ABSTRACTS FROM THE WORKSHOP ON
HOMOGENIZATION AND EFFECTIVE MODULI
OF MATERIALS AND MEDIA

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October 22-26, 1984
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INTRODUCTION

This workshop focused on the large-scale effects of small-scale variation in the properties of continua. From the mathematical point of view, this entails studying the solutions of linear or nonlinear partial differential equations with rapidly varying coefficients or boundary conditions. From a physical standpoint, the issues concerned determination of effective moduli of composite materials, both natural and man made. Areas receiving attention included:

- New methods for bounding effective moduli [Bergman, Berryman, Murat, Willis]
- Exact analysis of composites with specified structure [Carroll, Milton, Murat, Sheng, Vogelius]
- New, sometimes nonlocal, effects produced by rapid variation [Tartar]
- Nonlinear continuum models [Carroll, Papanicolaou, Pipkin, Willis]
- Effects on dynamical response [Papanicolaou]
- Applications to structural optimization [Bendsoe, Kohn]

This workshop was partially supported through grants from the Office of Naval Research and the National Science Foundation.

Jerry Ericksen
David Kinderlehrer
Robert Kohn
Jacques-Louis Lions
Pretzels: time periodic configuration of STP lubricated by water
Daniel Joseph
GENERALIZED PLATE-MODELS AND OPTIMAL DESIGN

by

Martin P. Bendsoe

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We consider the optimal design of linearly elastic, solid plates, that is, we seek the stiffest plate that can be made with a given amount of material. For large values of the ratio between the maximum and minimum allowable thickness, a stiffest plate does not exist within the class of plates with slowly varying thickness. This is basically caused by the cubic dependence of the rigidity tensor on the thickness.

We will describe an extended class of plate models that allow for fields of integral stiffeners, and we will show how one can obtain an effective (smeared-out) rigidity tensor for this type of plate by homogenization or continuity considerations. Finally, numerical results will be presented which indicate that use of the generalized plate model regularizes the optimization problem.

Reading list:


The bulk effective (generally complex) dielectric constant $\varepsilon_e$ of a composite medium has simple analytical properties when viewed as a function of the complex dielectric constants of the pure components $\varepsilon_1$, $\varepsilon_2$, ... The singularities of this function correspond to easily grasped resonances in the dielectric response. In the case of two-component systems, these singularities can be adequately represented as simple poles of a function $f \left( \frac{\varepsilon_1}{\varepsilon_2} \right) = \frac{\varepsilon_e}{\varepsilon_2}$, which depend only on the microgeometry, and a knowledge of these resonances suffices to determine $\varepsilon_e$ for any values of $\varepsilon_1$, $\varepsilon_2$ in a given microgeometry. Methods for calculating these resonances are described. If the resonances are used to parameterize the function $f(z)$, then various types of exact bounds can be derived for $f(z)$ based on partial information about the composite. This information can be in the form of microgeometric correlation functions or other structural features (e.g. volume fractions, rotational invariance), or in the form of one or more known values of the function $f(z)$. The principles and techniques for deriving such bounds will be described, and some newly derived bounds will be presented. Extension of this approach to composites made of more than two components will be mentioned, as will extensions to other properties, e.g. elastic moduli, scattering of electromagnetic waves, and the Hall effect.
Bibliography


VARIATIONAL BOUNDS ON PERMEABILITY

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General properties of variational bounds on Darcy's constant for slow viscous flow through porous media will be discussed. A new result on a phase-interchange relation for permeability will be derived using the variational method. Results of numerical computations for permeability of hard sphere and penetrable sphere models will be shown. The bound of Doi depending on two-point correlations and the analytical bound of Weissberg and Prager for the penetrable sphere model give comparable results in the low density limit, but the analytical bound is superior for higher densities. Prager's bound using the "constant" trial function and depending on three-point correlation functions is worse than the analytical bound at low densities but better (although comparable to it) at high densities. All of these bounds are disappointingly far (an order of magnitude) from empirical data on permeability of reservoir rocks. A procedure for methodically improving Prager's three-point bound has therefore been developed. By introducing a "Gaussian" trial function, the three-point bound is improved by an order of magnitude for moderate values of porosity. The new bounds are comparable in magnitude to the Kozeny-Carman empirical relation for porous materials. The end of the talk will be devoted to showing slides to illustrate recently developed methods for measuring one-point, two-point, and three-point spatial correlation functions from digitized photographs of surfaces of composite materials. These correlation functions are the ones which commonly arise in the variational bounds on conductivity and elastic constants as well as fluid permeability.
Reading list:

The development of constitutive theories to describe inelastic response of porous materials may rely on micromechanical models of void growth and collapse as well as on experimental data. A particular micromodel - the hollow sphere model - is described in some detail. Both closed form solutions and numerical solutions illustrate certain aspects of the macroscopic behavior, including rate effects, yield and failure.

Two "case histories", which involve both micromechanical and empirical treatments, are presented. The first is a theory to describe isostatic creep compaction of metal powders (at constant pressure and temperature). The second is a critical state plasticity theory to describe static triaxial response of porous reservoir rocks.
EFFECTIVE MODULI AND STRUCTURAL OPTIMIZATION

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Structural optimization is the discipline of choosing structural geometry to make the most efficient possible use of available materials. Composite materials - for example those with fiber reinforcements - have long been used to design efficient, light-weight structures. Recently a deeper relation between optimization and effective moduli has emerged from the work of Tartar, Murat, Cheng, Olhoff, Strang, Lurie, Cherkaev, and others. It appears that many problems of structural optimization can be solved only by allowing designs that vary on a microscopic scale. In other words, design optimization often requires the use of composite materials, even when this is not evident in the original formulation of the problem.

This discovery has practical consequences for the resolution of structural optimization problems. If a precise characterization can be given for all the composites that might appear in an optimal design, then the design optimization can be reformulated in "relaxed form," as an optimal control problem which has at least one solution. Approximate solutions must in practice be sought numerically.

The required information about composites is sometimes called a set of "optimal bounds" for the effective moduli. There must be no restriction of periodicity, randomness, or isotropy, since these will generally not hold for the optimal design. And they must be given in terms of the composite's "cost", usually a function of volume fraction. Such bounds have been obtained independently by Lurie and Cherkaev (R^2) and Murat and Tartar (R^n) in the case of a scalar equation (heat conduction), for mixtures of two isotropic materials. One hopes that further efforts will lead to optimal bounds in other contexts.
Information can also flow in the other direction. Strang and I have shown how to solve certain optimal design problems by constructively relaxing a corresponding variational integral. One can guess (and perhaps prove) bounds on effective moduli by examining which composites appear in the designs so obtained.

Reading list:


9) ______, "Estimation des coefficients homogeneises," in Lecture Notes in

10) ______, "Estimations fines de coefficients homogeneises," in proc. of con-
MODELLING THE PROPERTIES OF COMPOSITES BY LAMINATES

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It was first recognized by Schulgasser that the effective conductivity can be calculated exactly for a wide class of composites constructed via a laminating procedure. These laminate materials have inhomogeneities on multiple length scales: typically one begins by slicing two components and placing the slices in alternate order to form a multilayered sandwich. The sandwich in turn is sliced in a different direction (on a much larger length scale) and combined with, say, slices of another multilayered sandwich to produce a more complex sandwich. Of course, this process can be continued and thereby a tremendous variety of composites can be constructed. The transport properties of these laminates are easy to evaluate.

Laminate materials became the focus of attention when many of the well known bounds on the real and complex conductivity of composites, including polycrystals, were found to be attained by particular laminate materials. This led Lurie and Cherkaev to make the bold conjecture that the conductivity of any composite, with specified volume fractions of the components, can be modelled by an appropriate laminate material. Here we examine their conjecture and explore possible generalizations and ramifications of it. We show that the Hashin-Shtrikman bounds on the shear modulus are in fact attained by a quasi self-similar laminate material, constructed in a hierarchical manner. This demonstrates that the Hashin-Shtrikman bounds are, in fact, optimal. We also examine the question of what geometries produce the highest field concentrations, and find that laminates can produce higher concentrations than those found near sharp corners.

Reading List:
This lecture presents work in progress, being done in collaboration with Luc Tartar and Gilles Francfort. Our goals are two-fold: First, to give a new derivation of the Hashin-Shtrikman bounds in the context of isotropic elasticity; and second, to construct some special composites for which the bounds are sharp.

The first goal relates to bounds on the shear and bulk moduli of a macroscopically isotropic mixture of two isotropic elastic materials, in specified proportions. Our method is that of compensated compactness, which has recently been successfully applied to the corresponding problem for a scalar equation (heat conduction or electrostatics). Our exposition includes a discussion of what we mean by the "effective moduli of a composite" from the mathematical point of view. At present, we are only able to derive the Hashin-Shtrikman bound on bulk modulus, though we hope that further work will give the one on shear modulus as well.

We achieve the second goal by considering laminar composites. The construction is an iterative one, involving multiple scales and varying directions for the layers. At each stage one generates a new material, in general anisotropic, by inserting layers of a previously constructed composite into an isotropic matrix. We give a simple, algebraic expression for the result of this process. With the proper choice of parameters it gives an isotropic elastic material achieving simultaneously the Hashin-Shtrikman bounds on the bulk and shear moduli of the composite. The fact that both bounds can be attained seems to be new.
We will first derive a system of effective equations for wave propagation in a bubbly liquid. Starting from a microscopic description we obtain the effective equations by using Foldy's approximation in a nonlinear setting. We discuss in detail the range of validity of the equations. Small gas bubble volume fraction is a required condition.

We also derive effective equations at larger volume fraction in a small amplitude or linearized regime.

The work has been carried out jointly with R. Caflisch, M. Miksis and Lu Ting. The paper will appear in JFM. A related paper by J. Rubinstein will appear in JASA and a paper by Caflisch on the mathematical theory for the nonlinear effective equations will appear in CPAM.
SOME EXAMPLES OF CRINKLES

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In some energy minimization problems in continuum mechanics, a minimizing sequence of deformations exists but the energy attributed to the limiting deformation is not the same as the minimum energy. Two elementary mathematical examples are given to illustrate this state of affairs and to show what to do about it. A situation of this sort actually arises in the theory of membranes formed from inextensible fibers (i.e. cloth). There are easy examples in which minimizing sequences consist of highly folded states and the limiting deformation is continuously folded, a deformation that is not admissible in the original theory. The necessary modification of the energy functional and admissibility conditions is discussed. A brief review of the role of such sequences in elasticity theory is given.
The relationship between microstructure and electrical and elastic properties of heterogeneous composites is delineated through the effective medium description of four prototype structures. It is shown that while the concept of an effective medium implicitly implies non-resolution on the scale of individual inhomogeneities, the connectivity and shape of the component grains nevertheless have a profound influence on the values of the composite effective moduli. This structural sensitivity of the composite properties motivates the development of a first-principle approach to the calculation of effective moduli for periodic structures with arbitrary unit cell geometry. Based on the iterative solution of the inhomogeneous elastic wave equation in the long wavelength limit, the method uses the Fourier coefficients of a structure as inputs, and offers the advantage of circumventing the need for explicit boundary condition fitting across interfaces. The application of this method to the calculation of effective moduli is illustrated by examples.

Reading List:


REMARKS ON HOMOGENIZATION

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There are different definitions of homogenization. The one I have been advocating, in joint work with Francois Murat, consists in understanding the relations between oscillations in the coefficients of partial differential equations and the oscillations created in their solutions. One of the useful aspects of this mathematical theory is that it provides a mathematical model to describe the relations between microscopic and macroscopic scales which arise in many areas of continuum mechanics and physics. In this lecture I will give some typical examples of such mathematical questions:

1. An equation of Stokes type
\[
\begin{cases}
- \nabla \mu + u \times \text{curl}(v_{0} + \lambda v) = f - \text{grad} p
\
\text{div} u_{0} = 0
\end{cases}
\]
which gives a limiting equation
\[
\begin{cases}
- \nabla u_{0} + u_{0} \times \text{curl}v_{0} + \lambda^{2} Mu_{0} = f - \text{grad} p_{0}
\
\text{div} u_{0} = 0
\end{cases}
\]

2. An equation of diffusion type
\[- \text{div}(\alpha \text{grad} u) = f\]
which gives a limiting equation of the same type \[- \text{div}(\alpha_{0} \text{grad} u_{0}) = f\] but where an important question is to understand how statistical information on \(\alpha_{\varepsilon}\) gives constraints on the effective coefficient \(\alpha_{0}\).
3. A model equation

\[ -a_\varepsilon(t) \frac{d^2u_\varepsilon}{dx^2} + b_\varepsilon(t)u_\varepsilon = f \]

where the fluctuations of \( a_\varepsilon \) and \( b_\varepsilon \) create a nonlocal effect, the limiting equation being of the form

\[ -a_0(t) \frac{d^2u_0}{dx^2} + b_0(t)u_0 - H(x,t) * x u_0 = f \]
NEW MODELS FOR THIN PLATES WITH RAPIDLY VARYING THICKNESS

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This talk will discuss some new models for thin plates with rapidly varying thickness. The discussion includes highlights of the derivation of these models, as well as the analysis of their asymptotic relation to the equations of 3-D linear elasticity.

Plates with rapidly varying thickness are found as solutions to certain optimal design problems. I shall briefly indicate some implications of our findings for a problem of optimal thickness design.

Reading list:


VARIATIONAL ESTIMATES FOR THE OVERALL RESPONSE OF AN INHOMOGENEOUS NONLINEAR DIELECTRIC

J.R. Willis

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For any problem that can be formulated as a "minimum energy" principle, a procedure is given for generating sets of upper and lower bounds for the energy. It makes use of "comparison bodies" whose energy functions may be easier to handle than that in the given problem. No structure for the energy functions is assumed in the formal development, but useful results are most likely to follow when they are convex. When applied to linear field equations, the procedure yields the Hashin-Shtrikman variational principle, and so can be regarded as its generalization to nonlinear problems.

The procedure is applied explicitly to a boundary value problem for an inhomogeneous, nonlinear dielectric. Then a slight extension which describes randomly inhomogeneous media is applied, to develop bounds for the overall energy of a nonlinear composite, which reduce to the Hashin-Shtrikman bounds in the linear limit. Sample results are shown for a simple two-phase composite.
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