PROCREATION OF INNER PRODUCT SPACE FOR GENERALIZED B-FUNCTION

By

Anand Singh

and

H.S. Dhami

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INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

UNIVERSITY OF MINNESOTA

514 Vincent Hall
206 Church Street S.E.
Minneapolis, Minnesota 55455–0436

Phone: 612/624-6066 Fax: 612/626-7370
URL: http://www.ima.umn.edu

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Anand Singh & H. S. Dhami*
Department of Mathematics
University of Kumaun
Almora Campus
Almora (Uttaranchal) 263601
India

ABSTRACT

In an attempt to mucilage the bridge connecting special functions with generalized hypergcometric functions, here an attempt is being made to procreate inner product space for generalized B-function.

INTRODUCTION

With a quest to establish inter-relationship between two diversified fields of special functions and algebra, we have transformed generalized hypergeometric functions into a class of B-functions in our earlier studies [1633] & [1644]. We have ventured upto the generation of topological space in another communication [1605].

The present paper extends the knowledge is this direction, where inner product spaces have been procreated for B-functions by taking the parameters as matrices in terms of n-tuples.

P.T.O.

GENERAL FORMULATION

In the following result investigated during the course of generation of linear algebraic space for multiple hypergcometric [1633, 1.2]

$$B^{p\,r}_{q\,s}\begin{bmatrix}(a_p) & ; & (c_r) & ; & x\\ (b_q) & & (d_s) & & \end{bmatrix}$$

$$= I + \frac{(a_p)\,(d_s - I)}{(b_q)\,(c_r - I)}\,x + \frac{(a_p)\;(a_p+I)\;(d_s-I)\;(d_s-2I)}{(b_q)\;(b_q+I)\;(c_r-I)\;(c_r-2I)}\,x^2 + \ldots\ldots\,\infty \qquad \ldots\ldots\ldots (1)$$

Where (a_p), (b_q), (c_r) and (d_s) are matrices of order n x n.

Considering (a_p), (b_q), (c_r), (d_s) as scaler matrices so that

$$\left.\begin{matrix}(a_p) = k^p_1\, I\,, & (b_q) = k^q_2\, I\\ (c_r) = k^r_3\, I\,, & (d_s) = k^s_4\, I\end{matrix}\right] \qquad \text{-------------}\ (2)$$

and then setting

$k^q_2 > 0,\quad k^r_3 < 0\,,\quad k^p_1 \in I^-\quad$ and $\quad k^s_4 \in I^+$

we get

$$B^{p\,r}_{qs}\begin{bmatrix}(a_p) & (c_r) & & \\ & ; & ; & x\\ (b_q) & (d_s) & & \end{bmatrix} = I\,.\,B^{p\,r}_{q\,s}\begin{bmatrix}k^p_1 & k^r_3 & & \\ & ; & ; & x\\ k^q_2 & k^s_4 & & \end{bmatrix} \ldots\ldots\ldots\ldots\ (3)$$

$$\left|B^{p\,r}_{q\,s}\right| = \left(B^{p\,r}_{q\,s}\begin{bmatrix}k^p_1 & ; & k^r_3 & ; & x\\ k^q_2 & & k^s_4 & & \end{bmatrix}\right)^n$$

$$= \left(\sum_{m=0}^{\min(|k^p_1|, |k^s_4 - 1|)} A_m x^m \right)^n \qquad \ldots\ldots\ldots\ldots\ldots (4)$$

$$= \sum_{m=0}^{\lambda n} B_m x^m. \qquad \ldots\ldots\ldots\ldots (5)$$

Where $\lambda = \min(|k^p_1|, |k^s_4 - 1|)$

Above results can be easily justified with the help of following illustrative example.

Let us take the B-function as

$$B^{1\ 1}_{1\ 1}\left[\begin{matrix} \begin{pmatrix}-3 & 0\\ 0 & -3\end{pmatrix} & \begin{pmatrix}-4 & 0\\ 0 & -4\end{pmatrix} & \\ & ; & ; \ x \\ \begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix} & \begin{pmatrix}7 & 0\\ 0 & 7\end{pmatrix} & \end{matrix}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Whose value by result (3), shall be

$$I \cdot B^{1\ 1}_{1\ 1}\left[\begin{matrix} -3, & -4 & \\ ; & & ; \ x \\ 3 & 7 & \end{matrix}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$= I\left(1 + \frac{6}{5}x + \frac{1}{2}x^2 + \frac{2}{35}x^3\right)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Determinant of B-function in terms of n- tuples shall be

$$\left(1, \frac{12}{5}, \frac{61}{25}, \frac{46}{35}, \frac{271}{700}, \frac{2}{35}, \frac{4}{1225}\right) \qquad \ldots\ldots\ldots\ldots\ldots (9)$$

$$= \left(\frac{12}{5}, \frac{61}{25}, \frac{46}{35}, \frac{271}{700}, \frac{2}{35}, \frac{4}{1225}\right) \qquad \ldots\ldots\ldots\ldots\ldots (10)$$

where $\lambda = \min (|-3| , |6-1|) = 3$

Since $n = 2$

So the order of $\left| B^{1\ 1}_{1\ 1} \right| = 3 \times 2 = 6$

Considering

$$S = \left\{ \left| B^{p\ r}_{q\ s} \right| \Big/ \text{ by changing } k^{p}_{1} \text{ and } k^{s}_{4} \right\} \qquad \text{............... (11)}$$

and using following notations as binary operator

$$\left| B^{p\ r}_{q\ s} \right| = (B_1, \ B_2, \ldots\ldots\ldots\ldots , B_{\lambda n}) \qquad \text{............... (12)}$$

$$\left| B^{p\ r}_{q\ s} \right| + \left| B^{`\,p\ r}_{\ \ q\ s} \right| = ( B_1 + B^{`}_1, B_2 + B^{`}_2 , \ldots\ldots\ldots , B_{\lambda n} + B^{`}_{\lambda n}) \qquad \text{............. (13)}$$

and

$$\alpha \left| B^{p\ r}_{q\ s} \right| = (\alpha B_1, \alpha B_2, \ldots\ldots\ldots\ldots , \alpha B_{\lambda n}) \qquad \text{............................... (14)}$$

We observe that S is a vector space over the field of real number having dimension

$$= n \min (\lambda_1, \lambda_2, \ldots\ldots , \lambda_r) = n \lambda$$

and

Basis of $S = \{ (B^1_1 , B^1_2 , \ldots\ldots B^1_{\lambda n}) , (B^2_1, B^2_2 , \ldots\ldots , B^2_{\lambda n}), \ldots\ldots\ldots$

$\ldots\ldots , (B^{n\lambda}_1 , B^{n\lambda}_2 , \ldots\ldots , B^{n\lambda}_{n\lambda}) \}$ .... (15)

Where

$$B^j_i = 0 \quad \text{if } i \neq j , \quad i \geq 1$$
$$= 1 \quad \text{When } i = j \quad \& \quad B^j_0 = 1 \ \forall j \qquad \text{............ (16)}$$

Here zero vector of space is given by

$$\left| B_{q\ s}^{p\ r} \right| = (0,\ 0,\ \ldots\ldots\ldots,0) = 1 \qquad \ldots\ldots\ldots (17)$$

Taking

$$f(x) = A_0 + A_1 x + \ldots\ldots\ldots\ldots + A_{n\lambda} x^{n\lambda} \qquad \ldots\ldots\ldots\ldots (18)$$

Where A_0, A_1, ... , $A_{n\lambda}$ are matrices of order $n \times n$

It can be easily observed that A's given by the result (4) satisfy the necessary and sufficient condition of diagonalizibility in the modified form for ($m = \lambda n$).

Let us set

$$\alpha = (B^i_1,\ B^i_2,\ \ldots\ldots\ldots\ldots B^i_{\lambda n})$$

$$\beta = (B^j_1,\ B^j_2,\ \ldots\ldots\ldots\ldots, B^j_{\lambda n})$$

Where B_1, B_2, ... $B_{\lambda n}$ has been defined by result (12).

Defining

$$(\alpha, \beta) = \sum_{k=1}^{\lambda n} B^i_k\ \bar{B}^j_k \qquad \ldots\ldots\ldots (19)$$

We visualize that it is an inner product from S×S to F, it is evident from the following explanation.

$$\overline{(\beta, \alpha)} = \overline{\sum_{k=1}^{m} B^j_k\ \bar{B}^i_k}$$

$$= \sum_{k=1}^{m} \bar{B}^j_k\ B^i_k = (\alpha, \beta) \qquad \ldots\ldots\ldots\ldots (20)$$

Taking

$$\chi = (B^k_1, B^k_2, \ldots\ldots\ldots\ldots, B^k_m)$$

it can be easily seen that

$$(a\alpha + b\beta, \chi) = a(\alpha, \chi) + b(\beta, \chi) \qquad \ldots\ldots\ldots\ldots (21)$$

In case if

$$(\alpha, \alpha) = 0$$

$$\sum_{\ell=1}^{m} B^i_\ell \overline{B^i}_\ell = 0$$

$$\Rightarrow B^i_\ell = 0 \quad \forall \ell$$

Which further implies that

$$\alpha = (0, 0, \ldots\ldots, 0)$$

Which is a zero vector as defined in (20)

that is the function defined in (19) is an inner product space.

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