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COOPERATIVE ADVERTISING: A BIFORM GAME ANALYSIS

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INTRODUCTION

Cooperative advertising is one of the most important issues in the analysis of supply chain relationships. It is defined as an interactive relationship between two members in a manufacturer-retailer supply chain (Li et al., 2002). Within this relationship the parts have different objectives. The manufacturer invests in national brand advertising in order to promote its product and create a "brand equity" while the retailer invests in local advertising in order to increase the local demand (Xie and Neyret, 2008). Traditionally, cooperative advertising is achieved when the upstream party shares part of the downstream party´s advertising costs. The cooperative advertising has been also analyzed from a game theoretical point of view (Yue et al., 2006). These authors found out that if the upstream and downstream firms cooperate, they achieve cooperative advertising. They get this result, on a cooperative model, assuming that the total benefit of the supply chain is the total sum of the individual benefits.

Here we want to analyze if the manufacturer and the retailer can achieve cooperative advertising using a biform game model, where the cooperative stage will be designed following Von Neumann-Morgenstern Cooperative Game Theory (1944). It is also interesting to know if the supplier will share the advertising costs, once we assume specific investments and incomplete contracts and that he is the only one who makes specific investments.

To answer these questions we develop a model of cooperative advertising using a biform game model with three agents. The model is designed as such a way that the agents decide prices and quantities in the cooperative game stage. In the non-cooperative game stage, the downstream parties decide if they are going to invest in advertising while the supplier decides if he is going to share the advertising costs.

The contribution of this model is twofold: first, a mathematical contribution where we apply biform games using the maximin (see Marchi, E., 1967) approach to compute the characteristic functions of the model in the cooperative stage. In particular, we determine the expected benefits of the cooperative stage through the Shapley Value. Then, we calculate the expected benefits to the different player´s advertising strategies of the non cooperative stage. We proceed to solve the non-cooperative game with Nash equilibrium. Furthermore, we also consider a mathematical contribution using the von Neumann-Morgenstern approach in the biform games because, in this way, we are assuming how the players behave outside the coalitions. Our second contribution is an economic one. We show that the combination of the cooperative and non cooperative scenarios in a context of cooperative advertising theory implies that the supplier will share the advertising costs, achieve a Pareto optimum solution and generate higher benefits for the whole upstream-downstream chain. The upstream party will use its bigger bargaining power through the sharing costs to motivate sales at the retailer level and make the brand known in order to increase the total benefits for all the players. So, we get cooperative advertising as an endogenous result.

This paper proceeds as follows: in section 1, we present the literature review, while in section 2; the economic environment of the model is described. We develop a biform game model under incomplete contracts and specific investments framework. The core and the Shapley value are the solutions for the cooperative stage and the Nash equilibrum, the solution for the non-cooperative stage. Finally, in section 3, we present the conclusions of the model, followed by a description of some limitations and future researchs.

1- LITERATURE REVIEW

Models of supply chain relations are one of the most important topics in the management research literature. They deal with pricing, purchaising, production and inventories (Xie and Neyret, 2008). In pricing, or in determination of the wholesale prices, Jeuland and Shugan (1983, 1998) and Moorthy (1987) use quantity-discount price mechanisms and two part tariffs to achieve coordination.

Cachon and Lariviere (2005) explores different types of supply chain revenue -sharing contracts where the retailer pays the supplier a wholesale price for each unit purchased plus a percentage of the revenue generated by the retailer. These authors demonstrate that revenue-sharing contracts with one retailer coordinate the supply chain and the benefits generated by the retailer and the supplier. They also show that revenue sharing coordintates the supply chain when the retailers compete in quantities, for example the Cournot competition.

Other important issue in the analysis of models of supply chain relations is cooperative advertising. Cooperative advertising is defined as an interactive relationship between two members in a manufacturer-retailer supply chain (Li et al., 2002). It is usually analized in two member models where there is an uspstream party (supplier/manufacturer) who invests in national brand advertising in order to promote its product and to create a “brand equity”, and a downstream party, a retailer, who invests in local advertising in order to increase the local demand (Xie and Neyret, 2008). Traditionally, cooperative advertising is achieved when the upstream party shares part of the downstream party´s advertising costs. This sharing cost is the manufacturer´s participation rate (Bergen & John, 1997). The upstream party uses the cooperative advertisement to motivate sales at the retailer level and national advertising to capture potencial consumers in order to make the brand known.

Yue et al. (2006) present a paper where the cooperative advertising can be seen from a game theoretical point of view. They pose that in a two member´s model, cooperative advertising can be seen as cooperative or non-cooperative relationships. On the one hand, in the non-cooperative point of view, the party who has a manipulative power in the chain controls the other party, who becomes the follower. It is defined as a Stackelberg´s relation where there is a leader (the manufacturer), who plays first, and there is a follower (the retailer), who plays second (Gaski, 1984; Munson and Rosenblatt, 2001). It is also known as vertical cooperative advertising. One example of this case is Mc Donalds imposing strong control over its franchises (Love, 1986). On the other hand, in the cooperative point of view, the upstream and downstream parties cooperate to each other in order to determine prices, quantities and advertising strategies with the objective to achieve higher benefits for the whole chain. One example is the case of Wal Mart and Procter & Gamble explained by Huang et al (2002) paper. These authors explained that in Wal Mart earlydays, Procter & Gamble acted as the leader indicating Wal Mart the quantity to sell it, the prices of its product and the terms of the negotiation. There where no information sharing, no joint planning and no system coordination. However, Wal Mart has grown to the point where its revenues where three times those of P&G, this Wal Mart´s growth impulsed the creation of a partnership and full coordination between WM and P&G that had increased the benefits for both parties. As Huang et al. explained, through coordination, Wal Mart and P&G have turned a win-lose situation into a win-win situation of reduced costs and increased revenues for both partners.

There is a new tendence in cooperative advertising: the leading power is shifting from suppliers (upstream parties) to retailers (downstream parties) (Huang and Li, 2000), as the case of Wal Mart and P&G commented above. Huang and Li developed three models comparing: the classical leader-follower advertising model, an advertising model relaxing the structure of leader-follower and a fully cooperative advertising model. The result of these authors is that the fully cooperative advertising model achieves a Pareto optimum, so it is better for the parties to cooperate efficiently.

In summary, the study of cooperative advertising has been analized through two-agent models under non-cooperative scenarios (Stackelberg) and coordinated scenarios, where the agents cooperate maximizing the total surplus generated by both (supplier and retailer). This study makes us wonder what happens if we introduce another downstream party and analyze the cooperative advertising through biform games that combines cooperative and non-cooperative stages, where the cooperative stage is going to be model through the cooperative game theory created by Morgenstern and Von Neumann (1944). It is also interesting to know what happens if we introduce the cooperative game to determine the prices and quantities, given that the supplier has a greater bargaining power and makes specific investments, and how this supplier´s bigger bargaining power under incomplete contracts, will influence in the advertising strategy decisions in a non cooperative scenario.

To answer these questions we develop a model of cooperative advertising using a biform game model with three agents, where they will decide prices, quantities and advertising strategies under incomplete contracts and specific investments. In the cooperative game stage, the agents will decide the prices and the quantities and in the non-cooperative game stage the downstream parties will decide if they are going to invest in advertising and the supplier will decide if he is going to share the advertsing costs made by the downstream parties, given that he has a higher bargaining power.

We proceed now to mention briefly some important features of the Biform Games, before describing the model.

1.1- BIFORM GAMES

Biform game models are a new theoretical concept created by Brandenburger and Stuart in 1993. A biform game model is a hybrid of cooperative and noncooperative game model designed for modeling business interactions (Stuart, 2005). It is a strict generalization of both: the strategic-form noncooperative and Transferable Utility (TU) cooperative game models (Brandenburger and Stuart, 1993).

The biform game models include two stages: a first non cooperative stage where players take their strategic decisions that are determined collectively in a second cooperative stage. Brandenburger and Stuart (2003) mentioned some examples of biform games as: in the first stage, the player might decide the capacity to install and in the second cooperative stage, the players bargain over the division of the economic value that their first-stage choices have created. These authors explained that biform game models are quite similar to two-stage models but the difference lies in the inclusion of the cooperative game theoretic structure in the second stage. They proposed as a solution of the cooperative game the Core solution, although they explained that different types of solution can be applied depending on the context of the research. For example Hart and Moore (1990) used the Shapley value because they modeled agreements among players who jointly create some value, meanwhile Grossman and Hart (1986) used Nash bargaining solution for bilateral agreements in the second stage (Brandenburger and Stuart, 2003).

These authors believe that biform models overcome some of the limitations that cooperative theory has. As they posed, one of the problems of the cooperative game theory is handling of the situations where the value created by a subset S of players - $v(S)$ - may depend on what the players outside do. This concept was created by von Neumamm and Morgenstern (1944). They made a specific assumption of how the players outside S behaved. They assumed: the minimax behavior. Von Neumann and Morgenstern posed that the players in S assumed the worst about the players outside S.

The biform game models avoid this limitation by “positing that each strategy profile in the first stage non-cooperative game leads to a different, and independently specified second-stage cooperative game”. The second difficulty that the authors mentioned in cooperative theory is the presumption of efficiency: all potential value is created. On the contrary, the biform games permit inefficiency. Brandenburger and Stuart said that true efficiency is created in the second cooperative stage but this efficiency is related to the strategic choices already made in the first non cooperative stage. They posed that “Overall efficiency would demand that the first stage strategic choices maximize the overall value created in the ensuing second-stage game”. Brandenburger and Stuart (1996) defined the overall value created equal to the willingness to pay by buyers minus the opportunity cost of the seller (to sell quantities, to install capacity, etc). They also defined the added value of the players,.when there are many players, as the difference between the value created by all players minus the value created by all other players (except the one in question).

These authors pointed out that their treatment of biform game models employs three conditions: 1) *Adding Up (AU)*: that implies, in the second cooperative stage, that the sum of the marginal contribution of the players equals the overall value created; 2) *No Externality (NE)*: which means that each player´s strategic choice does not affect the value that the remaining players can create (without that player) and 3) *No Coordination (NC)*: which means when one player switches strategy, the sign of the effect on the overall value created is independent of the other player´s strategic choices. They also included, in the biform game definition, the use of confidence indexes. The confidence index measures the player´s optimistic or pesimistic view in capturing most or little value that is to be divided in the residual bargaining. This confidence index takes values between 0 and 1. When the confidence index is near or equal to 1, the player´s view is optimistic, otherwise when the confidence index is near or equal to 0, the player´s view is pesimistic. The authors gave some examples where one or all of the conditions mentioned above are not satisfied. They also showed, in a monopolist´s case, that the AU condition is not satisfied but the Nash equilibrium is efficient in the first stage if the seller can anticipate capturing most of the value that it will be divided in the residual bargaining. It is worthy to clarify the confidence indexes are usefull when the core projections are given by intervals of imputations but not in the case that the core solution is a point or whether the players have singleton strategies. It is also important to remark that the confidence indexes are subjective and this is the reason why the authors do not insist in Nash equilibrium as the only non cooperative solution of the game. They also point out that other (non-cooperative) solution concepts are quite compatible with their approach, for example dominant strategies. They explained that for achieving a Nash equilibrium it is sufficient that each player is assumed to be rational and assigns probability one to the actual strategy choices of other players.

Now, we describe the economic environment of our model.

2- THE ECONOMIC ENVIRONMENT

This model is developed under a biform game framework and the presence of incomplete contracts and specific investments. A biform game model is a hybrid of cooperative and noncooperative game model designed for modeling business interactions (Stuart, 2005). Players take their decisions of buying, producing and advertising depending on their expected benefits. To compute those expected benefits, we have chosen the Shapley value solution for the cooperative second stage because it takes into account what each player could reasonably get before the game starts, and the Nash solution for the non cooperative first stage.

More specifically, we consider a model with three agents, two downstream parties and one upstream party. The upstream party is a supply firm which sells its product to the downstream parties. The supplier has to make specific investments to obtain the product q, which has no alternative use out of these relationships. We assume the existence of a parameter, λ, that measures the degree of investment specificity, where $\lambda \in [0,1]$. The higher the value of λ, the more specific the investment. If $\lambda = 1$, the investment is fully specific, while a value of $\lambda = 0$ indicates that the investment becomes totally general. Furthermore, this product q requires a quality process that is not observable to third parties and, therefore, it is not possible to write down an enforceable contract. The supplier gets this q at a cost λc_s.

The downstream parties buy the product from the supplier and they sell it in different markets. Each firm is a monopoly in its market. These downstream firms have incentives to merge or to be vertically integrated with the supplier because this would ensure them the required quality. So, we can say the downstream firms compete for the supplier giving him bargaining power, in the sense that both downstream firms need the supplier. In this model, the bargaining power responds to the existence of specific investments. The downtream parties have incentives to invest in advertising in order to increase the demand of q and sell more quantity while the supplier has to decide if he is going to share the advertsing costs in order to make the brand known.

As a result, the incentive problem is twofold: on the one hand, the downstream firms want the supplier to produce under a given quality process and to share the advertising costs. On the other hand, the supplier, after having made the specific investments, has to decide if he will share the advertising costs facing the possibility that the downstream firms may argue that the product has not reached the required quality and, consequently, they will pay less for the product or, in the worst case, they will refuse to buy it.

We consider two-stage supply transaction. The first stage is a non cooperative one, where the downstream parties will decide if they are going to invest in advertising and the suplier will decide if he is going to share the advertising costs, given the expected benefits determined in the second cooperative stage. In the second cooperative stage the downstream firm 1 buys q_1 to the supplier and the second firm buys q_2, such that $q_1 + q_2 = q$ and $q_1, q_2 > 0$. The supplier has to make specific investments to produce these quantities at a cost $\lambda c_{s1}(1+h_1(\mu_1)), \lambda c_{s2}(1+h_2(\mu_2))$, where $(1+h_1(\mu_1))\, and (1+h_2(\mu_2))$ are the sharing costs of advertising. The parameters μ_1, μ_2 take value 0 or 1, when they take value 0, it means that the supplier does not share the advertising cost, the contrary when they take value 1. The supplier sells the quantities at prices w_1, w_2, being $w_1, w_2 \geq 0$. The downstream parties buy the quantities at prices w_1, w_2 and sell them at p_1, p_2, respectively. The prices p_1, p_2 are positive and they are decided by each firm. The downstream firm 1 faces a demand[1] function given by $q_1 = f_1(p_1, \alpha_1) = F_1(\alpha_1)\beta_1 - \gamma_1 p_1 G_1(\alpha_1)$ and the downstream firm 2 faces the demand function $q_2 = f_2(p_2, \alpha_2) = F_2(\alpha_2)\beta_2 - \gamma_2 p_2 G_2(\alpha_2)$, where $\alpha_1 \, and \, \alpha_2$ represent the

[1] For simplicity, it is assumed the demand function is known by the downstream parties. We know this is a strong assumption but if we assume expected demand functions, our results will remain unchanged.

decision to invest in advertising. The parameters α_1 *and* α_2 take the value 0 or 1. When α_i, where i=1,2, takes value 0 means that the downstream party decide not to invest in advertising, meanwhile when it takes value 1 implies that the downstream party invest in advertising. F_1, F_2, G_1, G_2 are positive functions that measure the increase/decrease in the demand function respect to the advertising decisions.

For simplicity, we assume linear demand functions. Additionally, to guarantee that the inverse demand functions exist and they are well defined, we further assume that:

1. the demand functions have negative slopes
2. f_1, f_2 are differentiable functions whose first derivatives are strictly negative and finite for any $p_1 \in \left[0, \bar{p}_1\right]$ and $p_2 \in \left[0, \bar{p}_2\right]$ such that $f_1, f_2 > 0$.

Given the demand functions, $w_1 \in \left[0, \frac{F_1(\alpha_1)}{G_1(\alpha_1)}\right]$ and $w_2 \in \left[0, \frac{F_2(\alpha_2)}{G_2(\alpha_2)}\right]$ and $p_1 \in \left[0, \frac{F_1(\alpha_1)}{G_1(\alpha_1)}\right]$, $p_2 \in \left[0, \frac{F_2(\alpha_2)}{G_2(\alpha_2)}\right]$. We also assume that $0 \le w_1 \le p_1 \le \frac{F_1(\alpha_1)\beta_1}{G_1(\alpha_1)\gamma_1}$ and $0 \le w_2 \le p_2 \le \frac{F_2(\alpha_2)\beta_2}{G_2(\alpha_2)\gamma_2}$

The players of the model have transferable utilities and they are rational, so they take decisions that maximize their expected utilities. The respective utility functions are: $U_s = U(\Pi_s) + \varepsilon_s$ for the supplier, $U_1 = U(\Pi_1) + \varepsilon_1$ for the downstream firm 1 and $U_2 = U(\Pi_2) + \varepsilon_2$ for the downstream firm 2. Furthermore, $\varepsilon_s, \varepsilon_{1,} \varepsilon_2$ are random variables that follow a normal distribution with $E(\varepsilon_s) = E(\varepsilon_1) = E(\varepsilon_2) = 0$ and variances $\sigma_s{}^2, \sigma_1{}^2, \sigma_2{}^2$, respectively. For simplicity, we also assume the different players are risk neutral.

Finally, the players' expected benefits are given by the following equations.

$\Pi_1 = R_1(q_1, p_1) - w_1 q_1$, for the downstream firm 1, where the revenue is equal to $R_1(q_1, p_1) = p_1 * q_1$

$\Pi_2 = R_2(q_2, p_2) - w_1 q_2$, for the downstream firm 2, where $R_2(q_2, p_2) = p_2 * q_2$, and

$$\Pi_3 = q_1 w_1 + q_2 w_2 - \lambda c_{s1} q_1 (1 + h_1(\mu_1)) - \lambda c_{s2} q_2 (1 + h_2(\mu_2)),$$ for the supplier.

2.1 -THE TIMING

As we mentioned earlier, the players decide to buy, produce and advertise in terms of their expected benefits. The timing of the model becomes as follows:

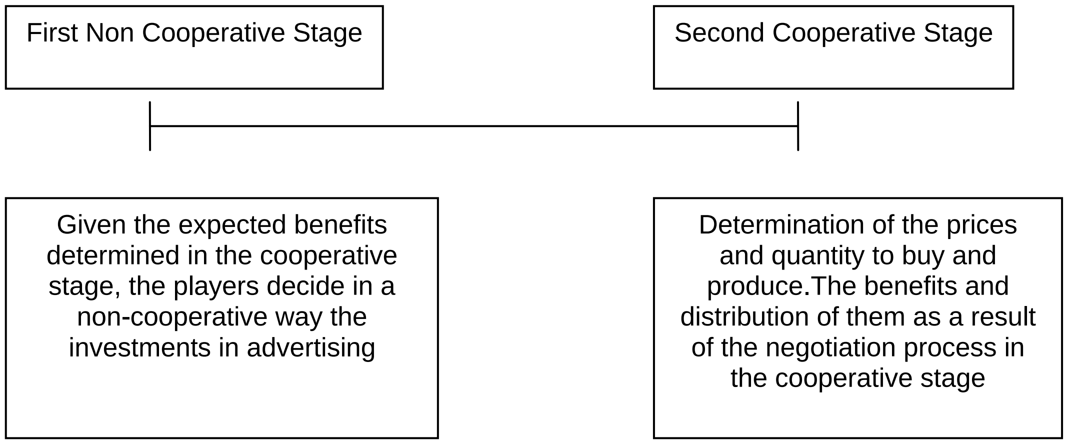

Figure 1: Timing of the Model

At second cooperative stage the downstream firms decide the quantity of product they will buy and the prices they are going to sell it. The supplier decides to produce this level of product, along with the quality and the price he will charge to the downstream parties. They are going to estimate the expected benefits given the advertising strategies of the first stage. At first stage, once the expected benefits are determined, the downstream parties will decide if investing in advertising and the supplier will decide if sharing the advertising costs in a non cooperative way. Due to the existence of incomplete information and incomplete contracts framework, the players decide output level and their prices as well as the investment in advertising as a function of their expected future benefits and their distribution.

2. 2- BENCHMARK

Under perfect information and complete contracts, we compute the first best. To do this, we assumed there is one player who has perfect information and is the owner of the assets. The firms act as one player who maximizes the total expected benefits (Π_T). The equation of the total expected benefit is:

$\Pi_T = p_1 q_1 - \lambda c_{s1}(1 + h_1(\mu_1))q_1 + p_2 q_2 - \lambda c_{s2}(1 + h_2(\mu_2))q_2$, being the demand functions $q_1 = f_1(p_1, \alpha_1) = F_1(\alpha_1)\beta_1 - \gamma_1 G_1(\alpha_1)p_1$; $q_2 = f_2(p_2, \alpha_2) = F_2(\alpha_2)\beta_2 - \gamma_2 G_2(\alpha_2)p_2$.

To assure the presence of a maximum, it must be satisfied that

$$f_x = f_y = 0$$

$$f^2{}_{xx} f^2{}_{yy} - f^2{}_{xy} > 0$$

$$f^2{}_{xx} < 0$$

So, the maximum is achieved.

$$\frac{\partial \Pi_T}{\partial p_1} = F_1(\alpha_1)\beta_1 - 2\gamma_1 G_1(\alpha_1)p_1 + G_1(\alpha_1)\lambda c_{s1}(1 + h_1(\mu_1))\gamma_1 = 0$$

$$\frac{\partial \Pi_T}{\partial p_2} = F_2(\alpha_2)\beta_2 - 2\gamma_2 G_2(\alpha_2)p_2 + G_2(\alpha_2)\lambda c_{s2}(1 + h_2(\mu_2))\gamma_2 = 0$$

From these derivatives, we get the prices and quantities that maximize the total net profit.

$$p_2 = \frac{F_2(\alpha_2)\beta_2 + G_2(\alpha_2)\lambda c_{s2}(1 + h_2(\mu_2))\gamma_2}{2\gamma_2 G_2(\alpha_2)}$$

$$q_2 = \frac{F_2(\alpha_2)\beta_2 - G_2(\alpha_2)\lambda c_{s2}(1 + h_2(\mu_2))\gamma_2}{2}$$

$$p_1 = \frac{F_1(\alpha_1)\beta_1 + G_1(\alpha_1)\lambda c_{s1}(1 + h_1(\mu_1))\gamma_1}{2\gamma_1 G_1(\alpha_1)}$$

$$q_1 = \frac{F_1(\alpha_1)\beta_1 - G_1(\alpha_1)\lambda c_{s1}(1 + h_1(\mu_1))\gamma_1}{2}$$

The total benefit in the benchmark case becomes:

$$\Pi_T = \frac{(F_1(\alpha_1)\beta_1 - G_1(\alpha_1)\lambda c_{s1}(1 + h_1(\mu_1))\gamma_1)^2}{4\gamma_1 G_1(\alpha_1)} + \frac{(F_2(\alpha_2)\beta_2 - G_2(\alpha_2)\lambda c_{s2}(1 + h_2(\mu_2))\gamma_2)^2}{4\gamma_2 G_2(\alpha_2)}$$

Next, we will find the $\alpha_1, \alpha_2, \mu_1, \mu_2$ that maximize Π_T. To do this, we are going to assume in this model that total benefits with advertising are higher than without advertising[2] for all players. So, the following conditions must be satisfied:

$$\begin{aligned}
&\frac{F_1(1)}{G_1(1)} > \frac{F_1(0)}{G_1(0)} \\
&\frac{F_2(1)}{G_2(1)} > \frac{F_2(0)}{G_2(0)} \\
&h_1(1) > h_1(0) \\
&h_2(1) > h_2(0)
\end{aligned} \qquad \text{(A)}$$

Under these conditions the total benefit is equal to:

$$\Pi_T = \frac{\left(F_1(1)\beta_1 - G_1(1)\lambda c_{s1}(1 + h_1(1))\gamma_1\right)^2}{4\gamma_1 G_1(1)} + \frac{\left(F_2(1)\beta_2 - G_2(1)\lambda c_{s2}(1 + h_2(1))\gamma_2\right)^2}{4\gamma_2 G_2(1)}$$

We compare this benchmark solution with the results obtained under incomplete contracts and a biform game framework.

2. 3 - INCOMPLETE CONTRACTS AND BIFORM GAME MODEL

Before developing the model, the biform game is defined (Brandenburger and Stuart, 1993). Fix a set of n players, to be indexed by $i = 1, \ldots\ldots, n$, and for each player i, a finite set A_i of strategies. Let $N = \{1, \ldots\ldots, n\}$, and let $A = A_1 \times \ldots\ldots \times A_n$. Consider:

a) a map V from A to the set of maps from 2^N to $\Re$, where $V(a)(\phi) = 0$ for every $a \in A$;

b) for each player i, a number $\alpha_i \in [0,1]$.

An *n-person biform game* is then a collection $(A_1, \ldots\ldots\ldots A_n; V; \alpha_1, \ldots\ldots\ldots \alpha_n)$

The biform game includes two stages: a cooperative and a non cooperative stage. These models are solved starting for the second cooperative stage in order to estimate the future benefits given the non cooperative strategies of the first stage, then, in the first stage, each player decide its strategies in a non cooperative way according to the expected benefits determined in the previuos cooperative stage.

[2] It can be also assumed that benefits with advertising are fewer than without advertising, we will have negative advertising strategies in that case. It can also be assumed combinations of advertising and negative advertising.

In our model the decisions taken in the second cooperative stage will depend on the non cooperative strategies of the first stage. For the cooperative stage we are going to use the Shapley Value solution. So, we get the baricenter of the core which is one point, therefor we won´t need to use confidex indexes. The solution proposed for the non cooperative stage will be the Nash equilibrium. In our model the three conditions AU, NE and NC are not satisfied because we are using the Von Neumann-Morgenstern cooperative game theory in which the value created by a subset S of players - $v(S)$ - will depend on what the players outside do or behaved. Von Neumann and Morgenstern posed that the players in S assumed the worst about the players outside S.

2.3.1- THE COOPERATIVE STAGE

In the cooperative stage or second stage, the three players decide the quantity to sell it and produce but these decisions will take into account the future benefits they expect to achieve. So, the players have to estimate their future benefits. To do this, we are going to use a cooperative game approach based on the subject of the Theorem of the Minimax created by von Neumann (1928). Under cooperative games, the players negotiate and compute their payoffs under different coalitions. The fact of using the maximin to compute the payoffs implies that, in the negotiation process, the actors estimate the least amount they can get if the other players play against them; in other words, they compute the payoffs in the worst scenarios.

We present the equations of the model and the computation of the characteristic functions.

$$\Pi_1(p_1, p_2, w_1, w_2) = (p_1 - w_1)(F_1(\alpha_1)\beta_1 - \gamma_1 G_1(\alpha_1)p_1) \qquad (1)$$

for downstream firm 1,

$$\Pi_2(p_1, p_2, w_1, w_2) = (p_2 - w_2)(F_2(\alpha_2)\beta_2 - \gamma_2 G_2(\alpha_2)p_2) \qquad (2)$$

for downstream firm 2, and equation (3)

$$\Pi_3(p_1, p_2, w_1, w_2) = (w_1 - \lambda c_{s1}(1 + h_1(\mu_1)))(F_1(\alpha_1)\beta_1 - \gamma_1 G_1(\alpha_1)p_1) + (w_2 - \lambda c_{s2}(1 + h_2(\mu_2)))(F_2(\alpha_2)\beta_2 - \gamma_2 G_2(\alpha_2)p_2)$$

for the supplier. The degree of investment specificity, λ, is high and it could be close to one in this model.

The possible coalitions of the model are:
$v(\phi), v(1), v(2), v(3), v(1,2), v(1,3), v(2,3), v(1,2,3)$.
The computation for the characteristic function for the downstream firm 1 becomes as follows:
First, we take equation (1) and apply the maximin

$$v(\alpha_1, \alpha_2, \mu_1, \mu_2)(1) = \max_{p_1} \min_{p_2, w_1, w_2} \Pi_1(p_1, p_2, w_1, w_2) = \max_{p_1} \min_{p_2, w_1, w_2} [(p_1 - w_1)(F_1(\alpha_1)\beta_1 - \gamma_1 G_1(\alpha_1)p_1)] \tag{4}$$

We must find the minimum values of p_2, w_1, w_2 that minimize this function for a given p_1, and then we will maximize that function with respect to p_1. In our context, the minimum value of w_1 that minimizes this function given p_1 is $w_1 = p_1$, so, we replace this value in the function

$$v(\alpha_1, \alpha_2, \mu_1, \mu_2)(1) = \max_{p_1} [(p_1 - p_1)(F_1(\alpha_1)\beta_1 - \gamma_1 G_1(\alpha_1)p_1)]$$

We get that the characteristic function for the downstream firm 1 becomes:

$$v(\{\alpha_1, \alpha_2, \mu_1, \mu_2\}\{1\}) = v(\alpha_1, \alpha_2, \mu_1, \mu_2)(1) = 0. \tag{5}$$

This is the payoff that the player 1 will have in his worst scenario. It means that if the supplier charges him the highest price that he can: $w_1 = p_1$, which is the highest cost that the downstream firm 1 can face, then the downstream firm gets zero profits.

The steps to compute the characteristic function to the downstream firm 2 are similar to the ones already computed for downstream firm 1. We proceed to simply write down the results and present the equation.

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(2)=\max_{p_2}\min_{p_1,w_1,w_2}\Pi_2(p_1,p_2,w_1,w_2)=$$
$$\max_{p_2}\min_{p_1,w_1,w_2}\left[(p_2-w_2)(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)\right] \quad (6)$$

The characteristic function or the payoff that downstream firm 2 will receive in case that the other players join themselves against him becomes

$$v(\{\alpha_1,\alpha_2,\mu_1,\mu_2\}\{2\})=v(\alpha_1,\alpha_2,\mu_1,\mu_2)(2)=0 \quad (7)$$

Equations (5) and (7) or the characteristic functions for the downstream firm 1 and the downstream firm 2, indicate the lowest value that they are able to get under the worst scenario.

Similarly, for the supplier, the characteristic function becomes:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(3)=\max_{w_1,w_2}\min_{p_1,p_2}\Pi_3(p_1,p_2,w_1,w_2)$$
$$\max_{w_1,w_2}\min_{p_1,p_2}\begin{bmatrix}(w_1-\lambda c_{s1}(1+h_1(\mu_1)))(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)p_1)+\\(w_2-\lambda c_{s2}(1+h_2(\mu_2)))(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)\end{bmatrix}$$

We have to look for the minimum values of p_1, p_2 that minimize the function. These values are:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(3)=\max_{w_1,w_2}\min_{p_1,p_2}\begin{bmatrix}(w_1-\lambda c_{s1}(1+h_1(\mu_1)))\left(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)\left(\frac{F_1(\alpha_1)\beta_1}{G_1(\alpha_1)\gamma_1}\right)\right)+\\(w_2-\lambda c_{s2}(1+h_2(\mu_2)))\left(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)\left(\frac{F_2(\alpha_2)\beta_2}{G_2(\alpha_2)\gamma_2}\right)\right)\end{bmatrix}$$
$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(3)=\max_{w_1,w_2}\left[(w_1-\lambda c_{s1}(1+h_1(\mu_1)))(0)+(w_2-\lambda c_{s2}(1+h_2(\mu_2)))(0)\right]$$
$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(3)=\max_{w_1,w_2}[0]$$

The characteristic function for the supplier is:

$$v(\{\alpha_1,\alpha_2,\mu_1,\mu_2\}\{3\})=v(\alpha_1,\alpha_2,\mu_1,\mu_2)(3)=0 \quad (8)$$

The equation (8) is telling us the value that, in the worst case, the supplier gets. We can think that this is the case that the downstream firms join together and argue that the quality is not the one required and they do not want the product. The supplier´s payoff is zero because the supplier cannot force the downstream parties to buy the product.

The characteristic function for the coalition $v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,2)$ is obtained considering the sum of $\Pi_1+\Pi_2$ given the non cooperative strategies of the first stage. Thus, the coalition $\{1,2\}$ gets the payoff of both players together.

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,2)=\max_{p_1,p_2}\min_{w_1,w_2}\left[\Pi_1(p_1,p_2,w_1,w_2)+\Pi_2(p_1,p_2,w_1,w_2)\right]=$$
$$\max_{p_1,p_2}\min_{w_1,w_2}\left[(p_1-w_1)(F_1(\alpha_1)\beta_1-\gamma_1G_1(\alpha_1)p_1)+(p_2-w_2)(F_2(\alpha_2)\beta_2-\gamma_2G_2(\alpha_2)p_2)\right]$$

Looking for the minimum values of w_1,w_2 that minimize the function and replacing them, we obtain (9):

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,2)=\max_{p_1,p_2}\min_{w_1,w_2}\left[\Pi_1(p_1,p_2,w_1,w_2)+\Pi_2(p_1,p_2,w_1,w_2)\right]=$$
$$\max_{p_1,p_2}\min_{w_1,w_2}\left[(p_1-p_1)(F_1(\alpha_1)\beta_1-\gamma_1G_1(\alpha_1)p_1)+(p_2-p_2)(F_2(\alpha_2)\beta_2-\gamma_2G_2(\alpha_2)p_2)\right]$$

The characteristic function for coalition (1, 2) is the following:

$$v(\{\alpha_1,\alpha_2,\mu_1,\mu_2\}\{1,2\})=v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,2)=0 \qquad (10)$$

This is the payoff that will get the coalition between the downstream parties in the worst scenario where it is assumed the supplier is against them. The supplier will charge them the highest price. The benefits they get are equal to zero in the worst condition. This equation points out the fact that if the downstream firms cooperate with each other, without any consideration of the third player (the supplier), they will be able to get 0. They cannot force the supplier to invest and sell them the product.

The characteristic function for the coalition $v(1,3)$ is:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,3)=\max_{p_1,w_1,w_2}\min_{p_2}\left[\Pi_1(p_1,p_2,w_1,w_2)+\Pi_3(p_1,p_2,w_1,w_2)\right]=$$

$$\max_{p_1,w_1,w_2}\min_{p_2}\begin{bmatrix}(p_1-\lambda c_{s1}(1+h_1(\mu_1)))(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)p_1)+\\(w_2-\lambda c_{s2}(1+h_2(\mu_2)))(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)\end{bmatrix}$$

Looking for the minimum value of p_2 that minimizes the function, we get:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,3)=$$

$$\max_{p_1,w_1,w_2}\min_{p_2}\begin{bmatrix}(p_1-\lambda c_{s1}(1+h_1(\mu_1)))(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)p_1)+\\(w_2-\lambda c_{s2}(1+h_2(\mu_2)))\left(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)\left(\frac{F_2(\alpha_2)\beta_2}{G_2(\alpha_2)\gamma_2}\right)\right)\end{bmatrix}$$

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,3)=\max_{p_1,w_1,w_2}\left[(p_1-\lambda c_{s1}(1+h_1(\mu_1)))(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)p_1)\right] \qquad (11)$$

Now, we have to find the value of p_1 that maximizes (11)

$$\frac{\partial[\Pi_1+\Pi_3]}{\partial p_1}=F_1(\alpha_1)\beta_1-2\gamma_1 G_1(\alpha_1)p_1+\lambda c_{s1}(1+h_1(\mu_1))\gamma_1=0$$

$$\frac{\partial^2[\Pi_1+\Pi_3]}{\partial p_1{}^2}=-2\gamma_1 G_1(\alpha_1)$$

$$p_1=\frac{F_1(\alpha_1)\beta_1+\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1}{2\gamma_1 G_1(\alpha_1)}$$

$$q_1=\frac{F_1(\alpha_1)\beta_1-\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1}{2}$$

Replacing these values in the equation (11), we get the characteristic function

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,3)=\frac{(F_1(\alpha_1)\beta_1-\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1)^2}{4\gamma_1 G_1(\alpha_1)} \qquad (12)$$

This is the payoff that the coalition between the downstream firm 1 and the supplier can get if the downstream firm 2 is assumed to oppose them. The supplier will produce $q_1=\frac{F_1(\alpha_1)\beta_1-\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1}{2}$ and the downstream will sell it at the price $p_1=\frac{F_1(\alpha_1)\beta_1+\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1}{2\gamma_1 G_1(\alpha_1)}$. The characteristic function implies that, under any circumstances, the downstream firm 1 and the supplier, together, are sure to obtain the least amount given by this equation. This result is the same as if the supplier and the downstream firm 1 were vertically integrated. It is necessary to note that this coalition makes sense if the investments are specific because, if the investments were general, the players neither need to form coalitions nor to be vertically integrated.

Doing the same for the coalition between the downstream firm 2 and the supplier, we get:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(2,3)=\max_{p_2,w_1,w_2}\min_{p_1}\left[\Pi_2(p_1,p_2,w_1,w_2)+\Pi_3(p_1,p_2,w_1,w_2)\right]=$$

$$\max_{p_2,w_1,w_2}\min_{p_1}\begin{bmatrix}(p_2-\lambda c_{s2}(1+h_2(\mu_2)))(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)+\\(w_1-\lambda c_{s1}(1+h_1(\mu_1)))(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)p_1)\end{bmatrix}$$

Looking for the minimum value of p_1 that minimizes the function, we get:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(2,3)=\max_{p_2,w_1,w_2}\min_{p_1}\left[\Pi_2(p_1,p_2,w_1,w_2)+\Pi_3(p_1,p_2,w_1,w_2)\right]=$$

$$\max_{p_2,w_1,w_2}\min_{p_1}\begin{bmatrix}(p_2-\lambda c_{s2}(1+h_2(\mu_2)))(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)+\\(w_1-\lambda c_{s1}(1+h_1(\mu_1)))\left(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)\left(\dfrac{F_1(\alpha_1)\beta_1}{G_1(\alpha_1)\gamma_1}\right)\right)\end{bmatrix}$$

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(2,3)=\max_{p_1,w_1,w_2}\left[(p_2-\lambda c_{s2}(1+h_2(\mu_2)))(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)\right] \quad (13)$$

Now, we have to find the value of p_2 that maximizes (13)

$$\frac{\partial[\Pi_2+\Pi_3]}{\partial p_2}=F_2(\alpha_2)\beta_2-2\gamma_2 G_2(\alpha_2)p_2+\lambda c_{s2}(1+h_2(\mu_2))\gamma_2=0$$

$$\frac{\partial^2[\Pi_2+\Pi_3]}{\partial p_2{}^2}=-2\gamma_2 G_2(\alpha_2)$$

$$p_2=\frac{F_2(\alpha_2)\beta_2+\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2}{2\gamma_2 G_2(\alpha_2)}$$

$$q_2=\frac{F_2(\alpha_2)\beta_2-\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2}{2}$$

Replacing these values in the equation (13), we get the characteristic function

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(2,3)=\frac{\left(F_2(\alpha_2)\beta_2-\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2\right)^2}{4\gamma_2 G_2(\alpha_2)} \qquad (14)$$

This is the payoff that the coalition between the downstream firm 2 and the supplier can get if the downstream firm 1 acts against them. The supplier will produce $q_2=\frac{F_2(\alpha_2)\beta_2-\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2}{2}$ and the downstream will sell it at the price $p_2=\frac{F_2(\alpha_2)\beta_2+\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2}{2\gamma_2 G_2(\alpha_2)}$. The characteristic function implies that, under any circumstances, the downstream firm 2 and the supplier, together, are sure to obtain the least amount given by this equation. As in the previous case, this result is the same as if the supplier and the downstream firm 2 were vertically integrated.

The characteristic function for the coalition $(1,2,3)$ is given by:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,2,3)=\max_{p_1,p_2,w_1,w_2}\left[\Pi_1(p_1,p_2,w_1,w_2)+\Pi_2(p_1,p_2,w_1,w_2)+\Pi_3(p_1,p_2,w_1,w_2)\right]=$$

$$\max_{p_1,p_2,w_1,w_2}\begin{bmatrix}(p_1-\lambda c_{s1}(1+h_1(\mu_1)))(F_1(\alpha_1)\beta_1-\gamma_1 G_1(\alpha_1)p_1)+\\(p_2-\lambda c_{s2}(1+h_2(\mu_2)))(F_2(\alpha_2)\beta_2-\gamma_2 G_2(\alpha_2)p_2)\end{bmatrix}$$

And now we must find the values p_1,p_2 that maximize the function.

$$\frac{\partial[\Pi_1+\Pi_2+\Pi_3]}{\partial p_1}=F_1(\alpha_1)\beta_1-2\gamma_1 G_1(\alpha_1)p_1+\lambda c_{s1}(1+h_1(\mu_1))\gamma_1=0$$

$$\frac{\partial^2[\Pi_1+\Pi_2+\Pi_3]}{\partial p_1{}^2}=-2\gamma_1 G_1(\alpha_1)$$

$$p_1=\frac{F_1(\alpha_1)\beta_1+\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1}{2\gamma_1 G_1(\alpha_1)}$$

$$q_1=\frac{F_1(\alpha_1)\beta_1-\lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1}{2}$$

$$\frac{\partial[\Pi_2+\Pi_2+\Pi_3]}{\partial p_2}=F_2(\alpha_2)\beta_2-2\gamma_2 G_2(\alpha_2)p_2+\lambda c_{s2}(1+h_2(\mu_2))\gamma_2=0$$

$$\frac{\partial^2[\Pi_2+\Pi_2+\Pi_3]}{\partial p_2{}^2}=-2\gamma_2 G_2(\alpha_2)$$

$$p_2=\frac{F_2(\alpha_2)\beta_2+\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2}{2\gamma_2 G_2(\alpha_2)}$$

$$q_2=\frac{F_2(\alpha_2)\beta_2-\lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2}{2}$$

Replacing these values in the equation we get that:

$$v(\alpha_1,\alpha_2,\mu_1,\mu_2)(1,2,3)=\frac{(F_1(\alpha_1)\beta_1-G_1(\alpha_1)\lambda c_{s1}(1+h_1(\mu_1))\gamma_1)^2}{4\gamma_1 G_1(\alpha_1)}+\frac{(F_2(\alpha_2)\beta_2-G_2(\alpha_2)\lambda c_{s2}(1+h_2(\mu_2))\gamma_2)^2}{4\gamma_2 G_2(\alpha_2)} \quad (15)$$

This expression is the characteristic function of the grand coalition. This is the maximum payoff that the grand coalition, or total coalition, can achieve if they decide to cooperate with each other.

Once we have computed the characteristic function for all the coalitions in the cooperative stage we are going to solve this cooperative stage using the Shapley value. The Shapely value is the baricenter of the core if the core is non-empty. One way to verify the non-emptiness of the core is demonstrating that the game is convex. To do this we have to satisfy the following condition (Shapley, 1971):

$v(S\cup T)+v(S\cap T)\geq v(S)+v(T)$ for all S and T[3] subsets of $\{1,2,3\}$

In our model, the following expressions are satisfied

[3]Shapley, Lloyd, 1971. Cores of convex games. International Journal of Game Theory 1, 11-26

$$v(1,2,3) \geq v(1) + v(2) + v(3)$$
$$v(1,2) \geq v(1) + v(2)$$
$$v(1,3) \geq v(1) + v(3)$$
$$v(2,3) \geq v(2) + v(3)$$
$$v(1,2,3) \geq v(1,2) + v(3)$$
$$v(1,2,3) \geq v(1,3) + v(2)$$
$$v(1,2,3) \geq v(2,3) + v(1)$$
$$v(1,2,3) + v(3) \geq v(1,3) + v(2,3)$$
$$v(1,2,3) + v(1) \geq v(1,2) + v(1,3)$$
$$v(1,2,3) + v(2) \geq v(2,3) + v(1,2)$$

This means that the game is convex so the core is non-empty. For a convex characteristic function, we determine how the benefits of the model can be split between the players under the negotiation process. Now, we can proceed with the application of the Shapley Value solution.

2.3.1.1 -THE SHAPLEY VALUE

A solution concept for the game under consideration is the Shapley value (Peleg and Sudhölter, 2003) and (Branzei and Tijs, 2005). Shapley (1953) looked at what each player could reasonably get before the game has begun. He put three axioms, which he called $\phi_i(v)$, player i´s expectation in a game with a characteristic function v, should satisfy the following:

S1: $\phi_i(v)$ is independent of the labeling of the players. If π is a permutation of 1, 2,...., n and πv is the characteristic function of the game, with the players numbers permuted by π, then $\phi_{\pi(i)}(\pi v) = \phi_i(v)$.

S2: The sum of the expectations should equal the maximum available from the game, so

$$\sum_{i=1}^{n} \phi_i(v) = v(N)$$

S3: If u, v are the characteristic functions of two games, $u + v$ is the characteristic function of the game playing both games together. ϕ must satisfy $\phi_i(u + v) = \phi_i(u) + \phi_i(v)$.

Given these assumptions, Shapley proved the following theorem:

Theorem. There is only one function which satisfies S1, S2 and S3, namely:

$$\phi_i(v) = \sum_{S:i\in S} \frac{(\#S-1)!(n-\#S)!}{n!}\left(v(S) - v(S-\{i\})\right)$$

Where the summation is over all coalitions S which contain player i and $\#S$ is the number of players in the coalition S. $\phi_i(v)$ is called the Shapley value. The quantity ϕ_i may be interpreted as the “equity value” associated with the position of the i-th player in the game (Shapley, 1965).

In our model, the Shapley value is:

For downstream firm 1,

$$\phi_1(v) = \frac{1}{2}\frac{\left(F_1(\alpha_1)\beta_1 - \lambda c_{s1}(1+h_1(\mu_1))G_1(\alpha_1)\gamma_1\right)^2}{4\gamma_1 G_1(\alpha_1)}$$

For downstream firm 2,

$$\phi_2(v) = \frac{1}{2}\frac{\left(F_2(\alpha_2)\beta_2 - \lambda c_{s2}(1+h_2(\mu_2))G_2(\alpha_2)\gamma_2\right)^2}{4\gamma_2 G_2(\alpha_2)}$$

For the supply firm

$$\phi_3(v) = \frac{1}{2}\frac{\left(F_1(\alpha_1)\beta_1 - G_1(\alpha_1)\lambda c_{s1}(1+h_1(\mu_1))\gamma_1\right)^2}{4\gamma_1 G_1(\alpha_1)} + \frac{1}{2}\frac{\left(F_2(\alpha_2)\beta_2 - G_2(\alpha_2)\lambda c_{s2}(1+h_2(\mu_2))\gamma_2\right)^2}{4\gamma_2 G_2(\alpha_2)}$$

The Shapley value indicates how the total benefits can be split in a fair way. We can see that the supplier gets more benefits compared with the benefits that each downstream firm gets. This indicates that the supplier has a greater bargaining power than the downstream parties because, as it was defined previously, the bargaining power of the players is determined by which players are most needed. In this model, the supplier is the most needed because the firms have incentives to merger or to be vertically integrated with the supply firm, for the reason that a merger or the vertical integration ensures them that the production will have the required quality. In other words, the bargaining power of the supplier relies on the specificity of the investments because it is this specificity what makes the cooperation credible. If the investments were general, non-specific, the cooperation framework would not be enforceable because each player could sell or produce outside the relationship. Such bargaining power is measured by the Shapley value and it is what makes possible the cooperation among the three players. Thus, this major supplier's bargaining power is what makes the upstream-downstream relationship an enforceable one.

Once the expected benefits of the cooperative second stage are determined the players will decide their non-cooperative strategies in the first stage.

2.3.2. - THE NON-COOPERATIVE STAGE

As the timing of the model indicates, once the expected cooperative benefits are determined, the players will decide if investing in advertising and sharing advertising costs. To do this, we are going to use the Nash equilibrium solution.

Definition: A Nash equilibrium of a strategic game is a profile of strategies $(\bar{s}_1,.....,\bar{s}_n)$, where $\bar{s}_i \in S_i$ (S_i is the strategy set of player i), such that for each player i, for all $s_i \in S_i$, $u_i(\bar{s}_i, \bar{s}_{-i}) \geq u_i(s_i, \bar{s}_{-i})$, where $s_{-i} = (s)_{j \in N/\{i\}}$ and $u_i : S \equiv \times_{j \in N} S_j \to \Re$.

In our model the Nash equilibrium for this game is defined:

$u_1(\bar{\alpha}_1, \bar{\alpha}_2, \bar{\mu}_1, \bar{\mu}_2) \geq u_1(\alpha_1, \bar{\alpha}_2, \bar{\mu}_1, \bar{\mu}_2)$ for all α_1

$u_2(\bar{\alpha}_1, \bar{\alpha}_2, \bar{\mu}_1, \bar{\mu}_2) \geq u_2(\bar{\alpha}_1, \alpha_2, \bar{\mu}_1, \bar{\mu}_2)$ for all α_2

$u_3(\bar{\alpha}_1, \bar{\alpha}_2, \bar{\mu}_1, \bar{\mu}_2) \geq u_3(\bar{\alpha}_1, \bar{\alpha}_2, \mu_1, \mu_2)$ for all μ_1, μ_2

We present the non cooperative game of the three players in two matrixes. The downstream firm 1 chooses the row, he decides if investing in local advertising $(\alpha_1 = 1)$ or not investing in advertising $(\alpha_1 = 0)$. The supplier chooses the column and decides if he will share the advertising cost $(\mu_1 = \mu_2 = 1)$ or he won´t do it $(\mu_1 = \mu_2 = 0)$. Finally, the downstream party 2 chooses the matrix and he will decide if investing in local advertising $(\alpha_2 = 1)$ or not investing in it $(\alpha_2 = 0)$. We solve the non-cooperative stage according to conditions (A) established in the benchmark section.

	$\mu_1=\mu_2=0$	$\mu_1=\mu_2=1$	$\mu_1=1;\mu_2=0$	$\mu_1=0;\mu_2=1$
$\alpha_1=0$	$\phi_1=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(0))\right)^2}{4\gamma_1G_1(0)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(0))\right)^2}{4\gamma_1G_1(0)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$	$\phi_1=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(1))\right)^2}{4\gamma_1G_1(0)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(1))\right)^2}{4\gamma_1G_1(0)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$	$\phi_1=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(1))\right)^2}{4\gamma_1G_1(0)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(1))\right)^2}{4\gamma_1G_1(0)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$	$\phi_1=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(0))\right)^2}{4\gamma_1G_1(0)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(0)\beta_1-\lambda c_{s1}G_1(0)(1+h_1(0))\right)^2}{4\gamma_1G_1(0)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$
$\alpha_1=1$	$\phi_1=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(0))\right)^2}{4\gamma_1G_1(1)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(0))\right)^2}{4\gamma_1G_1(1)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$	$\phi_1=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(1))\right)^2}{4\gamma_1G_1(1)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(1))\right)^2}{4\gamma_1G_1(1)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$	$\phi_1=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(1))\right)^2}{4\gamma_1G_1(1)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(1))\right)^2}{4\gamma_1G_1(1)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2G_2(0)}$	$\phi_1=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(0))\right)^2}{4\gamma_1G_1(1)}$ $\phi_3=\frac{1}{2}\frac{\left(F_1(1)\beta_1-\lambda c_{s1}G_1(1)(1+h_1(0))\right)^2}{4\gamma_1G_1(1)}+\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$ $\phi_2=\frac{1}{2}\frac{\left(F_2(0)\beta_2-\lambda c_{s2}G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2G_2(0)}$

$\alpha_2=0$

	$\mu_1 = \mu_2 = 0$	$\mu_1 = \mu_2 = 1$	$\mu_1 = 1; \mu_2 = 0$	$\mu_1 = 0; \mu_2 = 1$
$\alpha_1 = 0$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$
$\alpha_1 = 1$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(0)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)} +$ $\frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(0)\beta_2 - \lambda c_{s2} G_2(0)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(0)}$

$\alpha_2 = 0$

	$\mu_1 = \mu_2 = 0$	$\mu_1 = \mu_2 = 1$	$\mu_1 = 1; \mu_2 = 0$	$\mu_1 = 0; \mu_2 = 1$
$\alpha_1 = 0$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(1))\right)^2}{4\gamma_1 G_1(0)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(0)\beta_1 - \lambda c_{s1} G_1(0)(1+h_1(0))\right)^2}{4\gamma_1 G_1(0)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$
$\alpha_1 = 1$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$	$\phi_1^* = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3^* = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2^* = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(1))\right)^2}{4\gamma_1 G_1(1)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(0))\right)^2}{4\gamma_2 G_2(1)}$	$\phi_1 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)}$ $\phi_3 = \frac{1}{2}\frac{\left(F_1(1)\beta_1 - \lambda c_{s1} G_1(1)(1+h_1(0))\right)^2}{4\gamma_1 G_1(1)} + \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$ $\phi_2 = \frac{1}{2}\frac{\left(F_2(1)\beta_2 - \lambda c_{s2} G_2(1)(1+h_{12}(1))\right)^2}{4\gamma_2 G_2(1)}$

$\alpha_2 = 1$

To simplify the computation of the Nash equilibrium, we simplified the matrixes and put number 1 to the higher benefits and number 2 to the lower benefits for each player. So the matrixes become:

	$\mu_1 = \mu_2 = 0$	$\mu_1 = \mu_2 = 1$	$\mu_1 = 1; \mu_2 = 0$	$\mu_1 = 0; \mu_2 = 1$
$\alpha_1 = 0$	(2,2,2)	(2,1,2)	(2,2,2)	(2,2,2)
$\alpha_1 = 1$	(1,2,2)	(1,1,2)	(1,2,2)	(1,2,2)

$\alpha_2 = 0$

	$\mu_1 = \mu_2 = 0$	$\mu_1 = \mu_2 = 1$	$\mu_1 = 1; \mu_2 = 0$	$\mu_1 = 0; \mu_2 = 1$
$\alpha_1 = 0$	(2,2,1)	(2,1,1)	(2,2,1)	(2,2,1)
$\alpha_1 = 1$	(1,2,1)	(1,1,1)****	(1,2,1)	(1,2,1)

$\alpha_2 = 1$

We solve the non cooperative stage according to the estimated benefits in the cooperative stage and satisfying the following conditions:

$$\frac{F_1(1)}{G_1(1)} > \frac{F_1(0)}{G_1(0)}$$

$$\frac{F_2(1)}{G_2(1)} > \frac{F_2(0)}{G_2(0)}$$

$$h_1(1) > h_1(0)$$

$$h_2(1) > h_2(0)$$

There is only one Nash equilibrium and it is obtained when downstream firm 1 chooses $\alpha_1 = 1$ (invest in local advertising), downstream firm 2 chooses $\alpha_2 = 1$ (invest in local advertising) and the supplier chooses to share the advertising costs $\mu_1 = \mu_2 = 1$.

So, under incomplete contracts, specific investments and biform game framework the downstream firms will invest in local advertising and the supplier will share the advertising costs to make the brand known. The total benefit of the model is equal to the benchmark case, so a Pareto optimum is achieved. We show that using a biform game model with three agents to analyze cooperative advertising the downstream firms will invest in advertising and the supplier will share the cost of advertising. Our result is a combination of the vertical cooperative advertsing and the fully cooperative advertising because despite the supplier has a bigger bargaining power (as in vertical cooperative advertising) if he shares the advertising cost they will get higher benefits for the whole upstream-downstream chain (as in the fully cooperative advertising). The upstream party uses his majorr bargaining power through the sharing costs in the cooperative advertisement to motivate sales at the retailer level and make the brand known in order to increase the total benefits for all the players, achieving the first best. So, although the supplier has a higher bargaining power our result is the same as the fully cooperative case in the cooperative advertising theory.

3- CONCLUSION

The study of cooperative advertising has been analized through two-agent models under non-cooperative scenarios (Stackelberg) and coordinated scenarios, where the agents cooperate maximizing the total surplus generated by both (supplier and retailer). This division on cooperative and non cooperative approaches in the study of cooperative advertising make us wonder what happens if we analyze upstream-downstream relationships in cooperative advertising under a combination of both: cooperative and non cooperative scenarios. We also analyze what happens if we develop these relationships through a biform model where the cooperative stage is going to be model through the cooperative game theory created by Von Neumann and Morgenstern (1944)?. Will the supplier, despite he has a bigger bargining power and makes specific investments, share the advertising costs with the downstream firms?

To answer these questions we develop a model of cooperative advertising using a biform game model with three agents under incomplete contracts and specific investments. In the cooperative game stage, the agents will decide the prices and the quantities and in the non-cooperative game stage the downstream parties will decide if they are going to invest in advertising while the supplier will decide if he is going to share the advertsing costs made by the downstream parties, given that he has a bigger bargaining power.

We show that using a biform game model with three agents to analyze cooperative advertising the downstream firms will invest in advertising and the supplier will share the cost of it. Our result is a combination of the vertical cooperative advertsing and the fully cooperative advertising because despite that the supplier has a greater bargaining power (as in vertical cooperative advertising) if he shares the advertising cost they will get higher benefits for the whole upstream-downstream chain (as in the fully cooperative advertising). The upstream party uses his major bargaining power through the sharing costs in the cooperative advertisement to motivate sales at the retailer level and make the brand known in order to increase the total benefits for all the players, achieving the first best. So, although the supplier has more bargaining power our result is the same as the fully cooperative case in the cooperative advertising theory.

The contribution of this model is twofold: first, a mathematical contribution where we apply biform games, using maximin (see Marchi, E., 1967) to compute the characteristic functions of the model. Then, we determine the expected benefits of the cooperative stage through the Shapley value and the Nash equilibrium for the advertising strategies in the non cooperative stage. The second contribution is an economic one, where we show that the combination of the cooperative and non cooperative scenarios in the cooperative advertising implies that the supplier will share the advertising costs despite his bigger bargaining power, achieving a Pareto optimum and higher benefits for the whole upstream-downstream chain. The upstream party will use his greater bargaining power through the sharing costs in the cooperative advertising to motivate sales at the retailer level and make the brand known in order to increase the total benefits for all the players.

In this paper we develop the biform game model supposing that in the cooperative scenario the agents decide prices and quantities and in the non cooperative scenario the agents decide advertising strategies. We let for further research the development of a biform game model where the agents will decide the advertising strategies in the cooperative stage and in a non-cooperative stage the agents will decide prices and quantities.

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