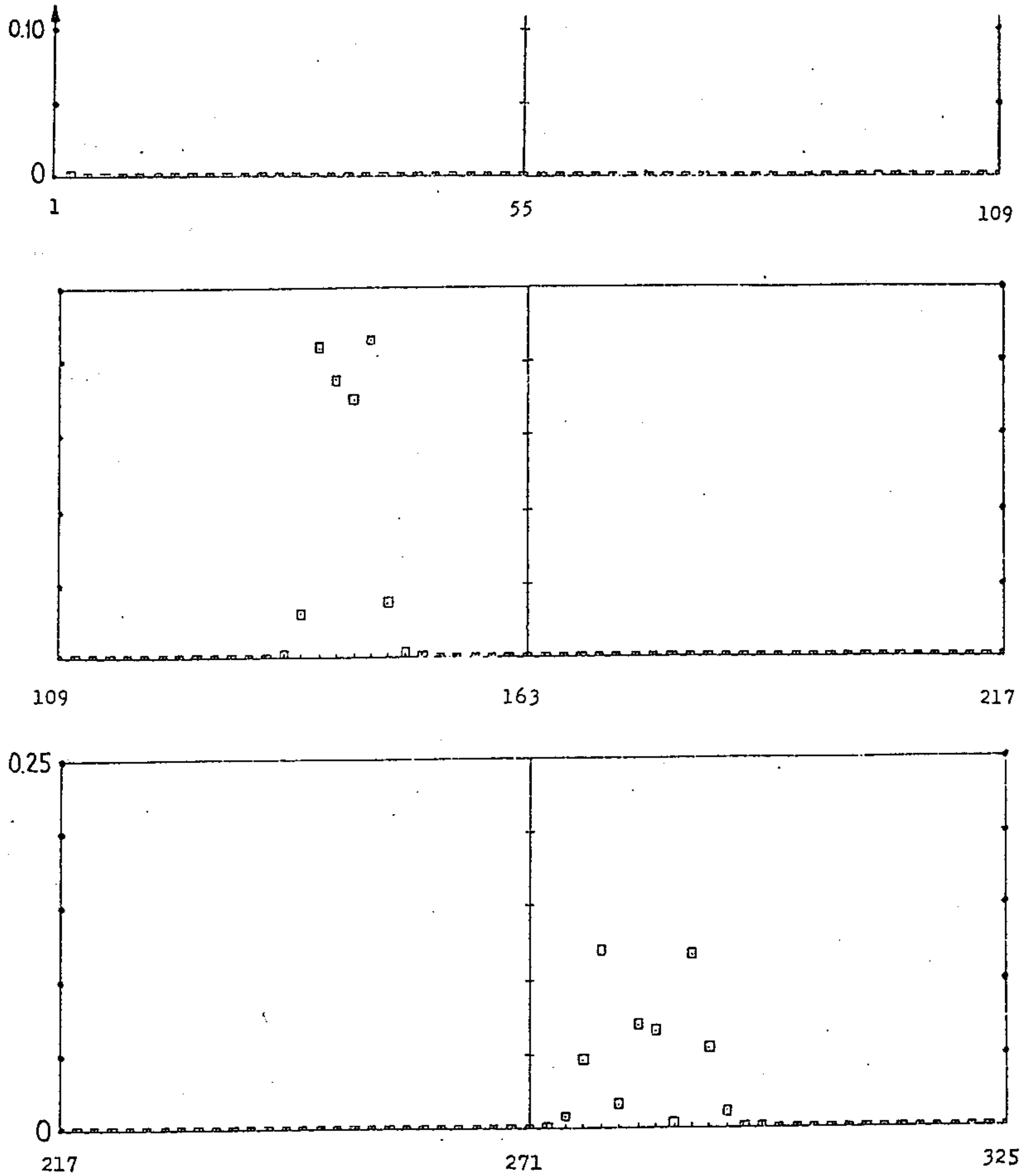

Fig. 8

It does not make sense for a practical realization of the converter to go higher than n because of the *resolution*, which manifests itself numerically with the *cancellation effect* in $\cos\alpha - \cos\beta$, where $\alpha \approx \beta$ (see (2.1)).

In the case $n = 71$, we already got 4 pairs of identical angles within the accuracy of 6 decimal digits, i.e. 81.1230^o.

In the practical realization (see [2], which includes laboratory results as well), 2048 steps for a quarter of a period was used. So one step correspond to $90^o/2048 = 0.0439^o = h$.

The quality of the resulting spectra, where the commutating angles t_i are changed by an amount $\leqq h/2$ is still similar to the one before. The algorithm has a ***good stability*** with reference *to perturbations of the* t_i (see Fig. 9).

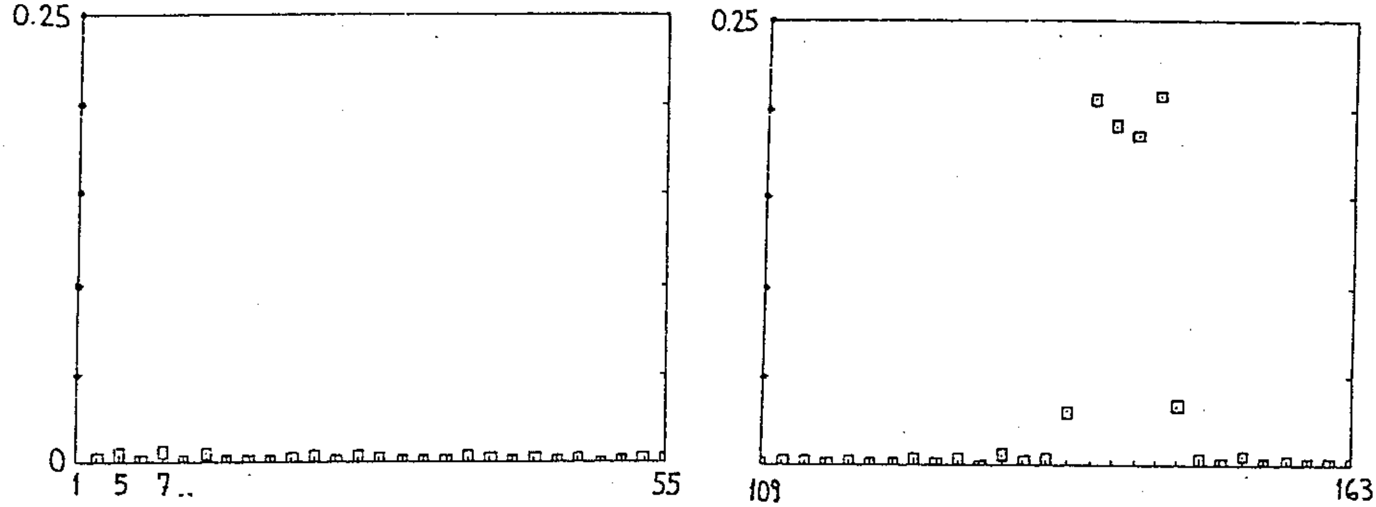

Fig. 9: Part of the spectra with perturbed t_i-values

The omitting part of the spectra is very much the same as in Fig. 8.

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