AN INVERSE KINEMATIC MATHEMATICAL MODEL USING GROEBNER BASIS THEORY FOR ARM SWING MOVEMENT IN THE GAIT CYCLE

By

Kimberly D. Kendricks

Adam M. Fullenkamp

Robert McGrellis

Jonathan Juhl

and

Ronald F. Tuttle

IMA Preprint Series # 2329

(July 2010)

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

400 Lind Hall

207 Church Street S.E.

Minneapolis, Minnesota 55455–0436

Phone: 612-624-6066 Fax: 612-626-7370

URL: http://www.ima.umn.edu

An Inverse Kinematic Mathematical Model Using Groebner Basis Theory for Arm Swing Movement in the Gait Cycle

30 August 2010

[1]Kimberly D. Kendricks, [2]Adam M. Fullenkamp, [1]Robert McGrellis, [1]Jonathan Juhl, and [1]Ronald F. Tuttle

[1]Center for MASINT Studies and Research
Air Force Institute of Technology
Wright Patterson Air Force Base, Ohio

[2]3D Human Signatures Laboratory
711th Human Performance Wing (711 HPW/RHPA)
Wright Patterson Air Force Base, Ohio

ABSTRACT

Two important indicators in the gait cycle: arm swing and foot placement, when isolated, provide key information about future behaviors in the gait cycle. Through video captured data, the INSPIRE (Integration of a Sensor Package for Identifying Radical Extremists) project seeks to identify gait characteristics in arm and leg swing movement to recognize threatening individuals. In particular, one approach discussed in this paper uses Groebner Basis Theory to model arm swing movement in the gait cycle. Through inverse kinematics, joint configurations of the shoulder and elbow are found to place the wrist at a specific position and orientation in the gait cycle.

1.0 Introduction

Conventional methods for human motion analysis often involve the manual palpation of predefined anatomical landmarks and the direct placement of active or passive fiducials. There are, however, applications for which the retrospective analysis of human movement from standard two-dimensional (2D) video imagery is required. Specifically, there is growing interest in the kinematic characterization of human activities from persistent surveillance footage for corporate security and law enforcement purposes (Niu et al., 2004; Pavan et al., 2008). The challenge for human movement scientists is to take a technological step backwards in terms of the image data source, but to provide a level of automated measurement fidelity comparable to contemporary motion capture systems. An inevitability of surveillance video-based person tracking is that certain anatomical landmarks and associated body segments will be more accessible than others (Andriluka et al., 2008). The question is how to address the issues of tracking ambiguity, and an obvious solution is the implementation of inverse kinematic (IK) techniques.

While a variety of IK- solutions have been proposed for modeling human motion, few have attempted to define motion models with a purely mathematical solution. In general, many IK- techniques involve some element of optimization or fitting (Meredith & Maddock, 2007; Mihelj, 2006; Todorov, 2007; Wang, 1999). The purpose of this report is to present a new approach for the application of Groebner Basis Theory in the development of an upper extremity (UE) motion model during gait through the inverse kinematic problem.

1.1 Inverse Kinematics

There are two types of kinematic problems. The forward kinematic (FK) problem is the computation of the position and orientation of a jointed flexible object in space, and the IK- problem is the process of determining all combinations of a jointed flexible object in order to achieve a desired position and orientation in space (Kendricks, 2007; Hartenberg, 1995). In robotics, the IK- problem is most widely used to determine object capability, efficiency, and accuracy for various robot manipulators. It's been proven that Groebner Basis Theory is a more efficient approach to solving the IK- robotics problem (Kendricks, 2007), and since certain aspects of human movement are comparable to robotic movement, a similar approach is used to solve the IK- problem for arm-swing movement in the gait cycle.

1.2 The INSPIRE Project

The goal of INSPIRE (Integration of a Sensor Package for Identifying Radical Extremists) is to determine gait signatures of human subjects carrying a concealed load and through these signatures, distinguish these subjects from others not carrying a load. This project is based upon the 2001 National Institute of Standards and Technology Human ID at a Distance Gait Program which focused on gait variance in a complex outdoor environment. INSPIRE, however, seeks to build a gait database of 100 human subjects, and record distinguishable gait characteristics due to a load, while also contributing to the national study of human identification.

Anatomical and demographic information (height, weight, race, ethnicity, lengths of upper and lower extremities, and any factors impacting gait) of the study's participants is collected. Volunteers walk through a circular 'race track' with and without a weighted vest simulating a load. A crossover platform and ramp are included in the race track scheme to provide a variation of the type of walking surface and to add more dynamics to the data sets. The ramp and crossover platform portion of the track are key features that augment fluctuations in the subject's gait as the subject's center of gravity changes when climbing up and down the ramp. The exact position of the crossover platform and ramp is 28 feet from the left side of the building, 25 feet from the right side of the building, and 6 feet away from the building. This positioning allows subjects to walk ten paces before the steps of the crossramp and ten paces after the ramp. The rationale for this is that ten paces are required for a human to reach normal gait. Flags are driven into the ground to restrict the walking motion so that the cameras can keep a Sagittal plane view of the subject at all times.

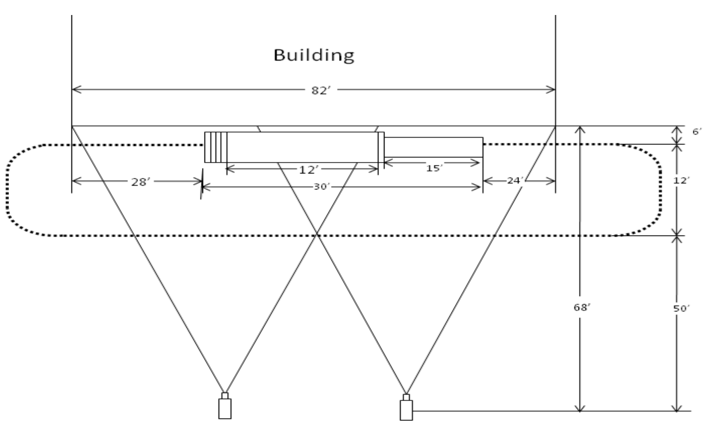

Figure 1. Birds-eye view of the INSPIRE race track drawn approximately to scale

2.0 Methods

2.1 Video Capture & Image Processing

To illustrate the IK-model using Groebner Basis Theory, a model is designed simulating movement captured on video for one human subject. The gait data was captured by two Canon GL2 Cameras on full-size tripods with pan-tilt heads used to mount the cameras. The Cannon GL2 is equipped with a 3CCD 1/4" pixel shift (charged coupled device) image sensor that provides a maximum of 480 x 720 pixels of resolution. Using the automatic recording program (e.g., standard settings of exposure, brightness, shutter speed, gain, and focus), video data was captured with the Canon GL2s at the NTSC standard, 29.97 frames per second, on mini Digital Video (mini DV cassette) media. Under this recording program, the camera automatically adjusts the focus, exposure, gain, aperture adjustment, and shutter speed, while giving the user the option of manually changing the white balance and focus. The recorded video was digitized and edited using Windows Movie Maker, and was later captured in DV AVI format. (This resulted in a resolution of 720 x 480 pixels.)

The DV AVI file was processed using the Point Light Visualization Developer (PLVD) designed by the Air Force Research Laboratory (Fullenkamp, 2009). The INSPIRE team utilized this tool to place markers on the shoulder, elbow, wrist, hip, knee, and ankle joints for one gait cycle. The PLVD tool allowed a user to view a frame of the AVI file and place marker points on a particular image in the frame. The PLVD tool fed the 'x' and 'y' coordinates of the points marked to an Excel spreadsheet organized according to the markers' name and frame number. (Such data was collected and processed at various parts of the track. However, for the development of this model, only those gait cycles which occurred on grass were considered.)

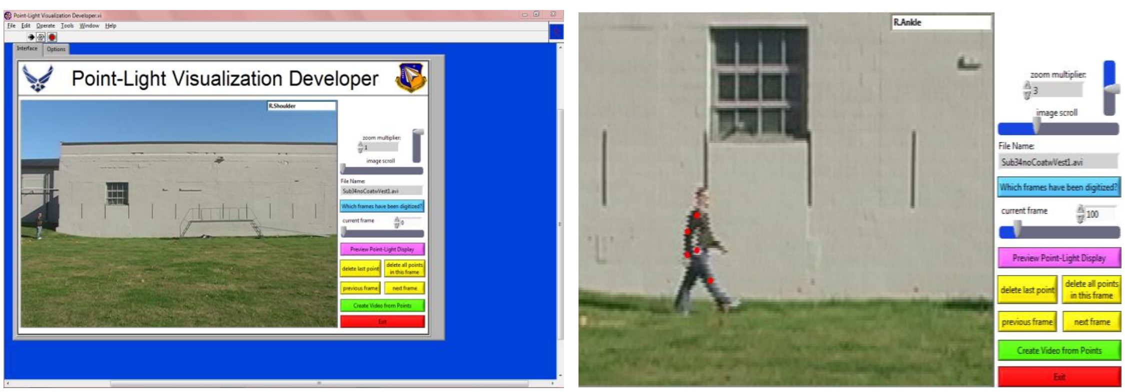

Figure 2. Example of Point Light Visualization Developer

2.2 Groebner Basis Theory

Developed in 1965 by Bruno Buchberger, Groebner Basis Theory evolved from an algorithm that applies computational algebra techniques to specific polynomial ideals, producing a Groebner basis that can be used to find solutions to a set of non-zero polynomials for a given ideal (Buchberger, 1985). By using lexigraphical order (lex) and back substitution, a solution to the variables in the set can be found (Adams & Loustaunau, 1994). More formally, there is the following definition.

Definition: A set of non − zero polynomials $G = \{g_1, \dots, g_s\}$ contained in an ideal I is called a $Groebner\ Basis\ for\ I$ if and only if for every $f \in I$ such that $f \neq 0$, there exists an $i \in \{1, \dots, s\}$ such that the $lm\ (gi) \mid lm(f)$. *(lm = leading monomial.)*

Terminology & Notation:

1. For a field k, a *Monomial Ordering* on $k[x_1, \dots, x_n]$ is a relation on the set of monomials x^α, $\alpha \in Z^n_{\geq 0}$ such that this relation is a linear ordering, and if $\alpha, \beta, \gamma \in Z^n_{\geq 0}$ such that $\alpha > \beta$, then $\alpha + \gamma > \beta + \gamma$. Moreover, this relation is also a well ordering.

2. *Lexigraphical order (lex order)*: Given a monomial ordering, let $\alpha = (\alpha^1, \dots, \alpha^n)$ and $\beta = (\beta^1, \dots, \beta^n) \in N^n_{\geq 0}$. We say $x^\alpha > x^\beta$ if $\alpha > \beta$.

3. Let k be any field and let $k \in [x_1, \dots, x_n]$ be ordered with respect to lex order, then for all $f \in k[x_1, \dots, x_n]$, with $f \neq 0$, $f = a_1 x^{\alpha_1} + a_2 x^{\alpha_2} + \cdots + a_r x^{\alpha_r}$, where $0 \neq a_i \in k[x_1, \dots, x_n]$ and x_i^α is ordered such that $x^{\alpha_1} > x^{\alpha_2} > \cdots > x^{\alpha_r}$.

S-Polynomials & A Groebner Basis:

Given the Division Algorithm in $k[x_1, \dots, x_n]$, the algorithm for a Groebner Basis is based upon the following definitions.

> **Definition**: Let $0 \neq f, g \in k[x_1, \dots, x_n]$. Let $L = lcm\big(lm(f), lm(g)\big)$. The polynomial $S(f,g) = \frac{L}{lm(f)} * f - \frac{L}{lm(g)} * g$ is called the $S-polynomial\ of\ f\ and\ g$.

S-polynomials then allow for the cancelation of leading monomials. Lastly,

> **Definition**: Let $G = \{g_1, \dots, g_t\}$ be a set of non − zero polynomials in $k[x_1, \dots, x_n]$. Then G is a *Groebner basis for the ideal* $I = < g_1, \dots, g_t >$ if an only if for all $i \neq j, S(g_1, g_t) \rightarrow {}^G_+ 0$. That is, $S(g_1, g_t)$ is divided by those $\{g_1, \dots, g_t\} \in G$, such that the remainder is zero.

In general, with the *Hilbert Basis Theorem* which states that every ideal $i \in k[x_1, \dots, x_n]$ has a finite generating set, the *variety (affine variety)*, then, of I will yield a set of solutions to the system of polynomial equations.

Forming a Groebner Basis

The geometry of the UE in Figure 2 was used to find polynomial equations to form the IK-model. (The data collected in the excel spreadsheet was used to determine the model's accuracy in predicting wrist placement in video captured data from the INSPIRE study. The authors discuss the results of this study in a separate paper.) A derivation of the IK-model is discussed below.

3.0 Results & Discussion

The IK- problem determines those joint combinations for the shoulder and elbow that will place the wrist at the arbitrary point *(a,b)* in the *XY*-coordinate plane. Let L_1 and L_2 denote the lengths of the upper arm and forearm respectively, and let cos θ_i and sin θ_i where $i = 1, 2$, denote the cosine and sine angle measures for the shoulder *(joint 1)* and elbow *(joint 2)*.

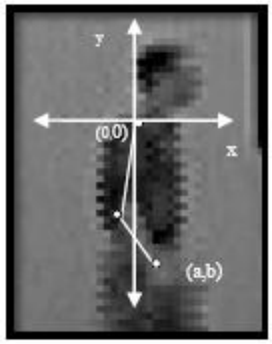

Figure 3. Projection of Sagittal Plane UE joint positions onto the XY-coordinate plane.

From Figure 3, a kinematic model is derived (Cox, Little and O'Shea, 1992) giving a system of four equations with four unknowns abbreviated as c_1, s_1, c_2, and s_2 (where *c* and *s* denote the cosine and sine angles for θ_1 and θ_2 respectively) describing the position and orientation of *(a,b)*.

(1) $a = L_2(c_1c_2 - s_1s_2) + L_1c_1$ **(2)** $b = -L_2(s_1c_2 + c_1s_2) + L_1s_1$

(3) $c_1^2 + s_1^2 = 1$ **(4)** $c_2^2 + s_2^2 = 1$

Using MAGMA, an algebraic computer software, the following Groebner basis is found using lex order (c_1, s_1, c_2, s_2):

(5) $$c2 + \frac{\frac{1}{2}*L1^2 + \frac{1}{2}*L2^2 - \frac{1}{2}*a^2 - \frac{1}{2}*b^2}{L1*L2}$$

(6) $$c1 - L2*\frac{b}{a^2+b^2}*s2 + \frac{-\frac{1}{2}*L1^2*a + \frac{1}{2}*L2^2*a - \frac{1}{2}*a^3 - \frac{1}{2}*a*b^2}{L1*a^2 + L1*b^2}$$

(7) $$s1 + L2*\frac{a}{a^2+b^2}*s2 + \frac{-\frac{1}{2}*L1^2*b + \frac{1}{2}*L2^2*b - \frac{1}{2}*a^2*b - \frac{1}{2}*b^3}{L1*a^2 + L1*b^2}$$

(8) $$s2^2 + \frac{\frac{1}{4}*L1^4 - \frac{1}{2}*L1^2*L2^2 - \frac{1}{2}*L1^2*a^2 - \frac{1}{2}*L1^2*b^2 + \frac{1}{4}*L2^4 - \frac{1}{2}*L2^2*a^2 - \frac{1}{2}*L2^2*b^2 + \frac{1}{4}*a^4 + \frac{1}{2}*a^2*b^2 + \frac{1}{4}*b^4}{L1^2*L2^2}$$

By setting the last equation equal to zero and solving for s_2, through back substitution, all joint combinations (c_1, s_1, c_2, s_2) placing the wrist at the point *(a,b)* are found, and the IK-problem is solved. Furthermore, by applying the joint parameters $-140° \leq \theta_1 \leq 90°$ *and* $-10° \leq \theta_2 \leq 140°$ (Luttegens and Hamilton, 1997), all reachable points within the subspace can be found (Kendricks, 2007). During the development of this model, the INSPIRE team was still collecting and processing the gait data. To determine whether the model was well-defined, a correspondence between FK-and IK-solutions was found using Vicon Motion data collected from two gait cycles from one human subject. Also, coordinate data from the PLVD along with prior information about the lengths of the upper and forearm were also used to validate the model for video captured data. (These results are discussed in a separate paper.)

4.0 Summary

The IK-approach using Groebner Basis Theory then is an alternate approach to arm swing movement in the gait cycle. To support future investigations, the authors are continuing to develop a motion capture and surveillance video database from which to begin transitioning idealized inverse kinematic models to more operationally relevant data sources. To enhance the richness of this database, new image processing techniques are being investigated and applied to the collected motion data to provide the clearest images for which this IK-model can be applied. Expanding the IK- model into three dimensions will allow the comparison of this method to other kinematic approaches. Further, by extending this model to the lower extremities, the INSPIRE project can create a robust model for which to identify gait signatures. The goal then is to validate this robust model as an alternative approach to characterizing video-based human motion for security and force protection applications.

5.0 Disclaimer

6.0 Acknowledgements

The authors would like to thank the West Virginia High Technology Consortium and Dr. Charmaine Gilbreath for supporting this research, the Air Force Research Laboratory at Wright Patterson Air Force Base, and the American Society for Engineering Education for supporting Dr. Kendricks' work with this project.

7.0 References

Adams, W., Loustaunau, P., 1994. An Introduction to Groebner Bases. Graduate Studies in Mathematics, 3. American Mathematical Society, USA.

Andriluka, M., Roth, S., Schiele, B, 2008. People-tracking-by-detection and people-detection-by-tracking. IEEE Conference on Computer Vision and Pattern Recognition, 1-8.

Buchberger, B., 1985. Groebner Bases: An algorithmic method in polynomial ideal theory. Multidimensional Systems Theory. Reidel Publishing Company, Dodrecht.

Cox, D., Little, J., O'Shea, D., 1992. Ideals, Varieties, and Algorithms: An Introduction to Computational Algebraic Geometry and Commutative Algebra. Springer- Verlag, New York Inc.

Fullenkamp, A.M., 2009. Point-Light Visualization Developer, Version 1.0 User Guide. Technical Report, U.S. Air Force Research Laboratory, Wright-Patterson Air Force Base, OH.

Kendricks, Kimberly D., 2007. Solving the Inverse Kinematic Robotics Problem: A Comparison Study of the Denavit-Hartenberg Matrix and Groebner Basis Theory. PhD. Thesis, Auburn University Libraries, Auburn, Alabama.

Meredith, M., Maddock, S., 2007. Approximating character biomechanics with real-time weighted inverse kinematics. Computer Animation and Virtual Worlds 18, 349-359.

Mihelj, M., 2006. Inverse kinematics of human arm based on multisensory data integration. Journal of Intelligent and Robotic Systems 47, 139-153.

Niu, W., Long, J., Han, D., Wang, Y., 2004. Human activity detection and recognition for video surveillance. IEEE International Conference on Multimedia and Expo 1, 719-722.

Pavan, T., Chellappa, R., Subrahmanian, V.S., Udrea, O., 2008. Machine recognition of human activities: a survey. IEEE Transactions on Circuits and Systems for Video Technology 18(11), 1473-1488.

Roth, B., 1994. Advances in Robot Kinematics. Advances in Robot Kinematics and Computational Geometry. Kaluwer Academic Publishers, Netherlands, 7-16.

Stifter, S., 1994. Algebraic methods for computing inverse kinematics. Journal of Intelligent and Robotic Systems. Springer, Netherlands 11(1-2), 79-89.

Wang, X., 1999. A behavior-based inverse kinematics algorithm to predict arm prehension postures for computer-aided ergonomic evaluation. Journal of Biomechanics 32, 453-460.