SEMINAR ABSTRACTS
Volume 2
WINTER QUARTER

IMA Preprint Series # 142
April 1985

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
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CONTINUUM PHYSICS AND PARTIAL DIFFERENTIAL EQUATIONS

Seminar Abstracts

Volume 2 Winter Quarter

The winter quarter calendar included workshops on Liquid Crystals and Amorphous Polymers and Non-Newtonian Fluids (IMA preprints 129 and 141) and a seminar series whose abstracts are collected here. We attempted to limit the seminars to two a week. They were informal and many were of an expository character. They exhibit some evidence of the brio of the participants and the high level of adrenalin which characterizes this program.

Haim Brezis
Constantine Dafermos
Jerry Ericksen
David Kinderlehrer

Scientific Committee
Global Bifurcation Problems from Nonlinear Elasticity

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Abstract

This lecture surveys a number of seemingly innocuous problems of nonlinear elasticity whose governing equations and solutions exhibit novel and complicated phenomena. A variety of new techniques are employed to handle the pathologies.

The problem of the whirling and drawing of an inextensible or elastic string, models the process by which fibers are manufactured. The governing equations contain two parameters, the whirling speed and the drawing speed. The equations have a troublesome singularity at the lowest point of the string. The basic governing equations are a fourth-order system of ordinary differential equations that can be written elegantly as a second-order system for a complex dependent variable \( u \). The multiparameter global bifurcation theory of Alexander and Antman is applied to a sequence of regularized problems. Connectivity results from point set topology are then used to prove that connected sheets of solution pairs converge to connected sets, each point of which has dimension \( z \), and that these limiting sets are sheets of solution pairs, with each sheet preserving the nodal properties it inherits from the corresponding sheets for the regularized problem. It is the number of zeros of \( |u| \) rather than those of its real or imaginary parts that is preserved on each sheet; the number of zeros of its real and imaginary parts vary along each sheet.

This analysis is valid for inextensible strings for all ranges of parameters, but for elastic strings only away from standing shocks. Available entropy conditions from the theory of hyperbolic conservation laws do not seem to offer physically reliable criteria to analyze possible shocks. To gain some insight into this problem, the exact equations for special shearing motions of a
viscoelastic medium are analyzed. Viscous shock profiles can be fully analyzed; they have qualitative properties quite different from those arising in fluid dynamics.

It is shown that the axisymmetric buckling of an anisotropic nonlinearly elastic circular plate under edge pressure leads to singular equations that can be treated by methods analogous to those used for the string. It is also shown that the solutions describing the axisymmetric buckling of spherical shells under hydrostatic pressure have a model structure that is a complicated analog of that for strings.
CONFORMAL DEFORMATIONS ON COMPLETE MANIFOLDS

Patricio Aviles

A basic problem in Riemannian geometry is that of studying the set of curvature on a manifold. A special case of this problem is that of conformal deformations of metrics. This problem might be described as follows. Let \((M^n, g)\) be an \(n\)-dimensional Riemannian manifold with a metric \(g\). Does there exist a metric \(\tilde{g}\) on \(M\) satisfying the conditions I-III:

(I) \(\tilde{g} = \lambda g\) where \(\lambda\) is a positive function. That is, \(\tilde{g}\) preserves angles. It is customary to refer to this property as a conformal deformation of the metric \(g\).

(II) The scalar curvature of \(\tilde{g} = \tilde{S}(x)\), where \(\tilde{S}(x)\) is a prescribed function on \(M\).

Finally if \((M^n, g)\) is complete then we would like to say

(III) \((M^n, \tilde{g})\) is also a complete Riemannian manifold.

This problem is studied by means of a partial differential equation. For two-dimensional manifolds, if we write \(\lambda = e^{2u}\), then the P.D.E. has the form

\[
\Delta g + \tilde{S} e^{2u} = S;
\]

If \(n > 3\) and \(\lambda = u^{4/(n-2)}\) then it has the form

\[
4\frac{(n-1)}{(n-2)} \Delta g + \tilde{S} u^{(n+2)/(n-2)} = Su,
\]

where \(S\) is the curvature of \(g\) (i.e. Gaussian curvature if \(n=2\) and scalar curvature if \(n > 3\)).

This problem has been studied extensively for compact manifolds with or without boundary. The most notorious open problem was the so-called Yamabe's problem, find \(u > 0\) solving \((2)\) with \(\tilde{S} = c > 0\) a constant, and where \(S > 0\), which was solved very recently by R. Schoen. Earlier progress was made by Aubin
Trudinger and Yambe. In general, for compact manifold (1) and (2) have been studied by Kazdan-Warner, Möser, Cherrier, R. Schoen and many other authors.

For noncompact manifolds little is known. In $\mathbb{R}^2$, with $S(x) < 0$ (1) was solved by Ni, McOwen and Ni and Kenig; with $S(x) > 0$ or of variable sign (1) was studied by the author. McOwen has also obtained some results in this last case. In $\mathbb{R}^n$, $n > 3$, the problem has been studied by Ni and Ni and Kenig. For topological reasons it is natural next to study complete manifolds with negative curvature.

In [1] we have studied (1) and (2) in this case. We mention two typical results obtained in [1].

**Theorem A.** Let $M$ be a simply connected, complete Riemannian manifold with sectional curvature $k$ satisfying $-a^2 < k < -b^2$, where $a^2 > b^2$ are arbitrary positive constants, and of dimension $n > 3$. If $\bar{S}$ is a nonpositive bounded function on $M$ satisfying $\bar{S}(x) < -d^2 < 0$ for $x \in M \setminus M_0$ where $M_0$ is a compact set and $d^2$ is also an arbitrary positive constant, then there is a unique complete metric $\bar{g}$, conformal to $g$, having $\bar{S}$ as its scalar curvature and such that the conformal factor is bounded.

It is easy to construct examples in which $u$ is not even continuous at the boundary of $M$. Hence it is natural to ask when is possible to find regular conformal factor?

**Theorem B.** Let $M$ and $\bar{S}$ be as in Theorem A and suppose $\bar{S}(x) = S(x) + H(x)$ where $\sup \{e^{\delta_0 \rho(x)} H(x) \text{ if } x \in M\} < \infty$ for some $\delta_0 > 0$.

Then there is a unique solution of (2) which satisfies

$$\lim_{\rho(x) \to \infty} u(x) = 1$$

where $\rho(x)$ denotes the distance of $x$ to a fixed point $0$.


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THE INITIATION AND GROWTH OF ADIABATIC SHEAR BANDS

R.C. Batra

Growth of perturbations in the simple shearing deforming of a softening thermo-visco-plastic solid have been examined by means of a finite element technique. It has been found that very small perturbations grow slowly at first, even though an underlying homogeneous stress-strain response has passed peak stress, but that eventually explosive growth occurs. During the slow growth phase, the stress follows the homogeneous response quite closely, but in the explosive phase, the stress drops precipitously. Large perturbations grow at a larger initial rate, and accelerate rapidly even before peak homogeneous stress. Inclusion of a dipolar plastic effect appears to be stabilizing.
R.C. Batra

The continuum theory of viscoelastic liquids as developed by Coleman, Markovitz and Noll in their book "Viscometric Flows of Non-Newtonian Fluids" will be presented. The proposed constitutive relation for viscoelastic fluids is capable of predicting the commonly observed phenomenon of climbing of a fluid on a rod immersed in a polymeric fluid contained in a rotating cylinder. Also the theory can predict the swelling of a fluid jet ejecting from a cylindrical tube.
ASYMPTOTIC SHAPES OF INFLATED NONCIRCULAR ELASTIC RINGS AND SHELLS

M. Carme Calderer

University of Delaware

We consider the asymptotic behaviour of large deformations of non-linearly elastic, noncircular rings under internal hydrostatic pressure. These rings can undergo flexure, extension, and shear. Their governing equations are the same as those for the inflation of cylindrical shells.

At first sight it might seem natural to take internal pressure $\bar{p}$ as a large parameter. It is easy to show, however, that there are circular elastic rings having deformed circular equilibrium configurations enclosing arbitrarily large areas, while the pressures needed to maintain any such configurations are bounded. Consequently we take the area enclosed by the ring as our large parameter.

Imposing very mild constitutive restrictions, we prove that the deformed configuration of a ring becomes circular as the enclosed area becomes infinite. We also show how to construct corrections to this asymptotic state and we illustrate the process with a specific example.

The difficulties of our analysis are greatly reduced by the absence of boundary layers due to the absence of boundaries. (The boundary layer theory, which requires a number of new ideas, is in preparation.) On the other hand the wealth of material response that we wish to encompass prevents us from making facile assumptions on the constitutive equations that would place our work in a routine setting. We accordingly present new techniques to handle the peculiarities of our problems. Our results throw light on a number of interesting questions of non-linear elasticity, such as the role of string theory in rod theory and membrane theory in shell theory.

This work was inspired by the results of Isaacson on the asymptotic shape of axisymmetric membranes and by Jai. Bernoulli's observation of 1691 that every equilibrium configuration of an inextensible string under hydrostatic pressure must be circular.
In our work we address the question of whether there actually is an asymptotic state, a question that has been overlooked in a number of previous studies for membranes. We emphasize that we furnish a rigorous justification for the procedure for correcting the asymptotic state. It is the availability of such readily constructed corrections that gives practical value to the study of asymptotic shapes.

Consider the following question: Can we design a thickness variation for an elliptical ring in such a way that every inflation of it by hydrostatic pressure has the shape of a homothetic ellipse? Our basic result that every asymptotic shape must be circular shows that the answer to this question is negative.
Continua with Microstructure

G. Capriz

A possible general approach to the dynamics of continuous bodies with microstructure is described. As in the topological theory of defects the model for the microstructure is a mapping of the place $R$ of the body into a differentiable manifold $M$ of finite dimension. An essential further step is the assignment over $M$ of the continuous group $G$ of rigid rotations (i.e., of the mapping of $M$ into itself caused by the rigid rotation specified by any proper orthogonal tensor $Q$); the knowledge of the infinitesimal generators of the group allows one to specify the concept of rigid generalized velocity and also to render explicit the condition of objectivity for the potential of internal actions.

The balance equations for momentum and micromomentum are obtained through a Hamiltonian principle; the corresponding developments can be viewed as a formal generalization of developments due to Toupin (for bodies with affine microstructure) and Ericksen (for liquid crystals). The condition of balance of moment of momentum is substituted by the condition that the power of internal actions vanish for all rigid velocity distributions; it leads, as in the special cases quoted above, to the specification of the antisymmetric part of the Cauchy stress in terms of the microstresses, with coefficients depending on the infinitesimal generators of $G$. A balance equation for energy and an entropy inequality are also proposed; they lead naturally to an interpretation, within the context, of well-known remarks of Ingo Müller regarding energy and entropy flux and the possible lack of objectivity of the energy flux.

A concept of latent microstructure is also proposed and consequent connections with recent work of Dunn and Serrin are indicated.
A concept of latent microstructure is shown to offer an interpretation of constitutive prescriptions involving displacement gradients of higher order, an interpretation which allows one to circumvent a known incompatibility of some of those prescriptions with the Clausius-Duhem inequality. The suggestion is in line with an early remark of Toupin on the possible identification of certain materials of second order with Cosserat continua whose microrotations are constrained. There is also a strict connection with a recent proposal of Dunn and Serrin regarding a modification of the equation of balance of energy, which allows one to put the constitutive equation for Korteweg fluids within the usual framework of consequences of the Clausius-Duhem inequality.
ASYMPTOTICS OF BLOW UP OF SEMILINEAR HEAT EQUATIONS

Yoshikazu Giga

IMA and Nagoya University

My talk concerns recent joint work with R. Kohn [2,3].

Classical solutions of nonlinear evolution equations may develop singularities even if the equation is parabolic. A simplest example is the initial value problem for a semilinear heat equation

\[ u_t - \Delta u - |u|^{p-1} u = 0 \quad (p > 1). \]

We are concerned with the asymptotic behavior of the solutions near singularities or blow up time. We characterize blow-up using similarity variables which come from the scaling consistent with the equation. To get asymptotics we classify self-similar solutions and study the rescaled equation. Our characterization shows that the diffusive effect still exists even near the blow-up time. Other aspect of blow-up, especially location of singularities are studied in [1,4,5].


In this lecture I will describe recent joint work with Ian Stewart on the existence of periodic solutions to symmetric systems of ODE, and the application of these ideas to the understanding of certain observed transitions in the Taylor-Couette system. This system consists of a fluid contained between two independently rotating concentric cylinders.

Independently, experimental and numerical work has indicated that some of the complexity in the observed time dependent states may stem from the interaction of two modes - the time independent Taylor vortices and the time dependent spiral cells. Specifically, experiments by Coles [1965] and, more recently, by Andereck et al [1984] have shown that when the outer cylinder counter rotates slowly Couette flow loses stability to Taylor vortices as the speed of the inner cylinder is increased. However, at higher speeds of counter-rotation Couette flow loses stability to a time dependent state called spiral cells. The experiment of Anderech et al [1984] indicate the existence of a unique critical speed of counter-rotation where Couette flow loses stability simultaneously to both modes. In addition, work of DiPrima and Crannick [1971] on the Navier-Stokes equation substantiates this observation.

Because of the circular symmetries in the Taylor-Couette systems the steady state transition to vortices is associated with a double zero eigenvalue in the linearized Navier-Stokes equations. Similarly, this symmetry causes the time dependent spiral cells to be associated with a pair of complex conjugate purely imaginary eigenvalues each of multiplicity two.

Our contribution is to analyse, using symmetry, the resulting flow on the six-dimensional center manifold obtained from the coalescence of the two modes described above. We show that it is possible to understand the observed secondary transitions, to wavy vortices and wavy spirals in the Taylor-Couette system, using this six dimensional model.
References

D. Andereck, Liv and H.L. Swinney [1984].

D. Coles [1965], Transition in circular Conette flow. JFM 93, 515-527.


ON THE NONLINEAR THEORY OF HYDRODYNAMIC STABILITY FOR
PARALLEL FLOWS IN THE SUBCRITICAL RANGE

Zhou Hengs

For parallel flows, transition often takes place in the range of Reynolds number that is subcritical for linear theory. So far, weakly nonlinear theory has not been able to deal with problems with the Reynolds number substantially smaller than the critical linear value. A method is proposed based on a suitable decomposition of the laminar velocity profile which can deal with the problem in the subcritical range.

For plane Poiseuille flow with two dimensional disturbances, threshold amplitudes for instability have been calculated by the proposed method, and were found to be close to the experimental results of Nishioka et. al.

For three dimensional problems, the proposed method can be used to find the resonant triad in the subcritical range. Using the same model as is adopted in the theory of secondary instability, comparable results were obtained by the proposed method. For three-dimensional perturbations the threshold amplitudes for instability were found to be almost one order of magnitude smaller than the threshold amplitudes of purely two dimensional disturbances with the same Reynolds number.
SOME DYNAMICAL PROBLEMS IN VISCOELASTICITY

by

William Hrusa

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The equation

\[(i) \quad u_{tt}(x,t) = \phi(u_x(x,t))_x + \int_0^t m(t-\tau)\psi(u_x(x,\tau))_x d\tau\]

\[x \in \mathbb{R}, t > 0,\]

provides a model for the motion of a homogeneous one-dimensional viscoelastic solid that occupies the interval $\mathbb{R}$ in a reference configuration, and has unit reference density. Here $\phi, \psi : \mathbb{R} \to \mathbb{R}$ are assigned smooth functions and $m : (0, \infty) \to \mathbb{R}$ is a given kernel. On physical grounds, it is natural to assume that $\phi' > 0$, that $\psi' > 0$, that $m$ is nonnegative, nonincreasing, and integrable, and that $\phi' - (\int_0^\infty m(s)ds)\psi' > 0$. If $m \equiv 0$ then (i) reduces to the quasilinear wave equation

\[(ii) \quad u_{tt} = \phi(u_x)_x.\]

It is well known that the initial value problem for (ii) does not generally have a global (in time) smooth solution no matter how smooth and small the initial data are.

If $m \not\equiv 0$, the memory term has a dissipative effect. For kernels that are smooth on $[0, \infty)$, equation (i), taken with appropriate initial and boundary conditions, has a unique globally defined classical solution; moreover, this solution tends to equilibrium as $t \to \infty$. If $m$ is smooth on $[0, \infty)$ and the initial data are suitably large, then the solution of (i) breaks down in finite time.

The situation is particularly interesting if $m(0^+) = \infty$. Such kernels are known to have a regularizing effect in the linear case, i.e. $\phi(u_x) = c^2 u_x$, and $\psi(u_x) = u_x$. This suggests that (i) should have better existence and regularity.
properties if $m(U^+) = \infty$. However, a singular kernel leads to significant complications in the analysis. Some existence results have already been established, but a number of interesting questions remain open.

References


2. W.J. Hrusa and M. Renardy, A nonlinear integrodifferential equation with a singular kernel.

THE CONVERGENCE OF NUMERICAL SCHEMES FOR HYPERBOLIC CONSERVATION LAWS

Mitchell Luskin

This talk is an introduction to numerical methods for nonlinear hyperbolic conservation laws and to the ideas which have been developed by many researchers to analyze them. It is well-known that conservation laws

\[ u_t + f(u)_x = 0, \quad -\infty < x < \infty, \quad t > 0, \]

1)

\[ u(x,0) = \phi(x), \]

do not have smooth solutions in general even if the initial data, \( \phi(x) \), is smooth. Thus, we must consider bounded, measurable solutions, \( u \), to 1) which satisfy

2)

\[ \int_{-\infty}^{\infty} \int_{0}^{\infty} (u w_t + f(u) w_x) \, dx \, dt + \int_{-\infty}^{\infty} \phi(x) \, w(x,0) \, dx = 0 \]

for all smooth test functions, \( w \), with compact support. A criterion that a difference scheme which is consistent for smooth solutions to 1) is consistent for weak solutions to 1) is that it be in conservation form, i.e., the approximation \( u_j^n \) to \( u(j \Delta x, n \Delta t) \) must satisfy

2)

\[ \frac{u_j^{n+1} - u_j^n}{\Delta t} + \frac{h_j^{n+1/2} - h_j^{n-1/2}}{\Delta x} = 0, \]

where

\[ h_j^{n+1/2} = h(u_j^n, u_{j-k+1}^n, \ldots, u_j^{n}) \]

and

\[ f(u) = h(u, \ldots, u). \]

It is also well-known that weak-solutions to 1) are not unique. Thus, a difference scheme in conservation form need not converge to the physically relevant
solution, and an additional criterion is needed. For scalar conservation laws, Harten, Hyman, and Lax [1] have shown that one such criterion is that the scheme be monotone, i.e.,

\[ H(u_j^{n-k}, \ldots, u_{j+k}^n) = u_j^n - \frac{\Delta t}{\Delta x} (h_{j+1/2}^n - h_{j-1/2}^n) \]

should be a monotone increasing function of each of its arguments.

Reference

AN OVERVIEW OF VISCOELASTIC MATERIAL FUNCTIONS: MEASUREMENT, TYPICAL BEHAVIOR FOR POLYMERS AND APPLICATION TO COATING FLOW PROBLEMS

by

Chris Macosko

Department of Chemical Engineering
and Materials Science
University of Minnesota

Summary

Polymeric liquids may be classified into dilute, semidilute and concentrated regimes as indicated in Figure 1 (de Gennes). Depending on the chain molecular weight the chains may become entangled as illustrated in Figure 2 (Graessley). The rheological behavior of dilute, isolated chains can be explained fairly well with bead spring models like those of Rouse or Zimm. Surprisingly the Rouse model also works well for concentrated but unentangled chains. Reptation theories like those of Doi and Edwards are making good progress on the rheology of entangled chains.

Rheological material functions depend on both time and strain, \( \tilde{\gamma} = F(t, \tilde{\gamma}) \). At small strain all material functions reduce to the shear stress relaxation modulus, \( G \), which is a function of time only, as illustrated in Figure 3. At larger strains \( G \) becomes dependent on strain and type of strain as illustrated in Figure 4. In a constant strain rate experiment, for steady motions the stresses become steady and time independent when the strains exceed the critical value of 10. Steady materials functions, such as the shear viscosity and uniaxial extensional viscosity, can be defined as indicated in Figure 5. The extensional viscosity shows particular diverse behavior for some materials, increasing rapidly at a critical rate, perhaps becoming unbounded. For dilute polymer solutions this has been identified as a transition from a random coil to an extended chain (Leal). Some associations like crystals may even form. The significance of this for the stability of some flows with free surfaces like roll coating and even simple tube flow (melt fracture) was discussed.

Figure 6 summarizes the three groupings of material functions and some ways they can be measured (Macosko).
Figure 1. Concentration regimes in good solvents.

Figure 2. Concentration – molecular weight diagram of viscoelastic regimes for polybutadiene in a good solvent.
Figure 3. Small strain ($B_{12} \leq 50\%$) stress relaxation. The same function is obtained for any type of strain.

\[ G = \frac{T_{12}(t)}{B_{12}} \]

\[ \log G \]
\[ \log t \]

Figure 4. Nonlinear stress relaxation. Different response for different strain fields: shear $B_{12}$, tension $B_{11}$, compression $-B_{11}$, etc.

\[ \log G(t, B_{12}) \]
\[ \log t \]

Shear viscosity:
\[ \eta = \frac{T_{12}}{B_{12}} \text{ or } \eta' \]
\[ (\lim_{t \to \infty}) \text{ steady uniaxial extension} \]

\[ \eta_c = \frac{T_{11} - \bar{B}_{12}}{B_{11} + \varepsilon} \]

Figure 5. Steady Straining material functions, independent of time.
functions at small strain, large strain and steady rate of strain.

Only homogeneous deformations can be used to determine material
decompositions. Classification of rheometers into homogeneous, nonhomogeneous and index type

Figure 6. Classification of rheometers into homogeneous, nonhomogeneous and index type.

NONHOMOGENEOUS

HOMOGENEOUS

NONHOMOGENEOUS

INDEXES

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- Fiber splitting (7.7)
- Entrance flows (7.8)
- Extrudate swell (6.5)
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- Failing ball (8.4)
- Failing disk (8.4)
- Force plate (8.1)
- Shear stress
- Steady strain
- Steady stress
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ON THE NEUMANN PROBLEM FOR SOME SEMILINEAR ELLIPTIC
EQUATIONS AND SYSTEMS†

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ABSTRACT

We derive a priori estimates for positive solutions of the Neumann problem for some semilinear elliptic systems (i.e., activator-inhibitor systems in biological pattern formation theory), as well as for semilinear single equations related to such systems. By making use of these a priori estimates, we show that under certain assumptions, there is no positive nonconstant solutions for single equations, or for activator-inhibitor systems when the diffusion coefficient (of the activator, in the case of systems) is sufficiently large; we also study the existence of nonconstant solutions for specific domains.

†) Research supported in part by NSF Grant # DMS 8200033A01.
DECAY TO EQUILIBRIUM AND STABILITY OF COEXISTENT PHASES IN ONE DIMENSIONAL NONLINEAR VISCOELASTICITY

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ABSTRACT

We study a mixed initial-boundary value problem modeling motion of a one-dimensional viscoelastic material with a non-monotone elastic stress-strain relation. We establish strong asymptotic convergence to equilibrium as $t \to \infty$, an issue left open by Andrews and Ball (J. Diff. Eqns. 44 (1982)). Stable and unstable equilibria are classified. The property of energy minimization is shown to be not necessary for stability of equilibria. In particular, "metastable" and "stable" states can coexist in stable, discontinuous asymptotic limits of smooth solutions. A viscosity criterion is proposed for the admissibility of waves in the associated elastic model. An account for the phenomenon of propagating phase boundaries in a loaded elastic bar is put forward. Lastly, a precise description of the smoothing of solutions in time versus space is derived from a simple existence theory.
MOLECULAR MODELS OF POLYMER MOTIONS

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Transport phenomena in amorphous polymer systems show how limits of widely differing behavior: the dilute solutions and the bulk polymers. In the former, each polymer coil is surrounded by a small-molecule solvent, whereas in the latter the environment consists of segments belonging to other polymer chains. Transport coefficients in dilute solution have, beginning with the pioneering work of Kirkwood and Riseman (1), been calculated with considerable success, especially so far as dependence or degree of polymerization is concerned. Here only interaction between polymer segments belonging to the same chain need be taken into account, but those include very long range hydrodynamic interactions, resulting from the motion of the segment perturbing solvent flow in the vicinity of another segment.

In the bulk polymer, the motion of an individual chain is much more restricted; for this case the so-called reptation model has been introduced by de Gennes (2). In simple reptation each chain is confined to a tube formed by its neighbor, and the only motion permitted is a quasi one-dimensional Brownian motion along that tube. This motion causes the tube at some initial time 0 to gradually disappear, starting from the ends, as sections are abandoned by the chain, to be replaced by sections of new tube. The reptator model has been particularly successful in predicting molecular weight dependence of self-diffusion coefficients in bulk polymers, but the viscosity-molecular weight relationship appears to present more of a challenge.

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