RESULTANTS AND INVERSION FORMULA
FOR N POLYNOMIALS IN N VARIABLES

By

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ABSTRACT. Based on the concepts of multivariate resultants and minimal polynomials, we
give an explicit inversion formula for n polynomials in n variables, which includes McKay and
Wang's formula for two polynomials in two variables as a special case. As a consequence, we
obtain a resultant criterion formula for the inverse of polynomial maps.

1. INTRODUCTION

The Lagrange Inversion Formula has been extended to the multi-variable case by several
authors. In a recent paper by Gessel [4], many of these formulas are examined and it shows
that they are all equivalent. Those formulas apply to a system of n formal power series in
n variables, for which the inverse system consists of n formal power series in n variables.

In this paper we study a system of n polynomials in n variables, for which the inverse
system is also n polynomials in n variables. By means of minimal polynomials [10] and
multivariate resultants [5], we obtain an explicit inversion formula, which includes Mckay
and Wang's formula (n = 2) in [7] as a special case.

Our inversion formula can be viewed as a generalization of the usual Cramer's rule to
the case of n polynomials equations in n variables.

Key words and phrases. Resultants, Inversion Formula, Jacobian Conjecture.
Although the theorems in this paper have been proved for polynomials over $C$, all the theorems remain true if $C$ is replaced by an arbitrary field.

I hope that the methods and results in this paper can be helpful for solving the Jacobian Conjecture. See [2].

2. Prelimineries

**Definition 1 (Macaulay [5, p.4-5; 6, p.3]).** The resultant of $n$ general homogeneous polynomials $f_1, \ldots, f_n$ in $n$ variables of degrees $l_1, \ldots, l_n$ can be defined as an integral function of the coefficients of the $f_i$, without repeated factors, whose vanishing is the sufficient and necessary condition that the $n$ polynomials should have a common solution. The resultant of $n$ nonhomogeneous polynomials in $n - 1$ variables is the resultant of corresponding homogeneous polynomials of the same degrees obtained by introducing a variable $x_0$ of homogeneity.

**Remark.** Macaulay [5] proved such a resultant is unique determined by the coefficients of $f_i$, and gave an explicit expression of the resultant as a polynomial of the coefficient of the $f_i$. See section 5 of this paper for details.

**Lemma 2** [5, p.5 and p.27]. Let

$$f_1, \ldots, f_n \in C[x_1, \ldots, x_{n-1}]$$

are general polynomials (i.e., all coefficients of the $f_i$ are indeterminates) and let all solution in $C^n$ of the system

$$f_2 = 0, \ldots, f_n = 0$$

are

$$\{(a_1^{(i)}, \ldots, a_{n-1}^{(i)}) | i = 1, 2, \ldots, d = deg f_2 \ldots deg f_n\},$$
Let
\[ R_{x_1 \ldots x_{n-1}}(f_1(x_1, \ldots, x_{n-1}), \ldots, f_n(x_1, \ldots, x_{n-1})) = \frac{Res_{x_1, \ldots, x_{n-1}}(f_1, \ldots, f_n)}{[Res_{x_1, \ldots, x_{n-1}}(f_2^+, \ldots, f_n^+) \deg f_i)^{deg f_i}} \]
where \( f_i^+ \) is the unique homogeneous form such that \( \deg(f_i - f_i^+) \leq \deg f_i \). Then
\[ R_{x_1 \ldots x_{n-1}}(f_1(x_1, \ldots, x_{n-1}), \ldots, f_n(x_1, \ldots, x_{n-1})) = \Pi_{i=1}^d f_1(a_1^{(i)}, \ldots, a_{n-1}^{(i)}) \]
and
\[ R_{x_1 \ldots x_{n-1}}(f_1(x_1, \ldots, x_{n-1}), \ldots, f_n(x_1, \ldots, x_{n-1})) = 0 \]
\[ \Leftrightarrow f_1(x_1, \ldots, x_{n-1}), \ldots, f_n(x_1, \ldots, x_{n-1}) \]
have a common zero.

**Lemma 3** [10, Theorem 4]. Let
\[ f = (f_1, \ldots, f_n) : C^n \to C^n \]
be a polynomial automorphism and let
\[ f^{-1} = g = (g_1, \ldots, g_n) \]
Suppose that
\[ h_i(x_1, \ldots, x_n) \]
is the minimal polynomial of
\[ f_1(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n), \]
\[ \ldots \]
\[ f_n(x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n). \]
Then
\[ g_i(x_1, \ldots, x_n) = \frac{h_i(x_1, \ldots, x_n)}{h_i(f_1(0, \ldots, 0, 1, 0, \ldots, 0), \ldots, f_n(0, \ldots, 0, 1, 0, \ldots, 0))}, \]
where 1 is at the \( i^{th} \) component.
3. Main Results

Theorem 4. Let

\[ u_1, \ldots, u_n \in C[t_1, \ldots, t_{n-1}] \]

be polynomials with zero constant terms such that

\[ u_2, \ldots, u_n \]

are algebraically independent, and let

\[ S(x_1, \ldots, x_n) = R_{t_1 \ldots t_{n-1}}(u_1 - x_1, \ldots, u_n - x_n) \]

Then

\[ S(x_1, \ldots, x_n) = [h(x_1, \ldots, x_n)]^q \]

where \( q \) is a positive integer and \( h \) is an irreducible polynomial in \( C[x_1, \ldots, x_n] \). Namely \( h \) is the minimal polynomial of \( u_1, \ldots, u_n \).

Proof. By lemma 2,

\[ S(x_1, \ldots, x_n) = \Pi_{i=1}^d (u_1(a_1^{(i)}, \ldots, a_{n-1}^{(i)}) - x_1), \quad (2) \]

\( S(x_1, \ldots, x_n) \) is a non-constant polynomial in \( x_1, \ldots, x_n \) such that

\[ S(u_1(t_1, \ldots, t_{n-1}), \ldots, u_n(t_1, \ldots, t_{n-1})) = 0, \]

so if \( h(x_1, \ldots, x_n) \) is the minimal polynomial of

\[ u_1(t_1, \ldots, t_{n-1}), \ldots, u_n(t_1, \ldots, t_{n-1}), \]

then \( h(x_1, \ldots, x_n) | S(x_1, \ldots, x_n) \).

On the other hand, let \( H(x_1, \ldots, x_n) \in C[x_1, \ldots, x_n] \) is an irreducible factor of

\[ S(x_1, \ldots, x_n) = R_{t_1 \ldots t_{n-1}}(u_1(t_1, \ldots, t_{n-1}) - x_1, \ldots, u_n(t_1, \ldots, t_{n-1}) - x_n). \]
If \((X_1, \ldots, X_n)\) is a zero of \(H(x_1, \ldots, x_n)\), then

\[ S(X_1, \ldots, X_n) = 0, \]

hence

\[ u_1(t_1, \ldots, t_{n-1}) - X_1, \ldots, u_n(t_1, \ldots, t_{n-1}) - X_n \]

have a common zero \((T_1, \ldots, T_n)\),

\[ \Rightarrow H(u_1(T_1, \ldots, T_{n-1}), \ldots, u_n(T_1, \ldots, T_{n-1})) = 0, \]

which means all points on the hypersurface \(H(x_1, \ldots, x_n) = 0\) are on the hypersurface \(h(x_1, \ldots, x_n)\). Since both \(h\) and \(H\) are irreducible, they are same.

In other words, up to a constant factor, \(S(x_1, \ldots, x_n)\) has only one irreducible factor in \(C[x_1, \ldots, x_n]\). Thus

\[ R(x_1, \ldots, x_n) = [h(x_1, \ldots, x_n)]^q. \]

**Remark.**

1. By Macaulay [5], we can express \(S(x_1, \ldots, x_n)\) in terms of the coefficients of \(u_1, \ldots, u_n\) in \(t_1, \ldots, t_{n-1}\).

2. Again by [5], \(R(x_1, \ldots, x_n)\) is not depend on the different choices of algebraically indenent \(u_2, \ldots, u_n\) among \(u_1, \ldots, u_n\) (up to a constant factor).

3. Theorem 4 gives efficient algorithm to obtain the minimal polynomials of \(n\) polynomials in \(n - 1\) vaviables which includ \(n - 1\) algebraically independent polynomials.

4. By lemma 2,

\[ S(x_1, \ldots, x_n) = \frac{Res_{t_1 \ldots t_{n-1}}(u_1 - x_1, \ldots, u_n - x_n)}{[Res_{t_1 \ldots t_{n-1}}(u_1^+, \ldots, u_n^+)]^\text{deg}f_i}. \]

When we specialize the coefficients of the \(u_1\), the right side could be

\[ \begin{matrix} 0 \\ \text{0} \end{matrix} \]

But we can overcome this difficulty by appropriate specialization, i.e., specializing some coefficients first, divising, then specializing the rest coefficients, we still can obtain \(S(x_1, \ldots, x_n)\)
as a non-constant polynomial. For details, see a forthcoming paper by S. S.-S. Wang and C. C.-A. Cheng.

**Theorem 5 (Inversion Formula).** If

\[ f = (f_1, \ldots, f_n) : \mathbb{C}^n \to \mathbb{C}^n \]

is a polynomial automorphism such that

\[ f^{-1} = g = (g_1, \ldots, g_n), \]

and

\[ f_{1,i} = f_1(t_1, \ldots, t_{i-1}, 0, t_{i+1}, \ldots, t_n), \]

\[ \ldots \]

\[ f_{k_i-1,i} = f_{k_i-1}(t_1, \ldots, t_{i-1}, 0, t_{i+1}, \ldots, t_n), \]

\[ f_{k_i+1,i} = f_{k_i+1}(t_1, \ldots, t_{i-1}, 0, t_{i+1}, \ldots, t_n) \]

\[ \ldots \]

\[ f_{n,i} = f_n(t_1, \ldots, t_{i-1}, 0, t_{i+1}, \ldots, t_n) \]

are algebraically independent. Let

\[ f_{k_i,i} = f_{k_i}(t_1, \ldots, t_{i-1}, 0, t_{i+1}, \ldots, t_n) \]

and

\[ S_i(x_1, \ldots, x_n) = R_{t_1 \ldots t_{i-1} t_{i+1} \ldots t_n}(f_{1,i} - x_1, \ldots, f_{n,i} - x_n). \]

Then

\[ [g_i(x_1, \ldots, x_n)]^q = \frac{S_i(x_1, \ldots, x_n)}{S_i(f_1(0, \ldots, 0, 1, 0, \ldots, 0), \ldots, f_n(0, \ldots, 0, 1, 0, \ldots, 0))}, \]

\[ i = 1, \ldots, n. \]
where 1 is at the $i^{th}$ component and

$$q = \deg_{x_i} S_i(f_1(0,\ldots,0,x_i,0,\ldots,0),\ldots,f_n(0,\ldots,0,x_i,0,\ldots,0)).$$

**Remark.** Here $S_i(x_1,\ldots,x_n)$ is just $S(x_1,\ldots,x_n)$ in theorem 5.

**Proof.** Since

$$C[x_1,\ldots,x_n] = C[f_1,\ldots,f_n],$$

we obtain

$$C[t_1,\ldots,t_{i-1},t_{i+1},\ldots,t_n]$$
$$= C[f_1(t_1,\ldots,t_{i-1},0,t_{i+1},\ldots,t_n),\ldots,f_n(t_1,\ldots,t_{i-1},0,t_{i+1},\ldots,t_n)].$$

$$i = 1,\ldots,n.$$ 

Therefore $f_{1,i},\ldots,f_{n,i}$ contain $n - 1$ algebraically elements.

By theorem 4,

$$S_i(x_1,\ldots,x_n) = [h_i(x_1,\ldots,x_n)]^q,$$

where $h_i$ is the minimal polynomial of

$$f_1(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_n),$$

......

$$f_n(x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_n).$$

By lemma 3,

$$S_i(x_1,\ldots,x_n) = c[g_i(x_1,\ldots,x_n)]^q,$$

hence

$$S_i(f_1(0,\ldots,0,x_i,0,\ldots,0),\ldots,f_n(0,\ldots,0,x_i,0,\ldots,0))$$
$$= c[g_i(f_1(0,\ldots,0,x_i,0,\ldots,0),\ldots,f_n(0,\ldots,0,x_i,0,\ldots,0))]^q = cx_i^q.$$
So
\[ q = \deg_{x_i} S_i(f_1(0, \ldots, 0, x_i, 0, \ldots, 0), \ldots, f_n(0, \ldots, 0, x_i, 0, \ldots, 0)) \]
and
\[ c = S_i(f_1(0, \ldots, 0, 1, 0, \ldots, 0), \ldots, f_n(0, \ldots, 0, 1, 0, \ldots, 0)). \]

Remark. Theorem 5 is a generalization of [7, Theorem 12], hence we have answered McKay and Wang’s [7, Question 16].

4. A RESULTANT CRITERION FORMULA FOR THE INVERSE OF POLYNOMIAL MAPS

In his book [1](also see [2] for a partial result), Abhyankar gives a nice

**Proposition 6.** Let \( k \) be a field and let \( f = (f_1, f_2) : k^2 \to k^2 \) be a polynomial map, and
\[
S_i(x_i, y_1, y_2) = \text{Res}_{\{x_1, x_2\} - \{x_i\}}(f_1 - y_1, f_2 - y_2), i = 1, 2,
\]
then

(1) \( x_i(i = 1, 2) \) are rational function of \( y_1, y_2 \) if and only if
\[
\deg_{x_i} S_i = 1 (i = 1, 2).
\]

Moreover,
\[
x_i = \frac{p_i(y_1, y_2)}{q_i(y_1, y_2)},
\]
where the \( \text{gcd}(p_i, q_i) = 1 \) and \( q_i \neq 0 \).

\[ \iff S_i(x_i, y_1, y_2) = c_i(q_i(y_1, y_2)x_i - p_i(y_1, y_2)), c_i \in k^*. \]

(2) \( x_i(i = 1, 2) \) are polynomials in \( y_1, y_2 \) if and only if
\[
\deg_{x_i} S_i = 1 (i = 1, 2).
\]
and the coefficients of $x_i$ in $S_i$ are constants. Moreover, $x_i = g_i(y_1, y_2), g_i$ are polynomials

$$\Leftrightarrow S_i(x_i, y_1, y_2) = c_i(x_i - g_i(y_1, y_2)), c_i \in k^*.$$ 

Abhyankar suggests that the similar results should be ture for general suituation. We prove in this paper that it is the case.

We now introduce some notations.

Let $f = (f_1, \ldots, f_n) : k^n \to k^n$ be a polynomial map and

$$f_{ki} = f_k(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n).$$

Suppose

$$f_{1i}, \ldots, f_{ki-1}, f_{ki+1}, \ldots, f_{ni}$$

are algebraically independent, we denote

$$R_i(f_1, \ldots, f_n) = \frac{Res_{x - \{x_i\}}(f_1, \ldots, f_n)}{[Res_{x - \{x_i\}}(f^+ - \{f_{ki}^+\})]^{m_i}},$$

where $x$ means $\{x_1 \ldots x_n\}$, $f^+$ means $\{f_1^+, \ldots, f_n^+\}$ and $m_i = \deg f_{ki,i}$.

**Theorem 7.** Let $f = (f_1, \ldots, f_n) : k^n \to k^n$ be a polynomial map, and let

$$S_i(x_i, y_1, \ldots, y_n) = R_i(f_1 - y_1, \ldots, f_n - y_n), i = 1, \ldots, n.$$ 

Then

$$x_i = \frac{p_i(y_1, \ldots, y_n)}{q_i(y_1, \ldots, y_n)},$$

where $gcd(p_i, q_i) = 1$ and $q_i \neq 0$

$$\Leftrightarrow S_i(x_i, y_1, \ldots, y_n) = c_i(q_i(y_1, \ldots, y_n)x_i - p_i(y_1, \ldots, y_n))^{s_i};$$

(2)

$$x_i = g_i(y_1, \ldots, y_n),$$
$G_i$ are polynomials

$$
\Leftrightarrow S_i(x_i, y_1, \ldots, y_n) = c_i(x_i - g_i(y_1, \ldots, y_n))^{s_i}.
$$

Both in (1) and (2), $c_i \in k^*$ and $s_i$ are positive integers.

**Proof.** We only prove (1), since (2) is a consequence of (1).

$\Leftarrow$: Suppose

$$S_i(x_i, y_1, \ldots, y_n) = c_i(q_i(y_1, \ldots, y_n)x_i - p_i(y_1, \ldots, y_n))^{s_i}, c_i \in k^*.$$

Substituting $y_i = f_i, S_i = 0$. Hence

$$x_i = \frac{p_i(y_1, \ldots, y_n)}{q_i(y_1, \ldots, y_n)}.$$

$\Rightarrow$: Suppose

$$x_i = \frac{p_i(y_1, \ldots, y_n)}{q_i(y_1, \ldots, y_n)},$$

$i = 1, \ldots, n$ are inverse function of $y_i = f(x_1, \ldots, x_n)$.

The functions

$$f_i(x_1, \ldots, x_{i-1}, \frac{p_i}{q_i}, x_{i+1}, \ldots, x_n) - y_i, i = 1, \ldots, n$$

considered as polynomials in

$$x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$$

with coefficients in $k(y_1, \ldots, y_n)$ have

$$\frac{p_k}{q_k}, k = 1, \ldots, i - 1, i + 1, \ldots, n$$

as a common solution, so

$$S_i(x_i, y_1, \ldots, y_n) = R_i(f_1 - y_1, \ldots, f_n - y_n)$$. 


considered as a polynomial in \(x_i\) with coefficients in \(k(y_1, \ldots, y_n)\) has a zero

\[
x_i = \frac{p_i}{q_i}.
\]

Hence

\[
R_i(f_1 - y_1, \ldots, f_n - y_n) = A_i(x_i - \frac{p_i}{q_i})^{s_i}, i = 1, \ldots, n,
\]

where \(A_i \in k(y_1, \ldots, y_n)\), comparing the coefficients in \(x^{s_i}\) of both sides, \(A_i \in k[y_1, \ldots, y_n]\).

Since

\[
k(x_1, \ldots, x_n) = k(f_1, \ldots, f_n),
\]

by proposition 2,

\[
A_i\left(\frac{p_i}{q_i}\right)^{s_i} = S_i(0, y_1, \ldots, y_n) = R_i(f_1 - y_1, \ldots, f_n - y_n)|_{x_i=0}
\]

are polynomials in \(k[y_1, \ldots, y_n]\). Hence \(A_i = c_i q_i^{s_i}\), so

\[
R_i(f_1 - y_1, \ldots, f_n - y_n) = c_i(q_i(y_1, \ldots, y_n)x_i - p_i(y_1, \ldots, y_n))^{s_i}, c_i \in k^*.
\]

As a consequence of theorem 7, we obtain

**Theorem 8.** (1) Let \(f : k^n \rightarrow k^n\) is a polynomial map, then \(f\) is a polynomial automorphism

\[
\Leftrightarrow
\]

there are some positive integers \(s_i\) such that

\[
\frac{d^{s_i}(R_i(f_1(x_1, \ldots, x_n), \ldots, f_n(x_1, \ldots, x_n)))}{dx_i^{s_i}} \in k^*, i = 1, \ldots, n.
\]

(2) Let \(\text{char}(K) = 0\), then the Jacobian conjecture holds

\[
\Leftrightarrow
\]

there are some positive integers \(s_i\) such that \(J(f) \in k^*\) implies that

\[
\frac{d^{s_i}(R_i(f_1(x_1, \ldots, x_n), \ldots, f_n(x_1, \ldots, x_n)))}{dx_i^{s_i}} \in k^*, i = 1, \ldots, n.
\]
5. Macaulay's construction of resultants

Macaulay [5] shows that the resultant equals the quotient of the determinant of a certain matrix $A$ whose entries are coefficients of the polynomials, and a minor of $A$. In this section we briefly mention Macaulay's construction of resultants.

Suppose we are given $n$ homogeneous polynomials $f_i$ in $n$ variables $x_i$, and that $f_i$ has degree $d_i$. We need some notation for monomials of $f_i$. Let $\alpha$ be an $n$-tuple of integers, we write $x^\alpha$ for the monomial $x_1^{\alpha_1} \ldots x_n^{\alpha_n}$.

The rows and columns of the matrix $A$ are indexed by the set of monomials in $x_1, \ldots, x_n$ of degree $d$ where

$$d = 1 + \sum_{i=1}^n (d_i - 1),$$

and letting $X^d$ denote the set of monomials of degree $d$, the cardinality of $X^d$ is

$$N = |X^d| = \binom{d + n - 1}{k}.$$

**Definition 9.** A polynomial is said to be reduced in $x_i$ if its degree (the maximum degree of its monomials) in $x_i$ is less than $d_i$. A polynomial that is reduced in all variables but one is said simply to be reduced.

Now consider the polynomial

$$f = C_1 f_1 + \cdots + C_n f_n, \quad (*)$$

where each $c_i$ is a homogeneous polynomial of degree $d - d_i$ with symbolic coefficients, which is reduced in $x_1, \ldots, x_i$. $f$ is a homogeneous polynomial of degree $d$, and so has $N$ coefficients. There also, in total, exactly $N$ coefficients in the $C_i$. To see this, image for the moment that each $f_i$ equals $x_i^{d_i}$. Then every monomial in $f$ is a multiple of a monomial from exactly one of the $C_i$'s. For the monomial $cx^\alpha$, let $j$ be the smallest index $i$ such that $x^\alpha$ is not reduced in $x_i$. Then $cx^\alpha$ is a multiple of a monomial from $C_j$ and from no other $C_i$. 
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Since the coefficients of \( f \) are linear function of the coefficients of the \( C_i \) via (*) , this determines a linear map \( a \) from coefficients of the \( C_i \) to coefficients of \( f \). Each non-zero entry in the matrix \( A \) is a coefficient of some \( f_i \). This defines the matrix \( A \) we mentioned earlier.

More precisely, if we index rows and columns of \( A \) by elements of \( X^d \), then the row corresponding to \( x^\alpha \) represents the polynomial

\[
\frac{x^\alpha}{x_i^d} f_i.
\]

The determinant of \( A \) vanishes if the \( f_i \) have a common zero, and it is therefore a multiple of the resultant \( R \) of the \( f_i \) [5]. We can write \( \text{det}(A) = MR \), where \( M \) is an additional factor which we should move. Macaulay shows that \( M \) is the determinant of a certain submatrix of \( A \), in fact the submatrix of elements whose row and column indices are not reduced. Hence \( R = \text{det}(A)/\text{det}(M) \).

REFERENCES

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