Online Resource Allocation Problems, with Applications to Revenue Management

David Simchi-Levi (MIT)

Papers

1. Tight Weight-dependent Competitive Ratios for Online Matching Problems
   - joint work with Will Ma
2. Dynamic Recommendation at Checkout under Inventory Constraint
   - joint work with Xi Chen, Will Ma, and Linwei Xin

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Extensions:

- can allow an *assortment* of items/prices to be offered to each customer
- can allow an item to have a *continuum* of potential prices
- can allow for *fractional* inventory consumption

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- algorithm makes no assumption on arrival sequence

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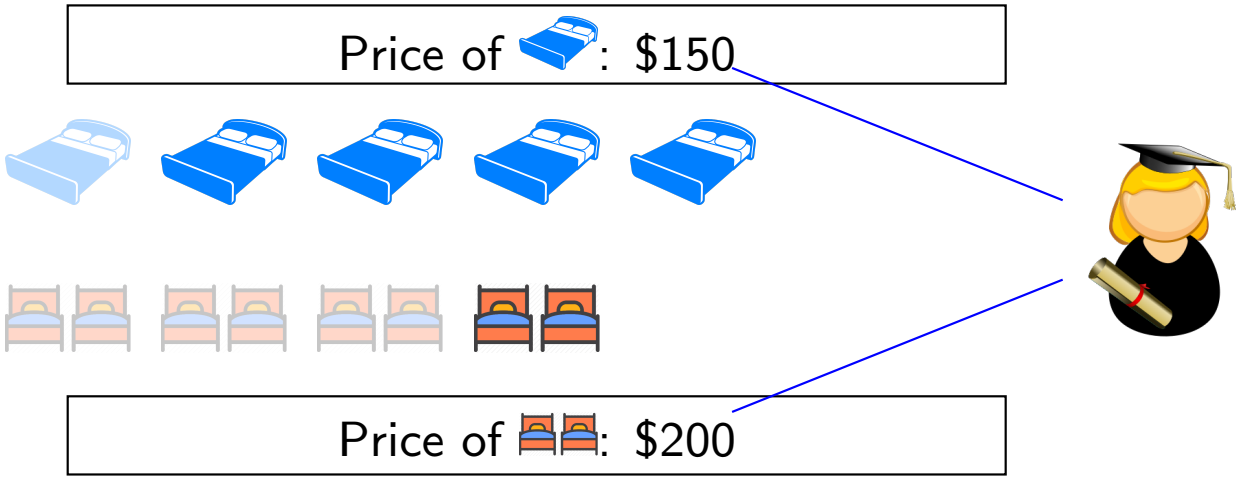

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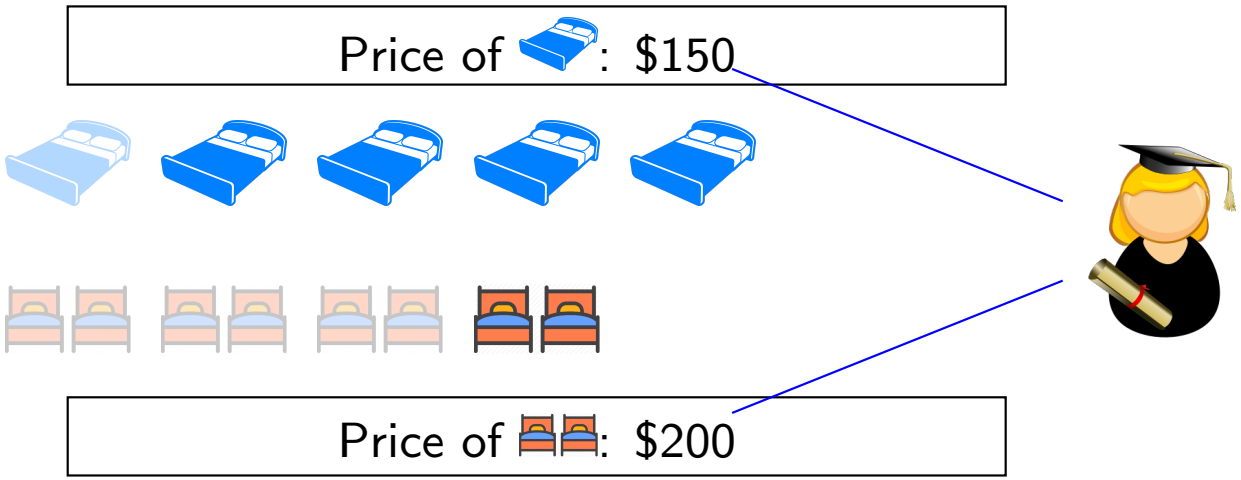

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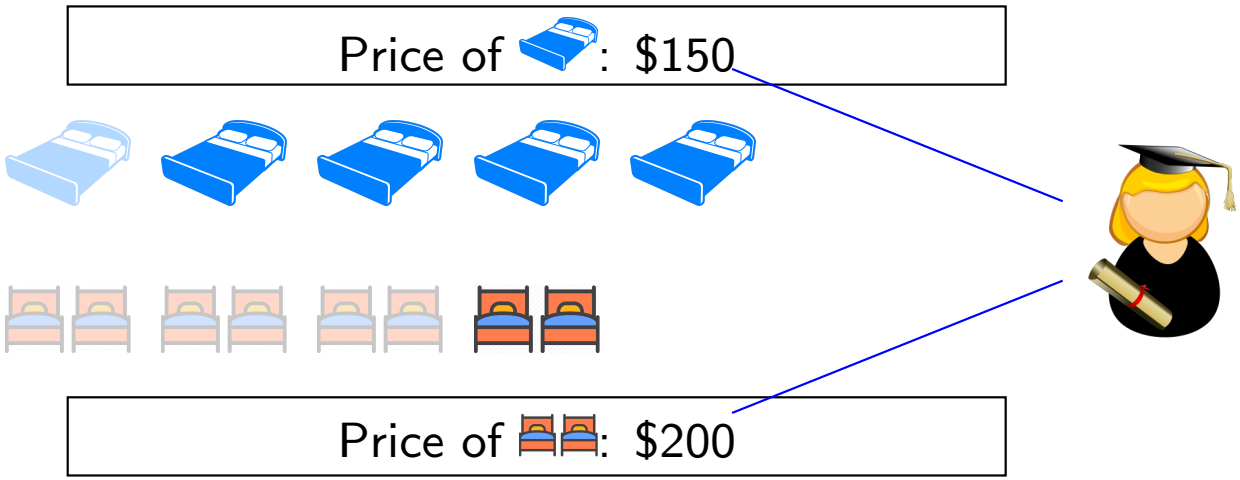

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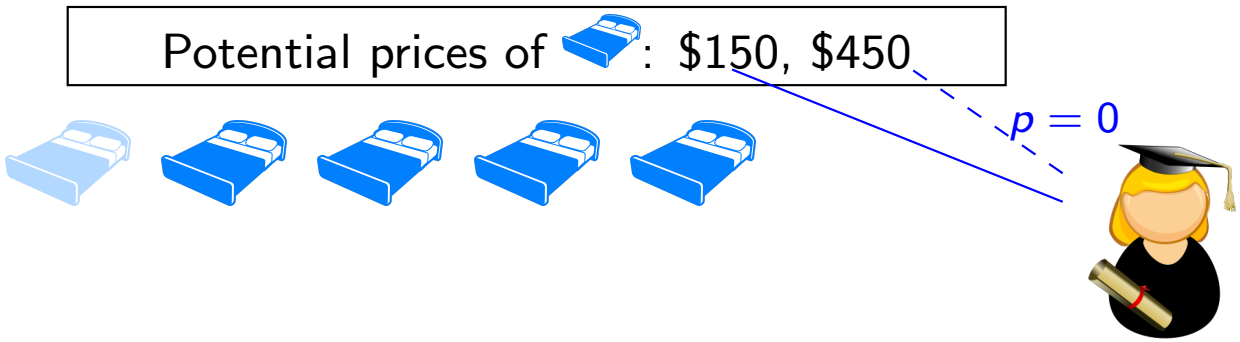
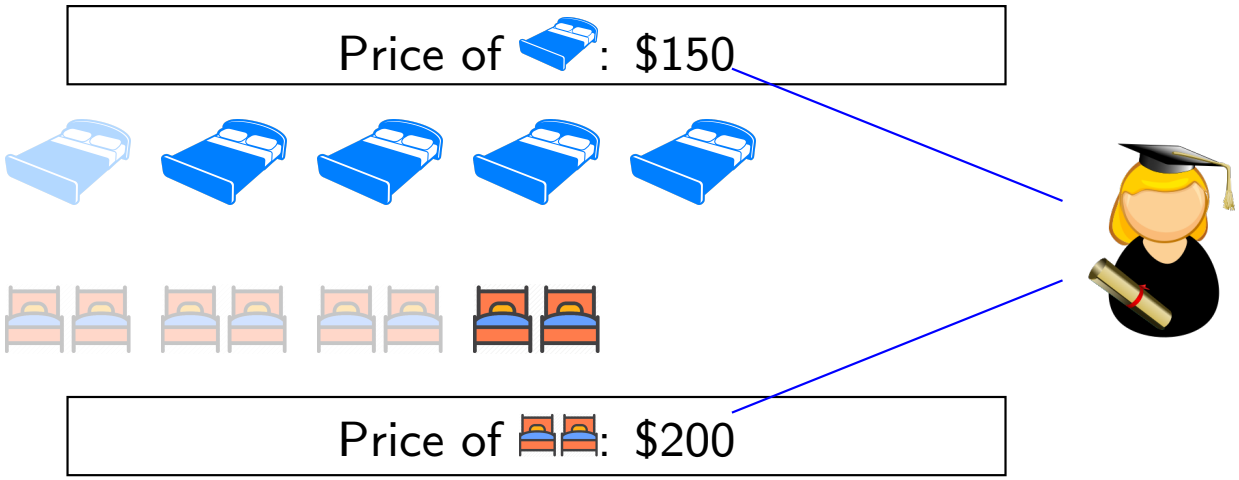


Challenge 1: Multiple Items



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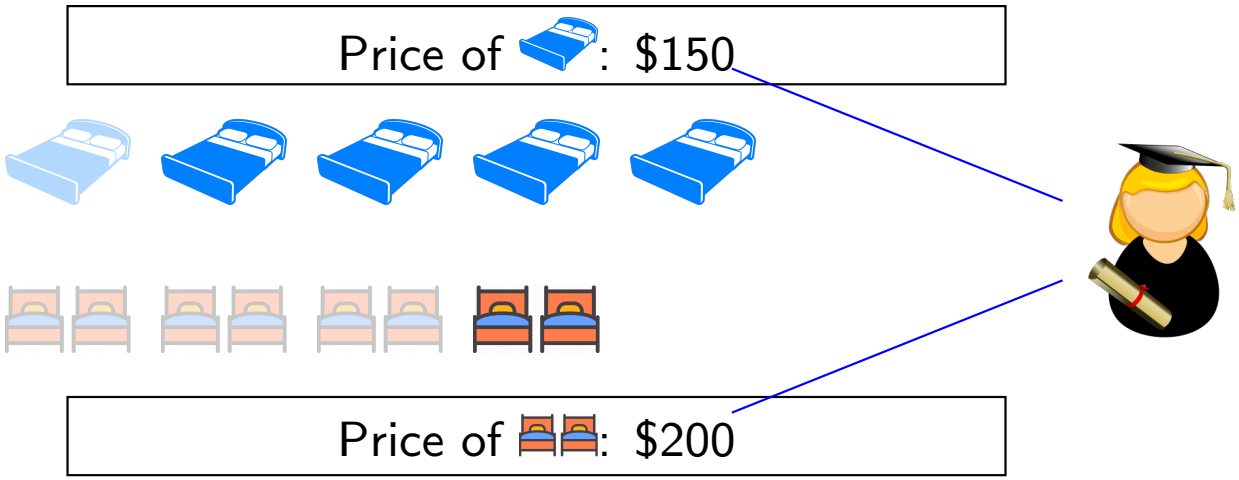

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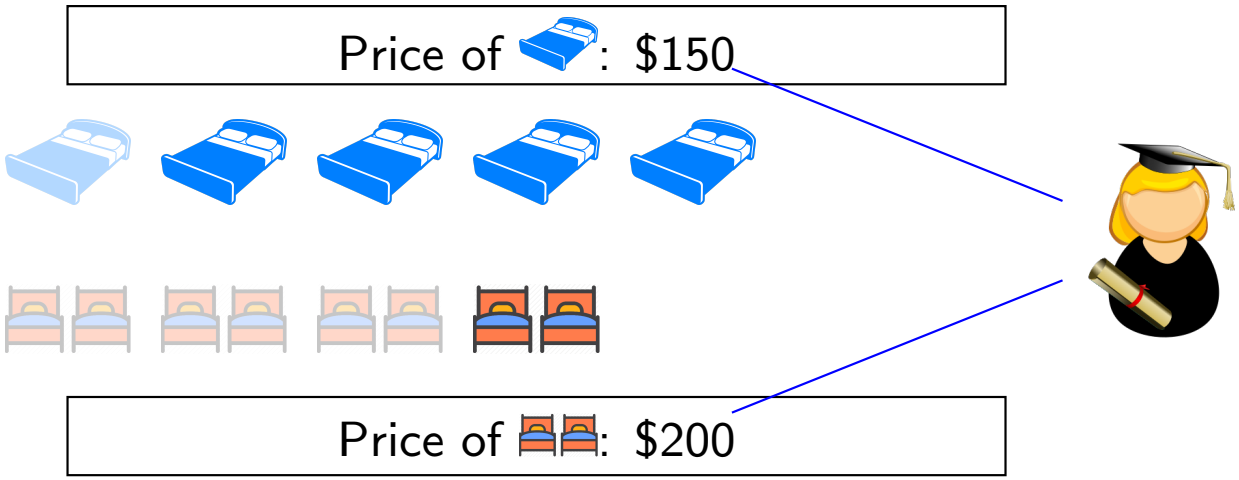

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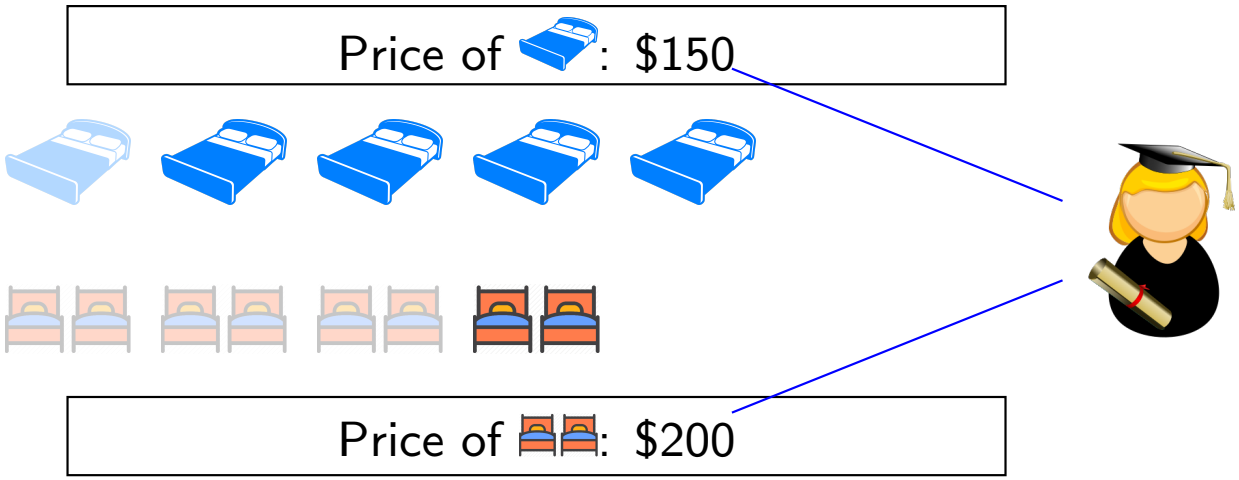
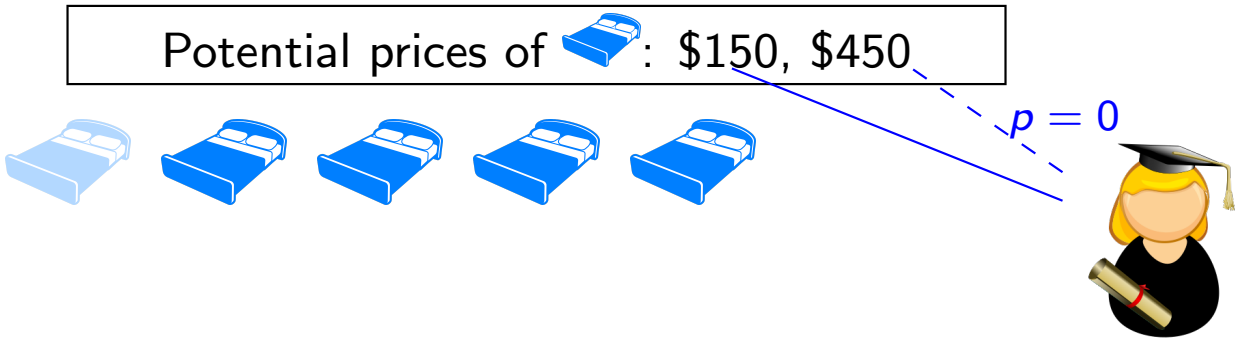


- **Solution**: "Inventory Balancing"
- *balance* the depletion of different items, relative to their prices
- idea has been used in the:
  - online *b*-matching problem (Kalyanasundaram/Pruhs '00)
  - Adwords problem (Mehta et al. '05, Buchbinder/Jain/Naor '07)
  - personalized assortment problem (Golrezaei/Nazerzadeh/Rusmevichientong '14)

Challenge 2: Multiple Prices



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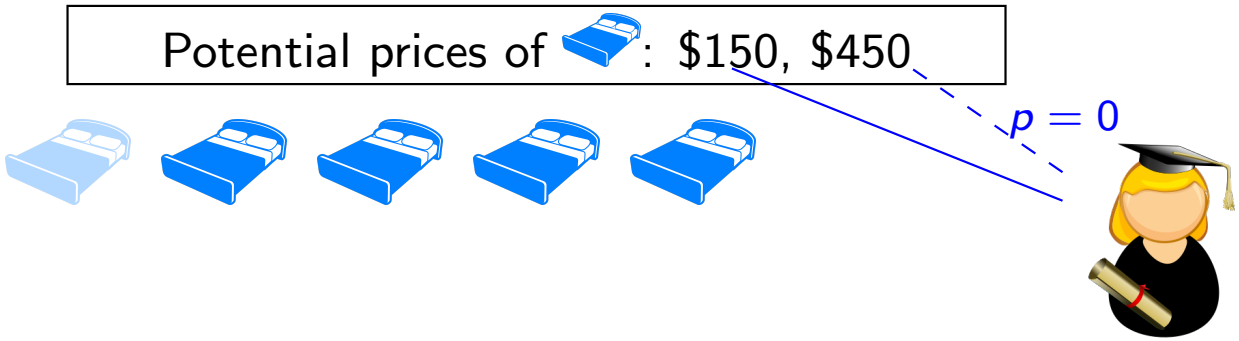

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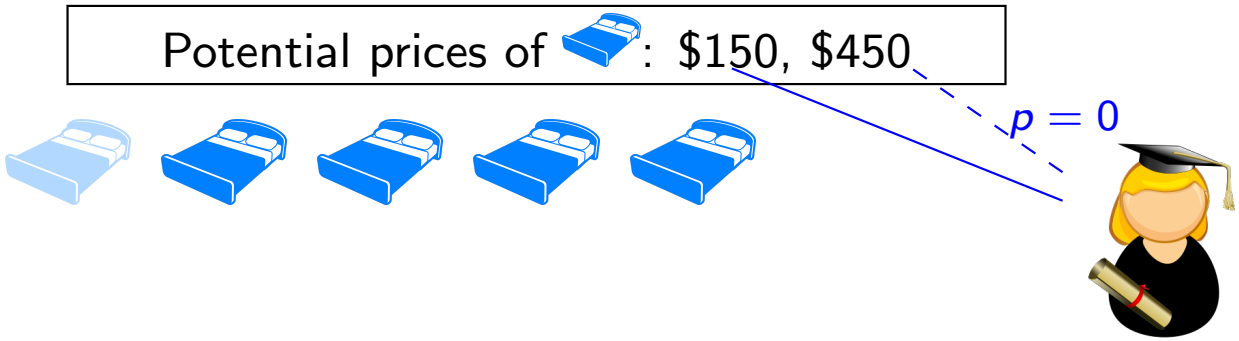

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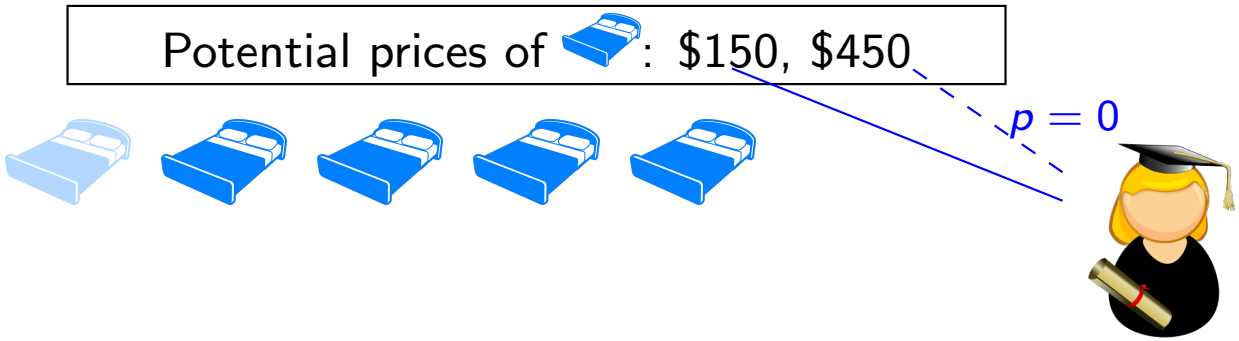

- **Solution**: "Booking Limits"
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- idea has been used in the:
  - one-way-trading problem (Lavi/Nisan '00, El-Yaniv et al. '01)
  - single-leg booking problem (Ball/Queyranne '09)
  - online dynamic pricing problem (Ma/S.-L./Teo '17)

Combining the Challenges

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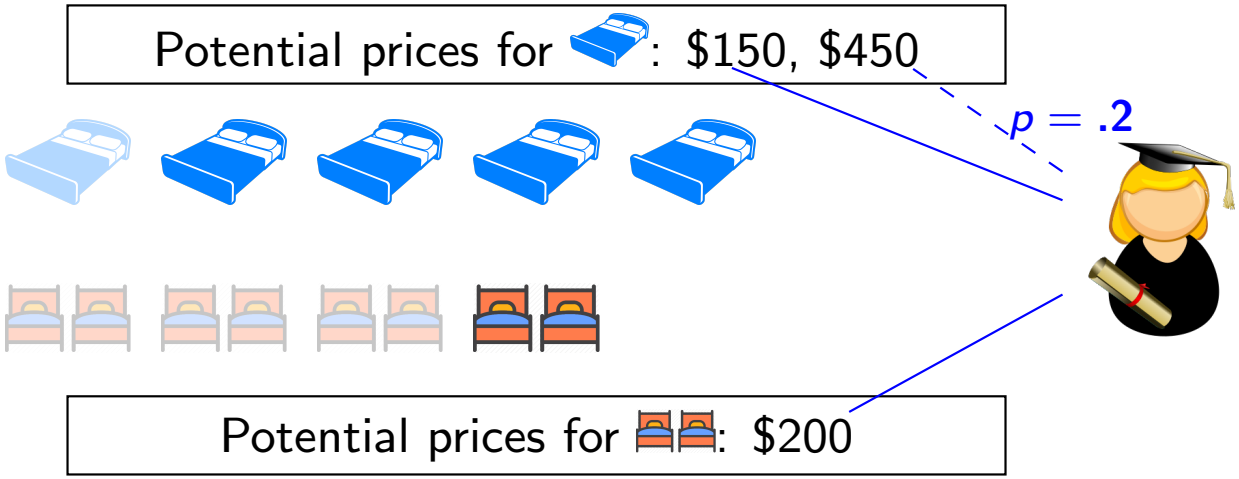

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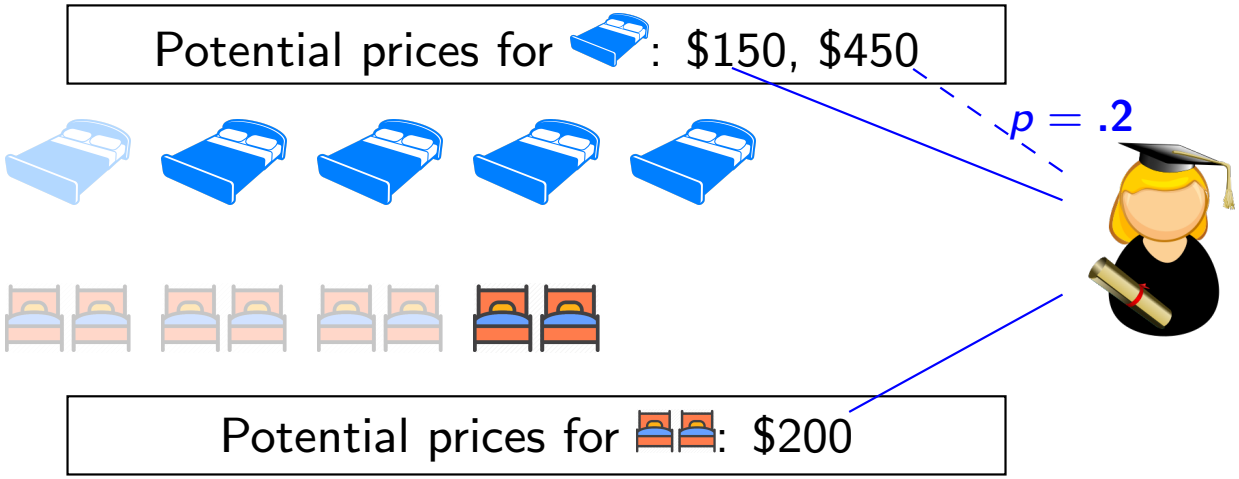

- **Bid-price Control Policy**: for each customer t, offer the item i and price j maximizing the expected "profit"

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where λ_i is the "cost", or bid price, for one unit of inventory of i

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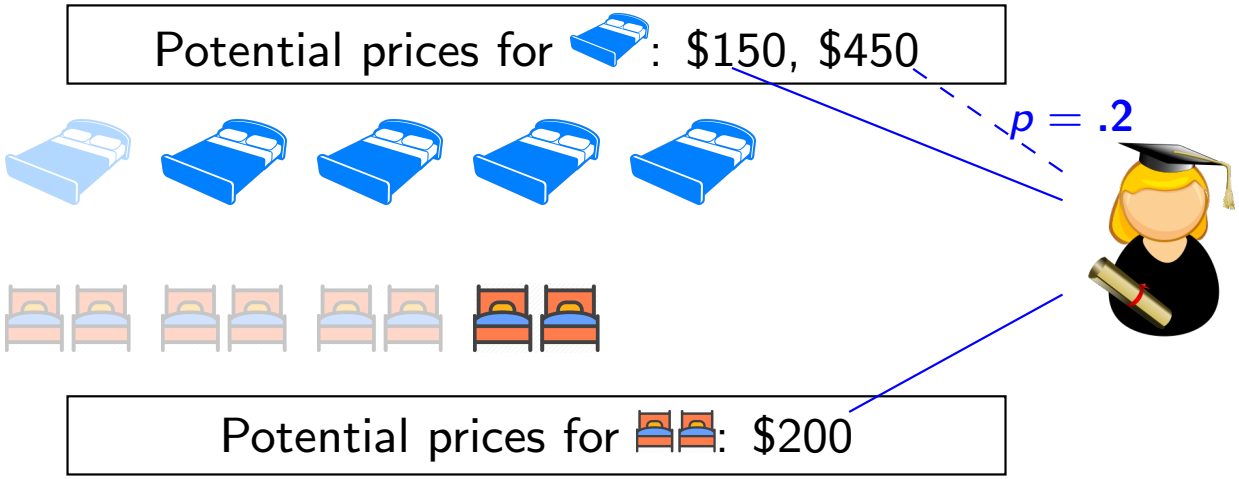

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- **Key Question**: what is the value of λ_i?

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- $\lambda_i = \Phi_i(w_i)$, where Φ_i is an increasing function

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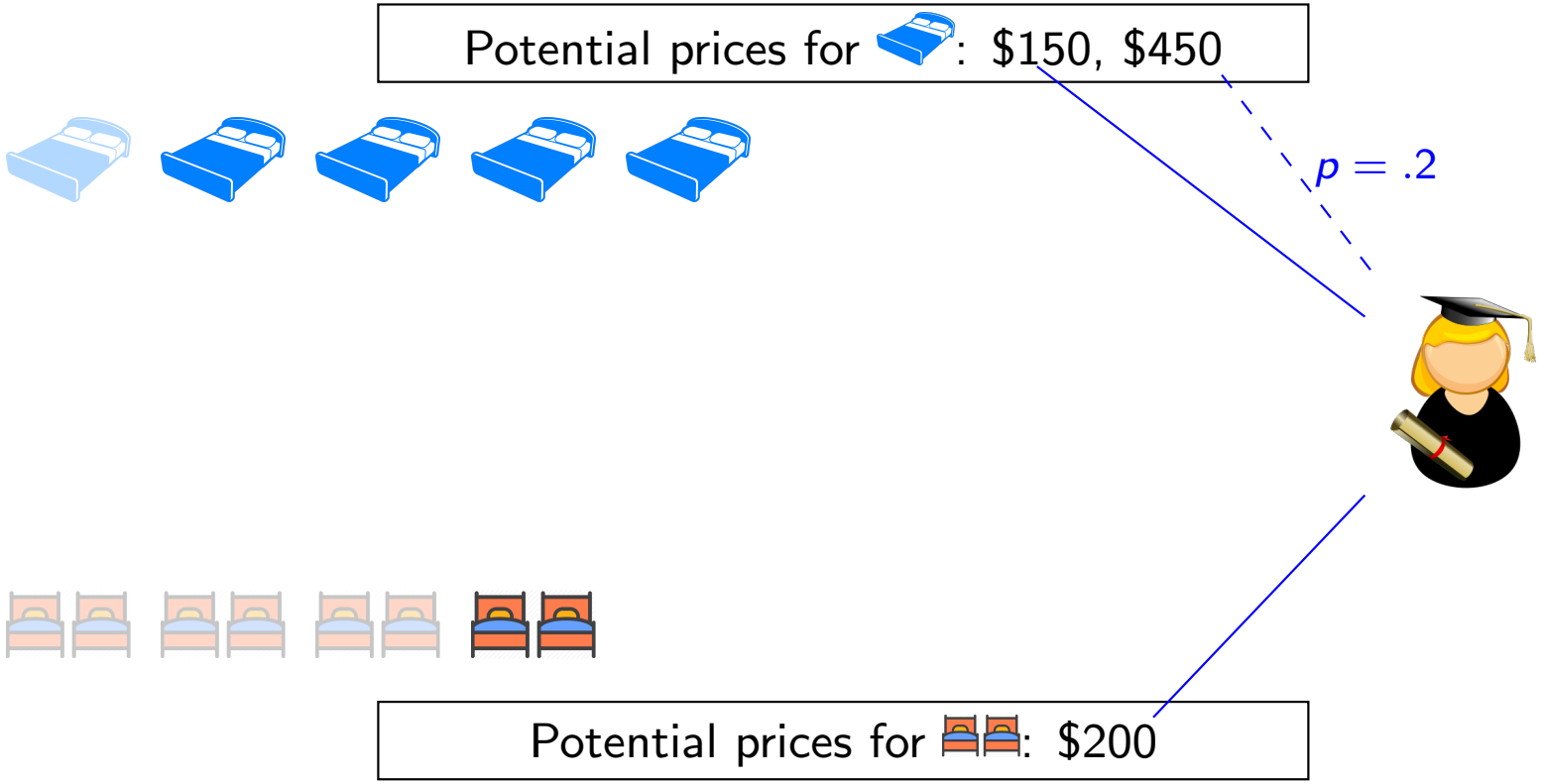

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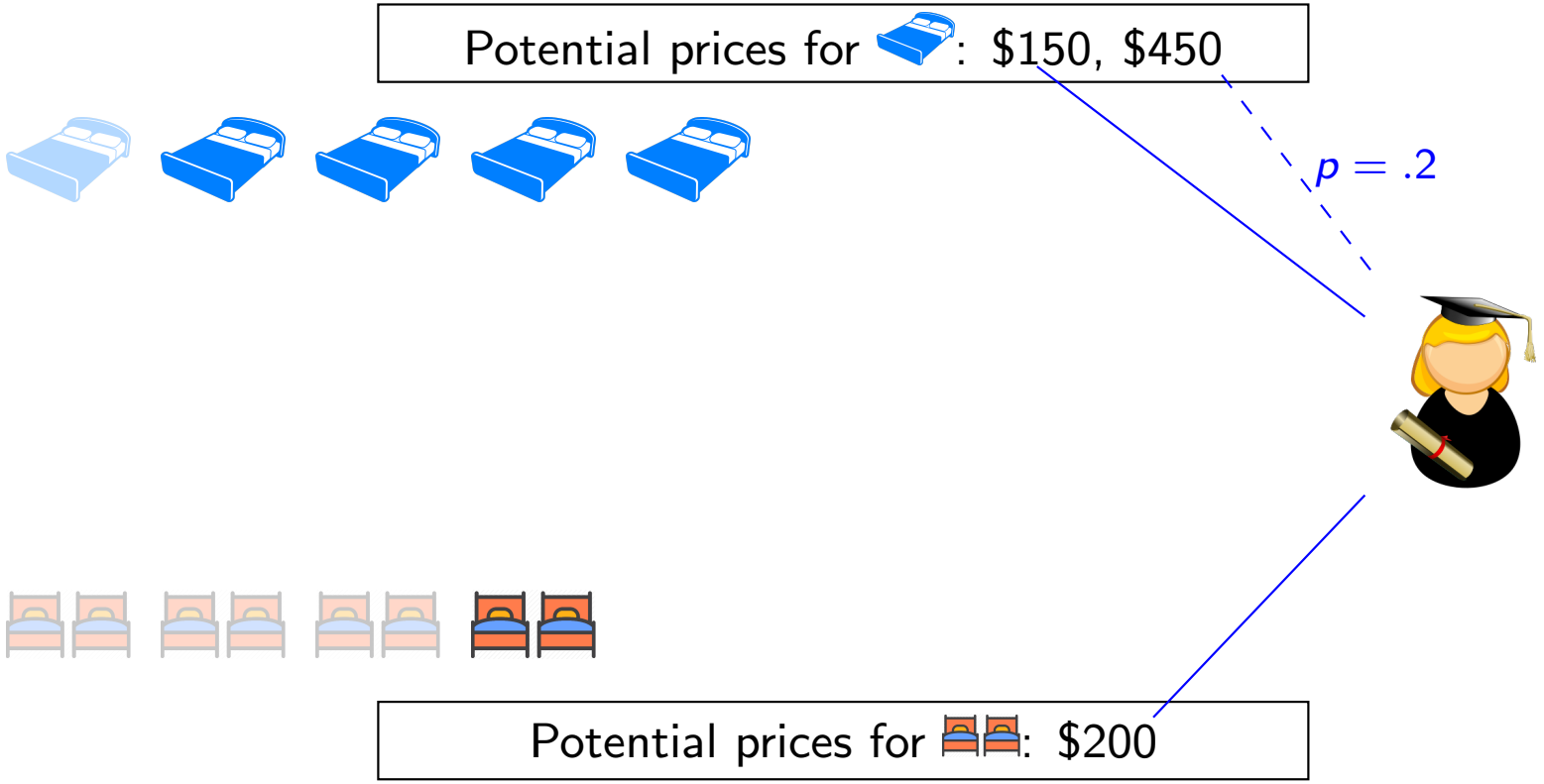

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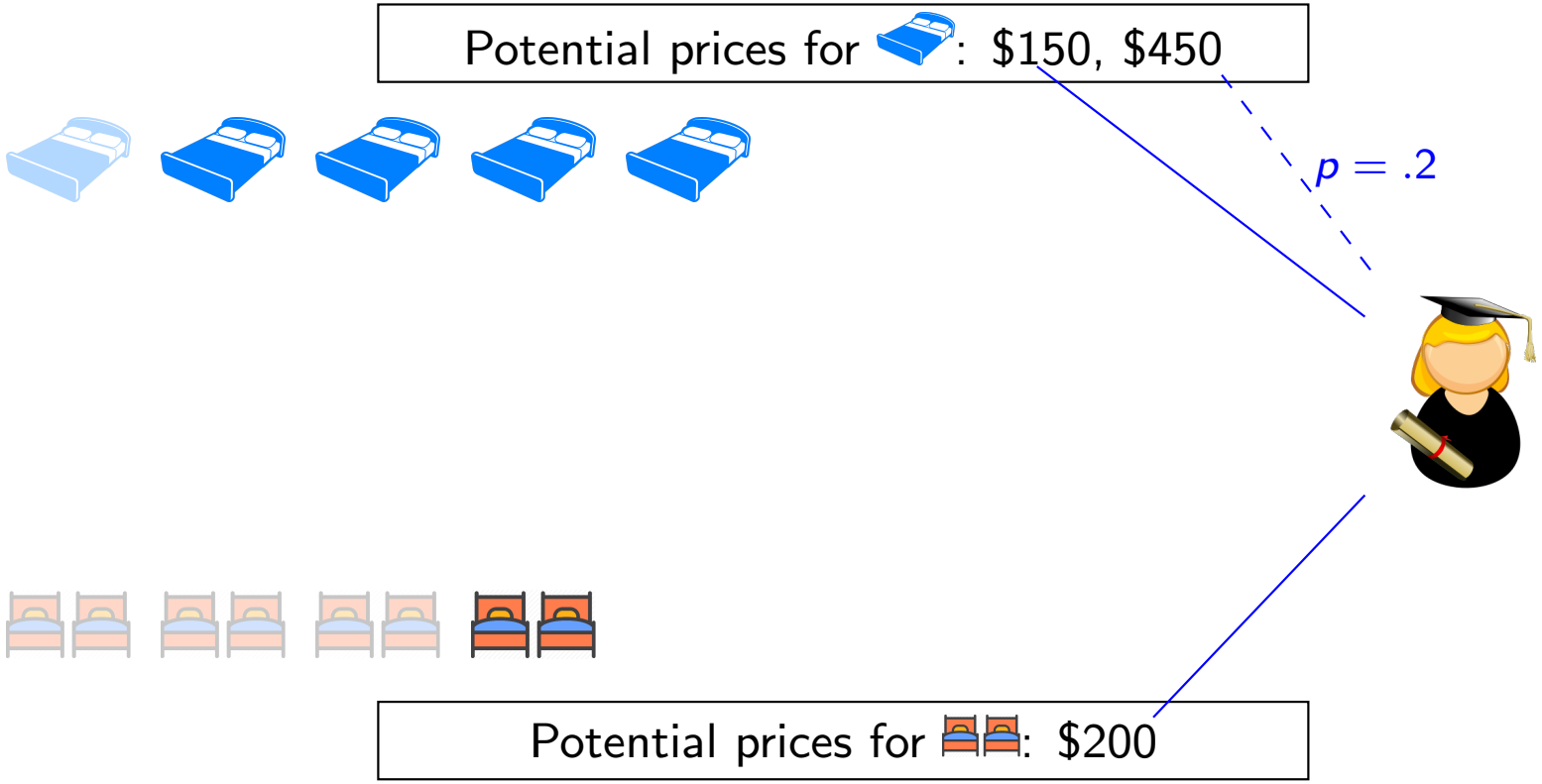

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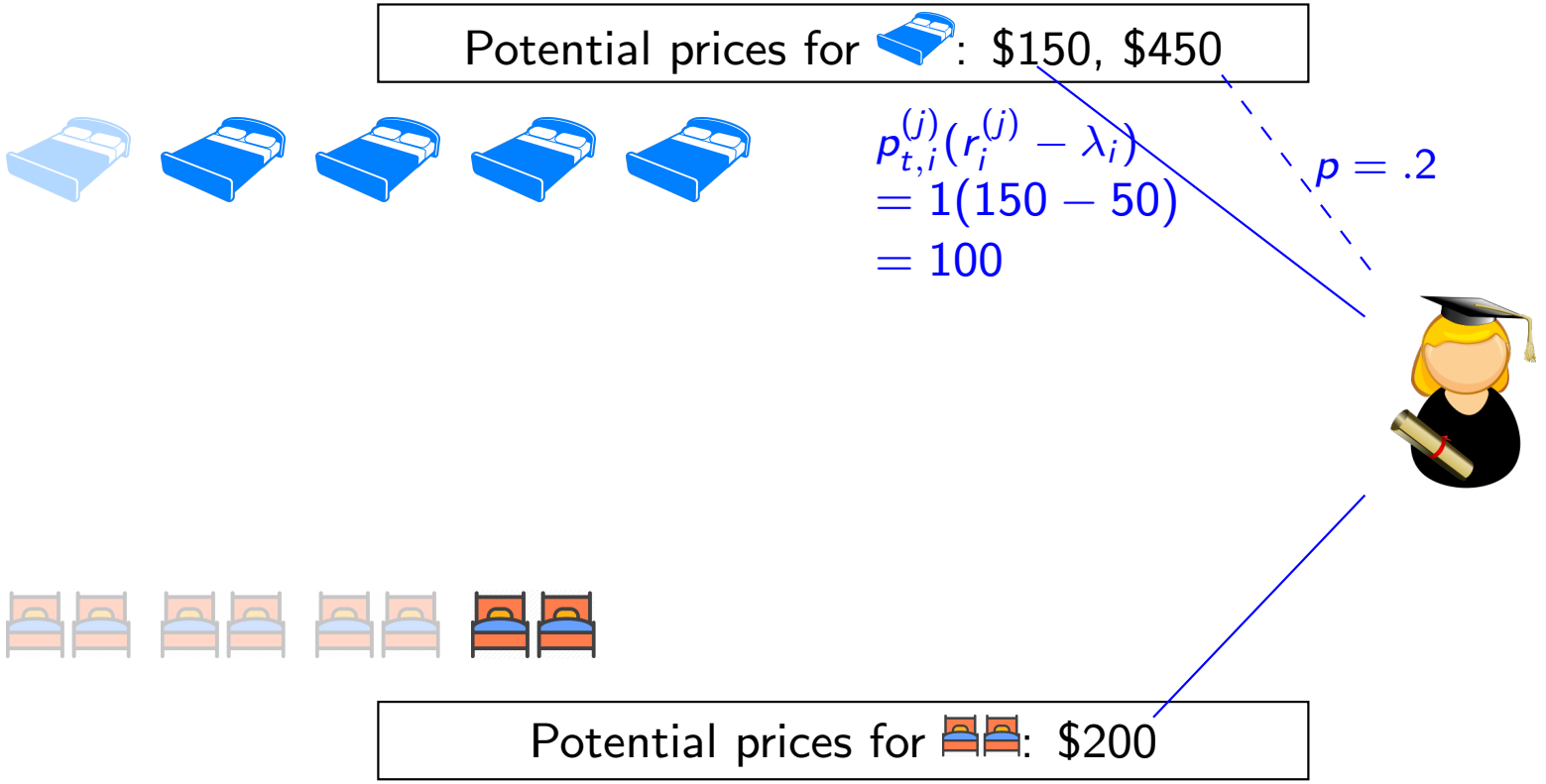

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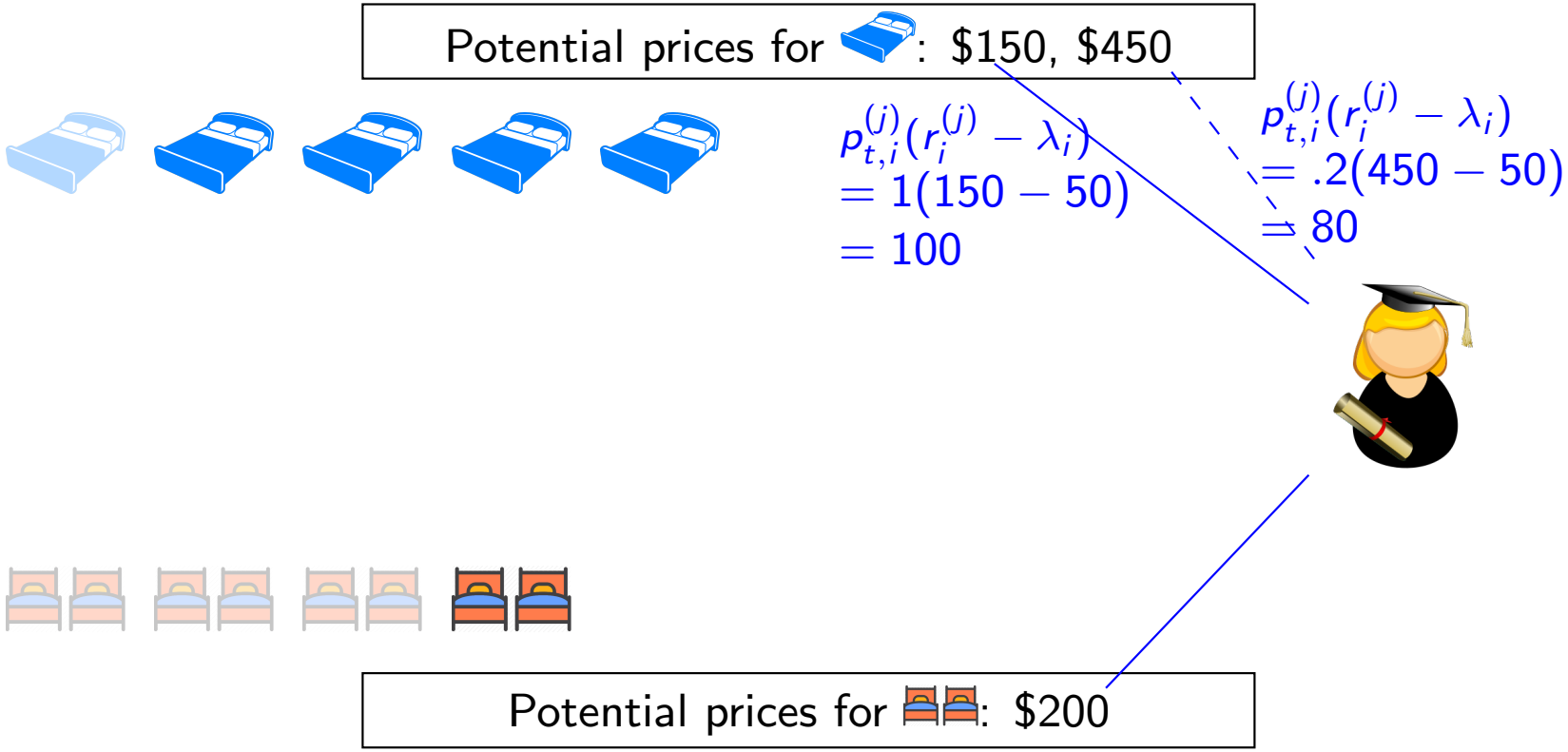

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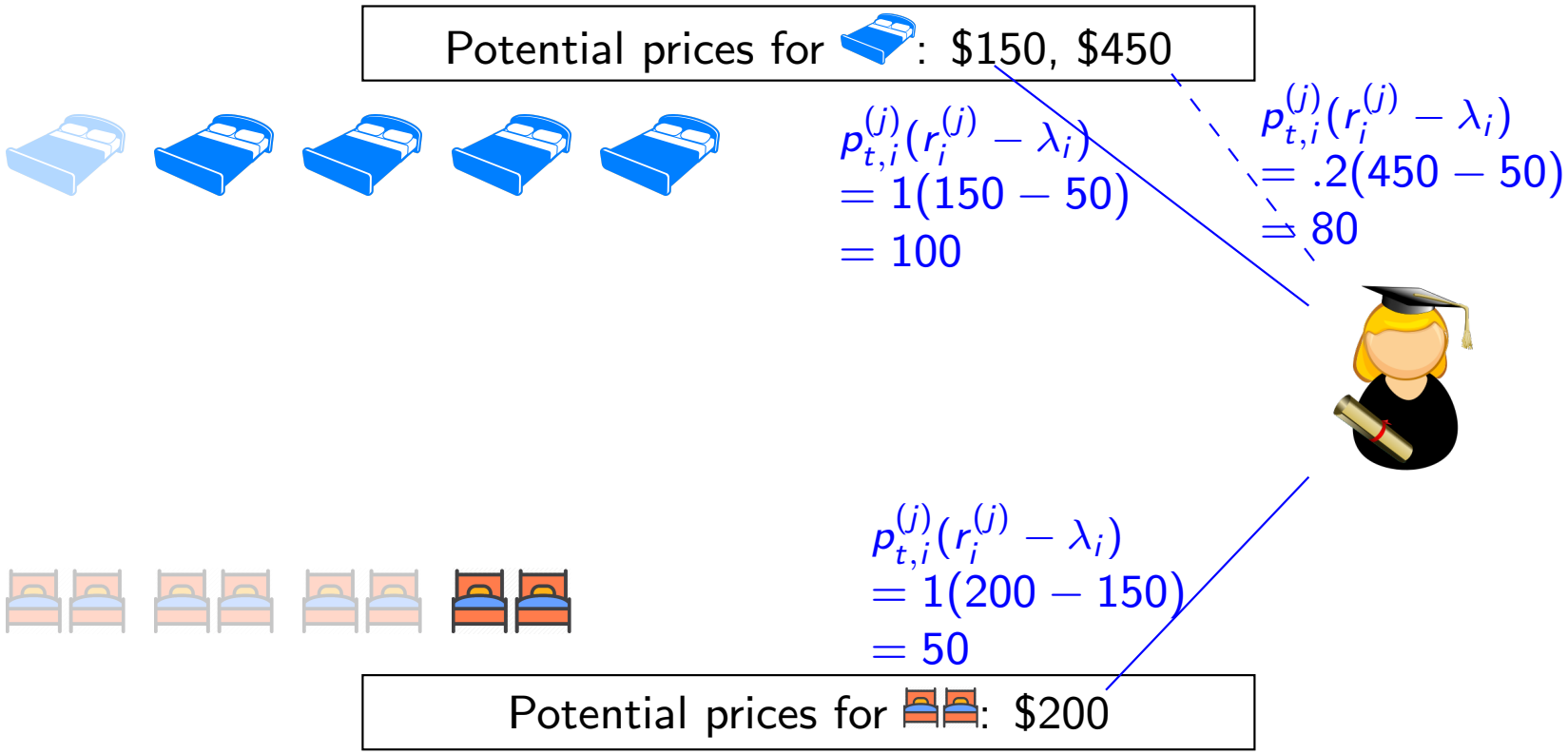

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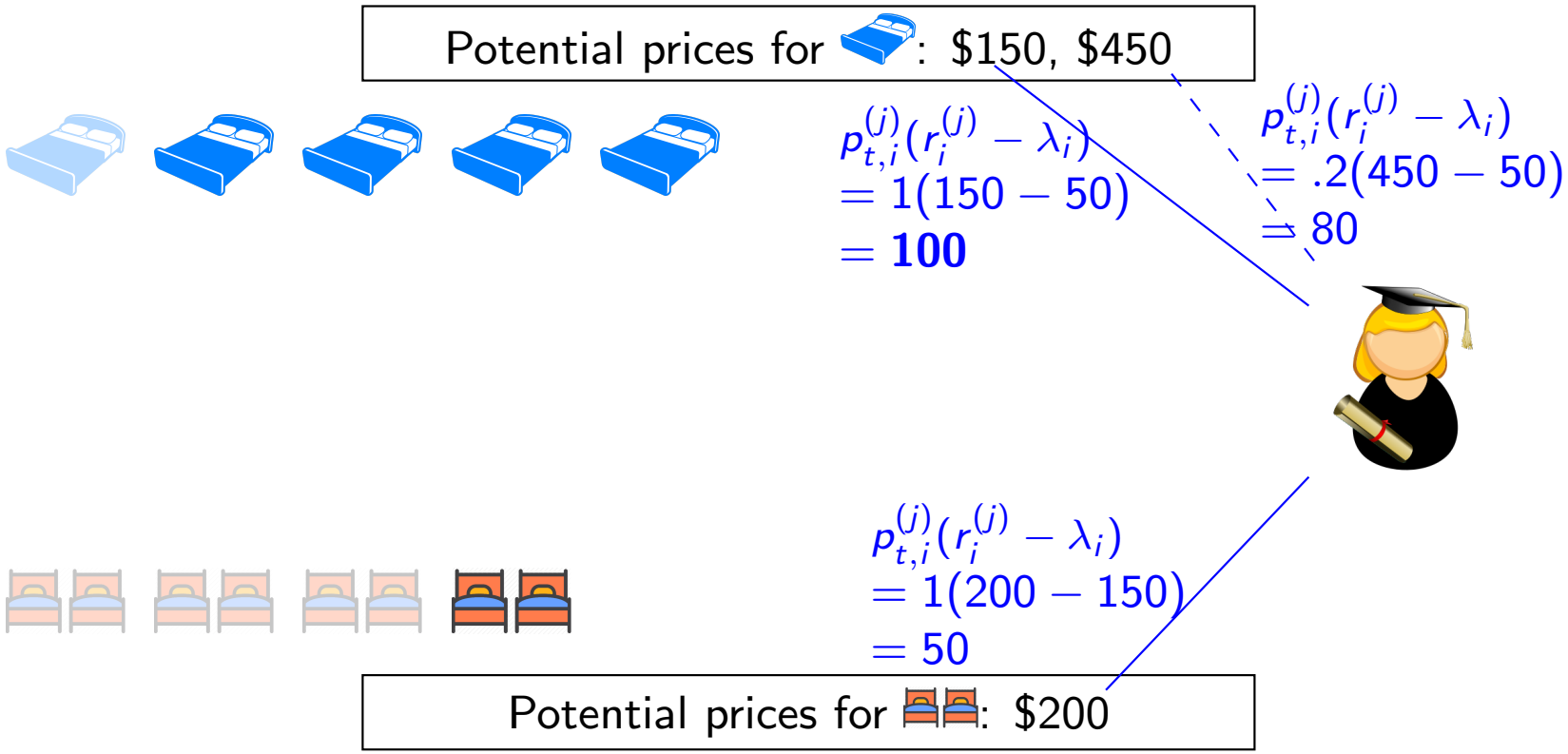

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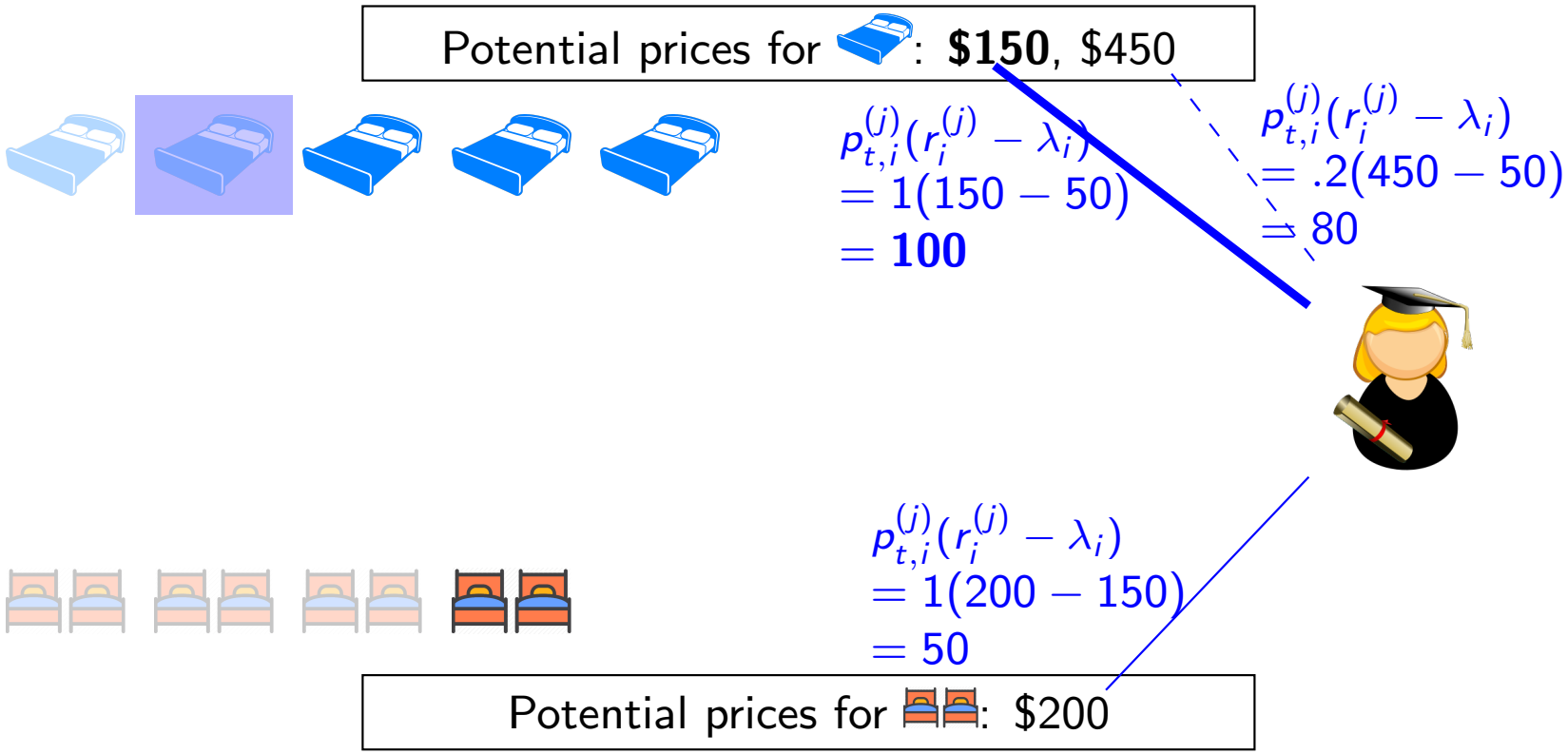

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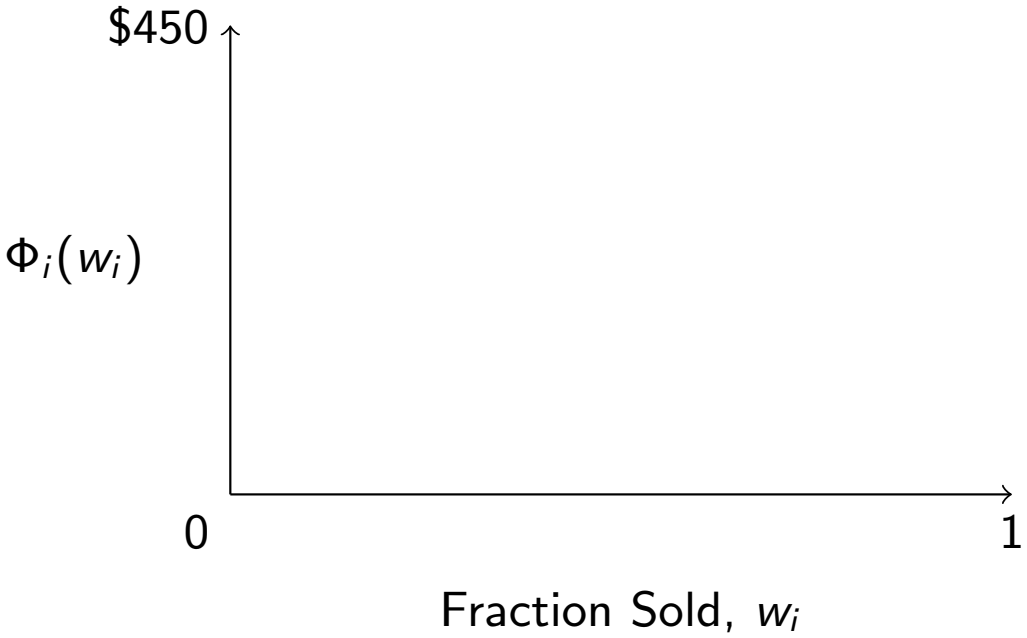

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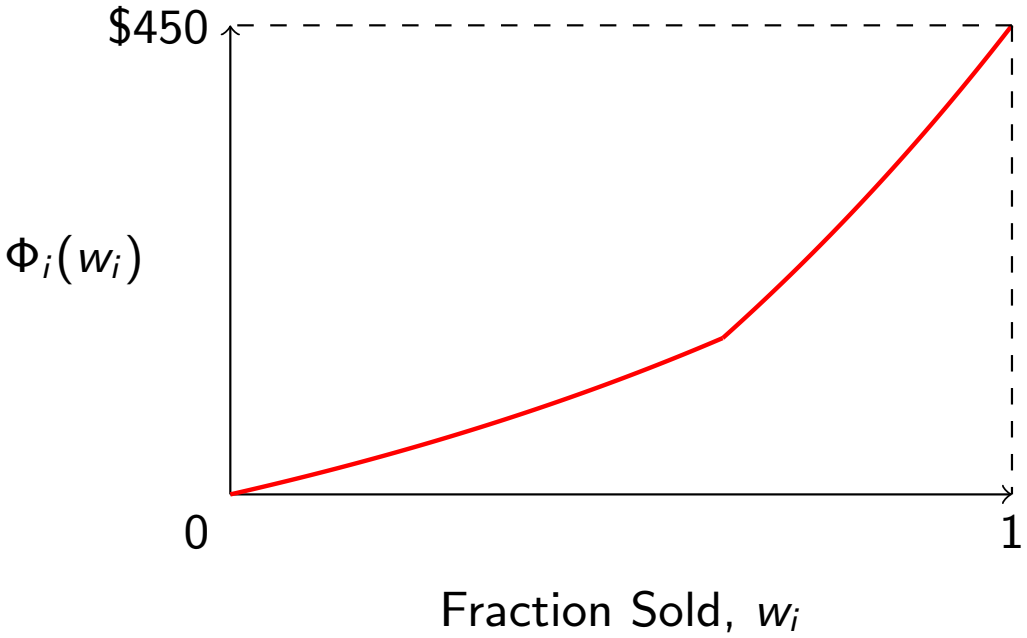

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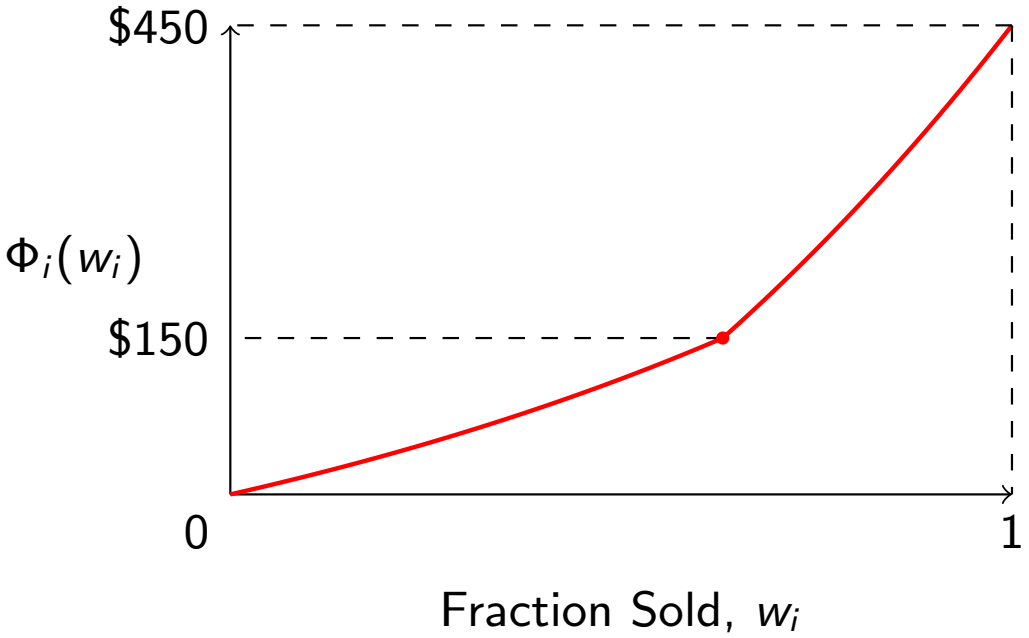

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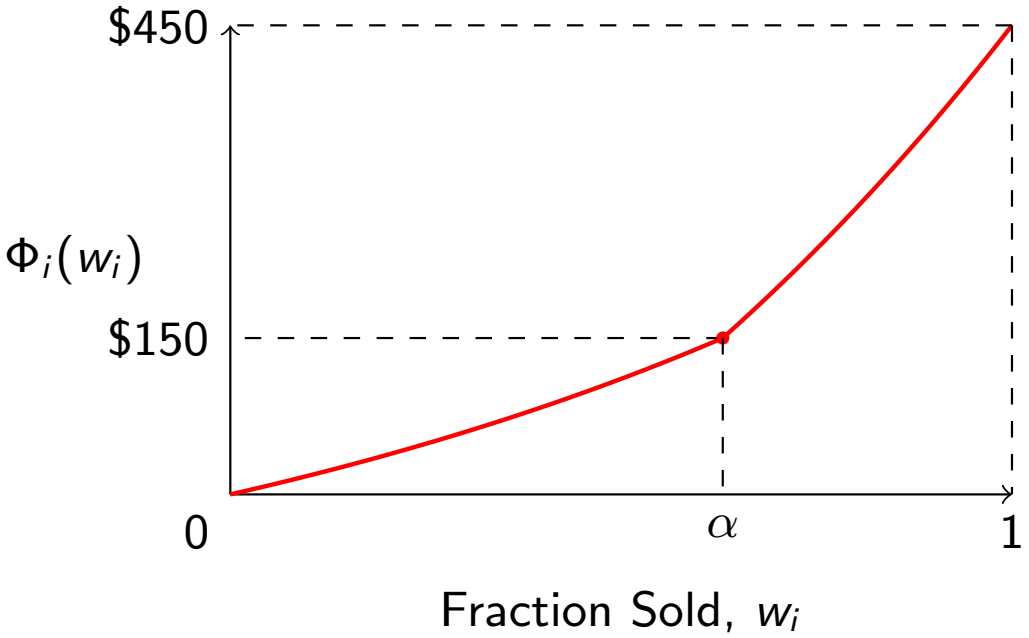

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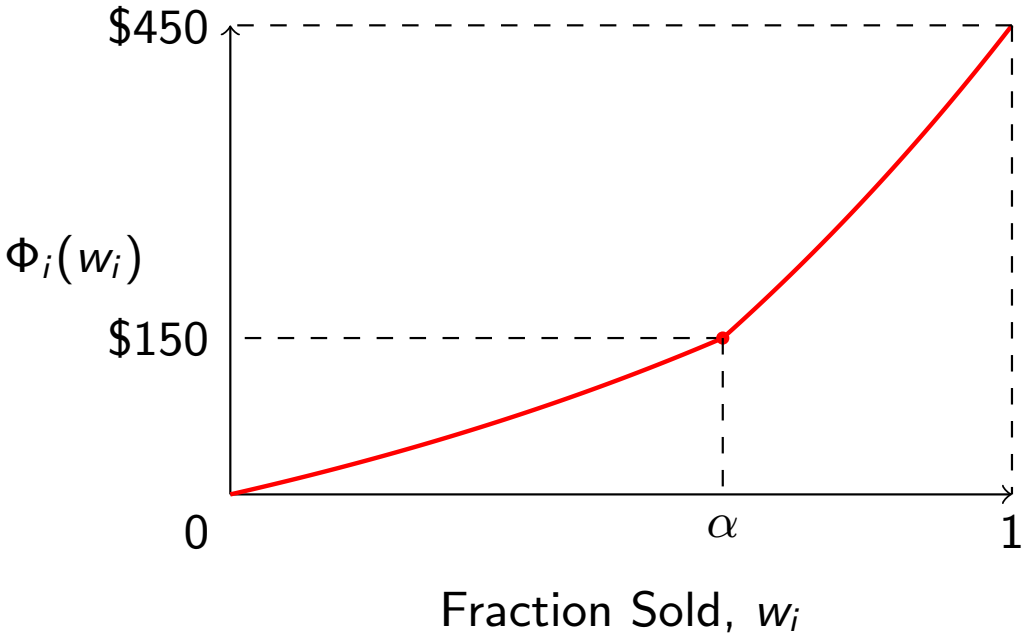

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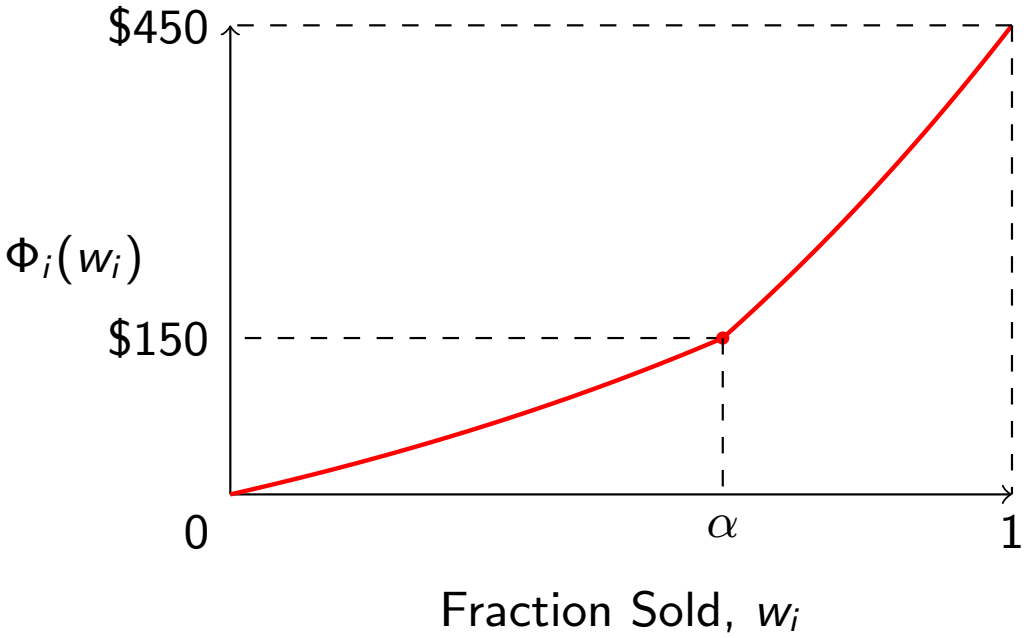

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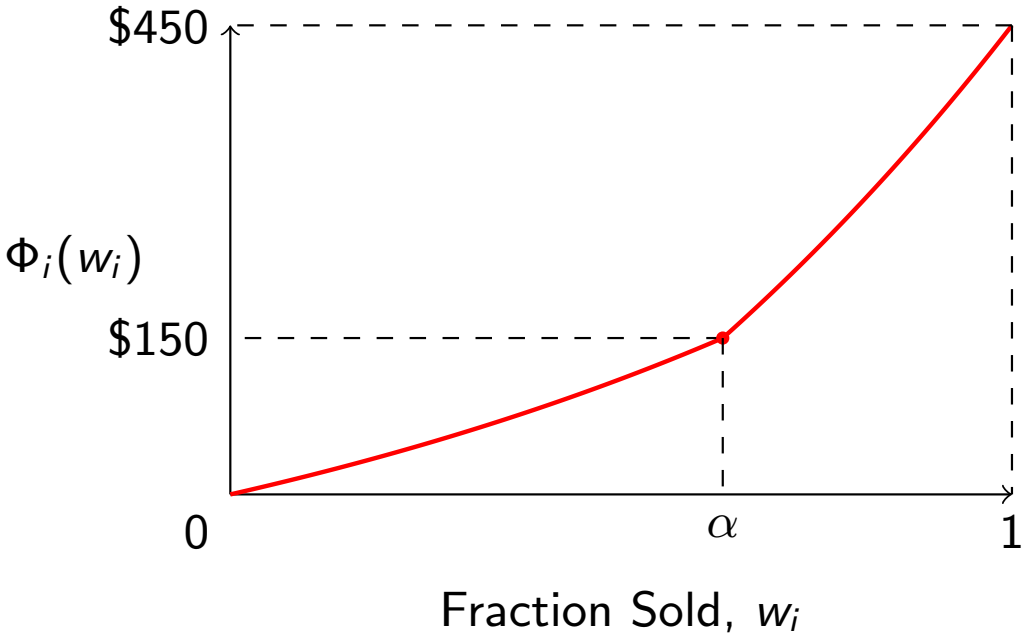

- $\alpha = \ln \frac{2(r-1)}{\sqrt{1+4r(r-1)/e}-1}$, where r is the ratio of high price to low price
- different than the optimal *single-item* booking limit $\sigma = \frac{r}{2r-1}$ of Ball/Queyranne

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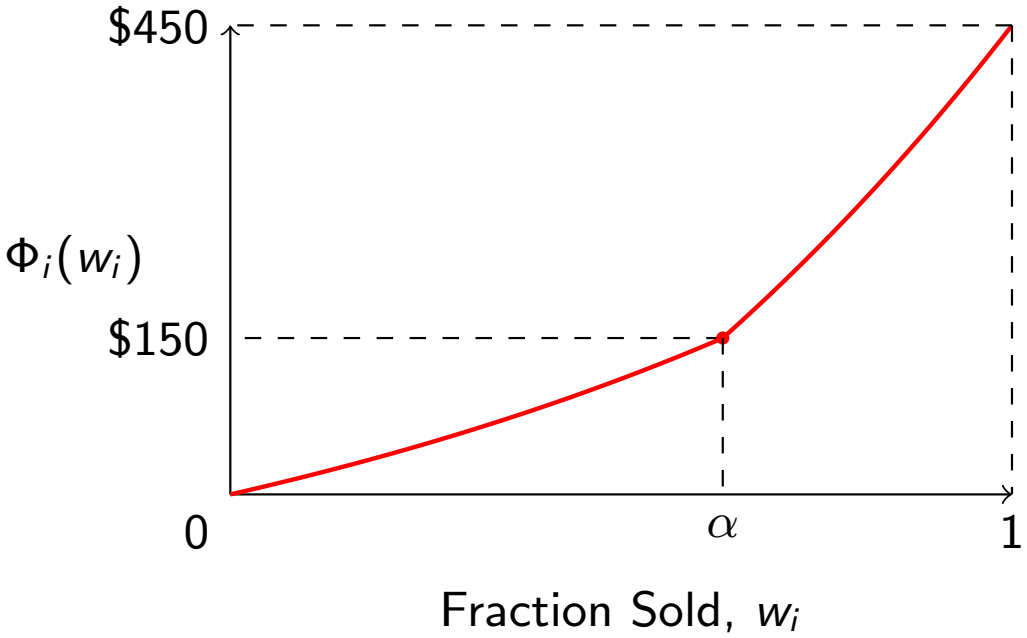

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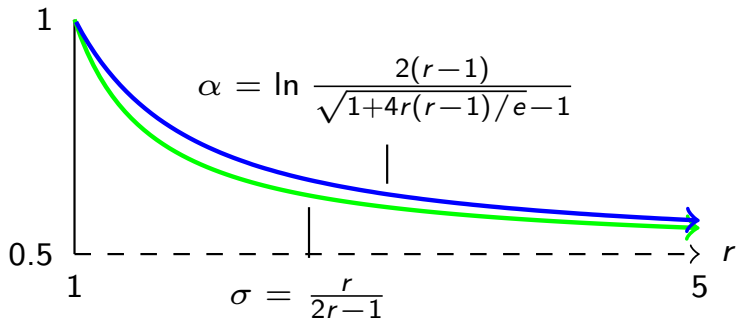
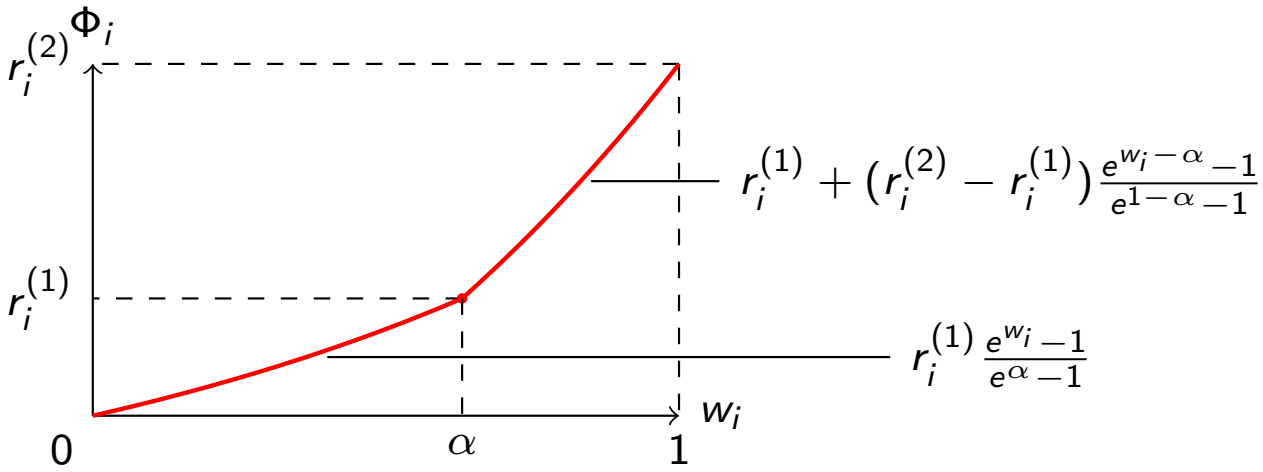
- $\alpha = \ln \frac{2(r-1)}{\sqrt{1+4r(r-1)/e}-1}$, where r is the ratio of high price to low price
- different than the optimal *single-item* booking limit $\sigma = \frac{r}{2r-1}$ of Ball/Queyranne



General Value Function Φ_j for 2 Prices

General Value Function Φ_i for 2 Prices

- when $m_i = 2$, Φ_i is piecewise-convex with 2 pieces, separated by α:



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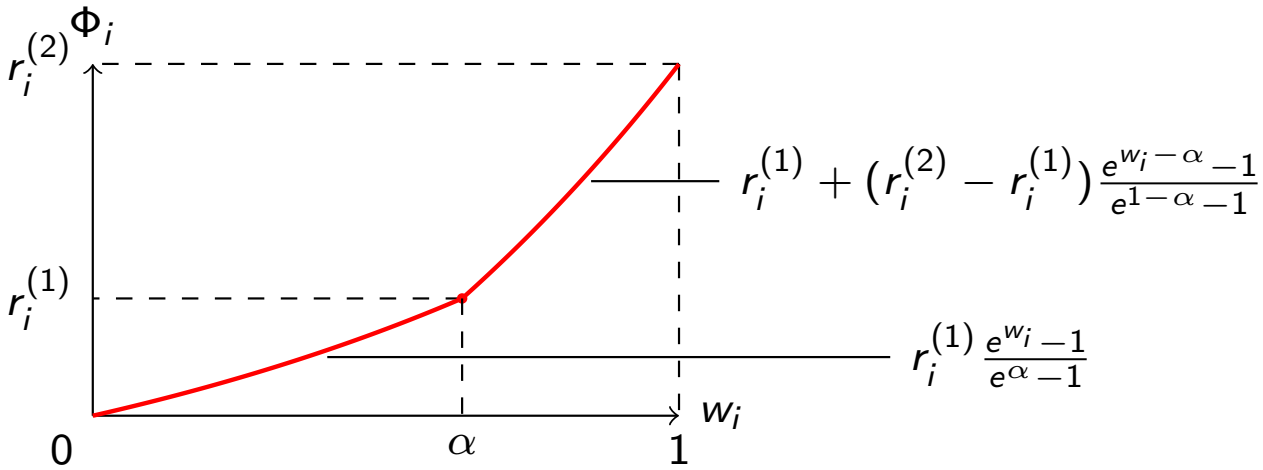

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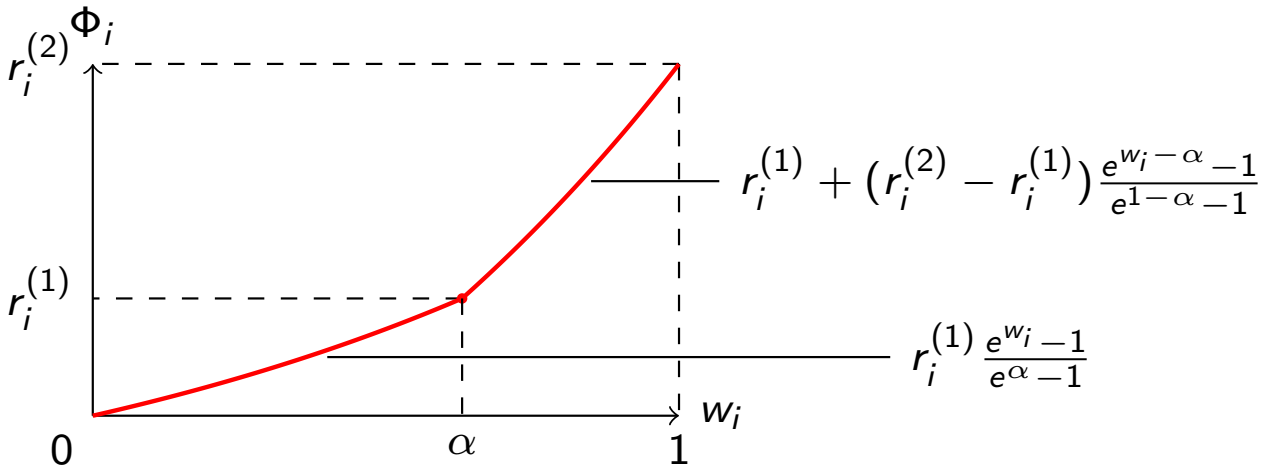

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- the competitive ratio associated with Φ_i, CR_i, is then $1 - e^{-\alpha}$

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Bid-price Control Policy:

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The Competitive Ratio

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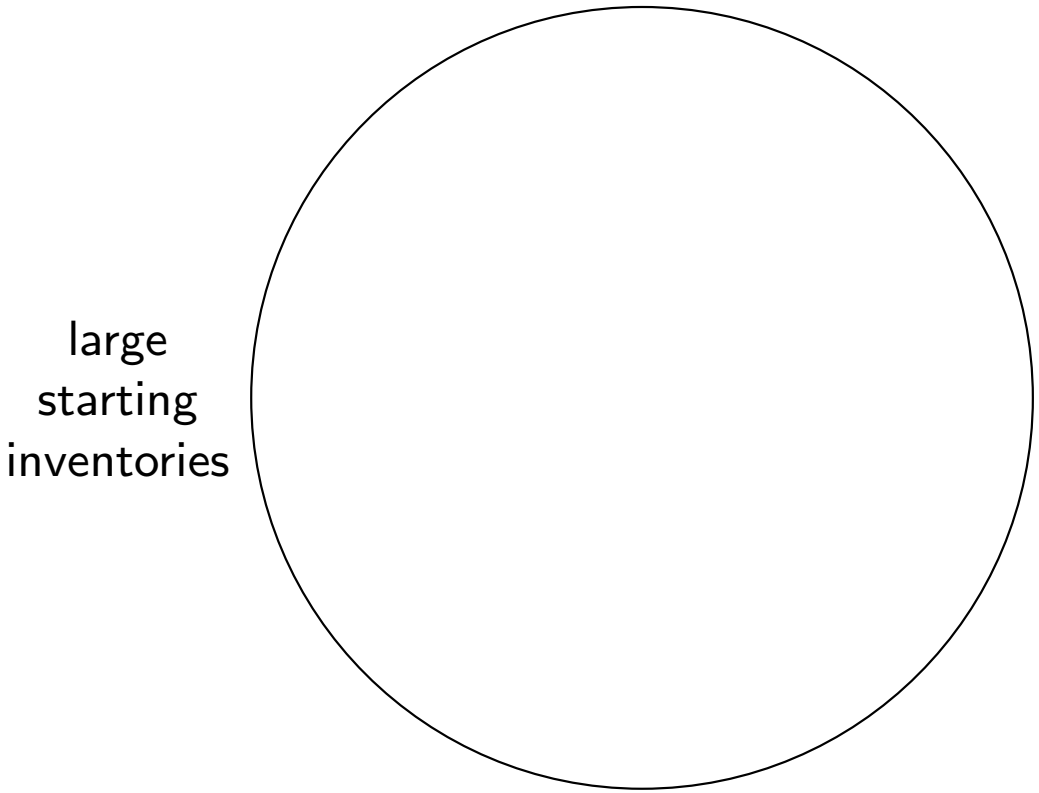
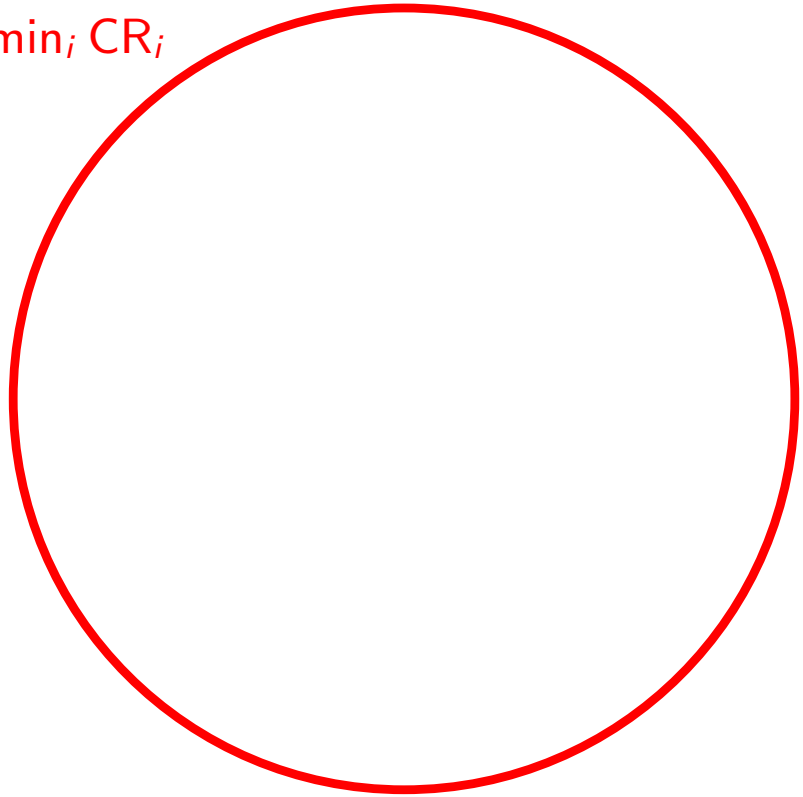


The Competitive Ratio

Algorithm 1
achieves $\min_i CR_i$

large
starting
inventories



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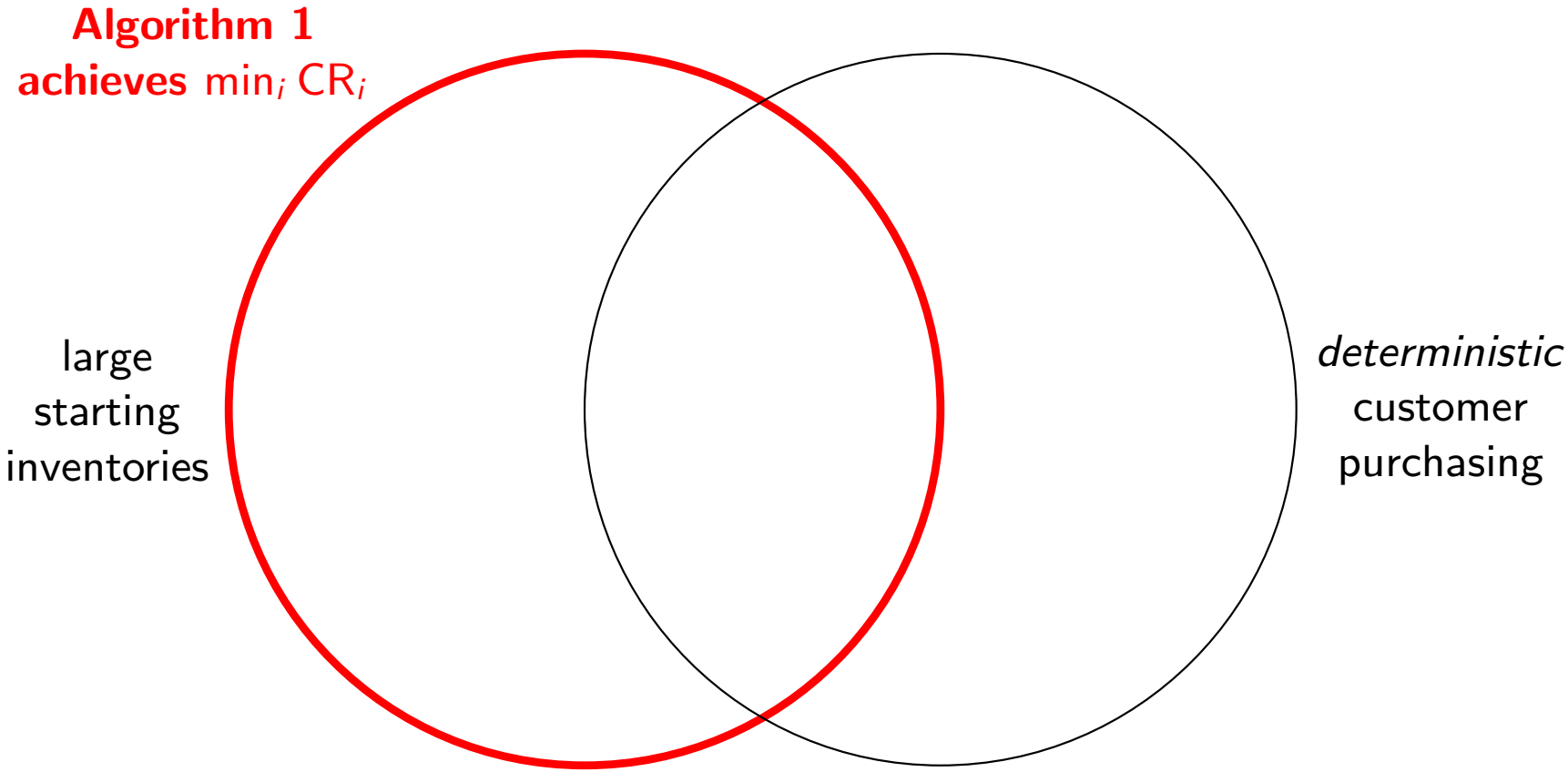

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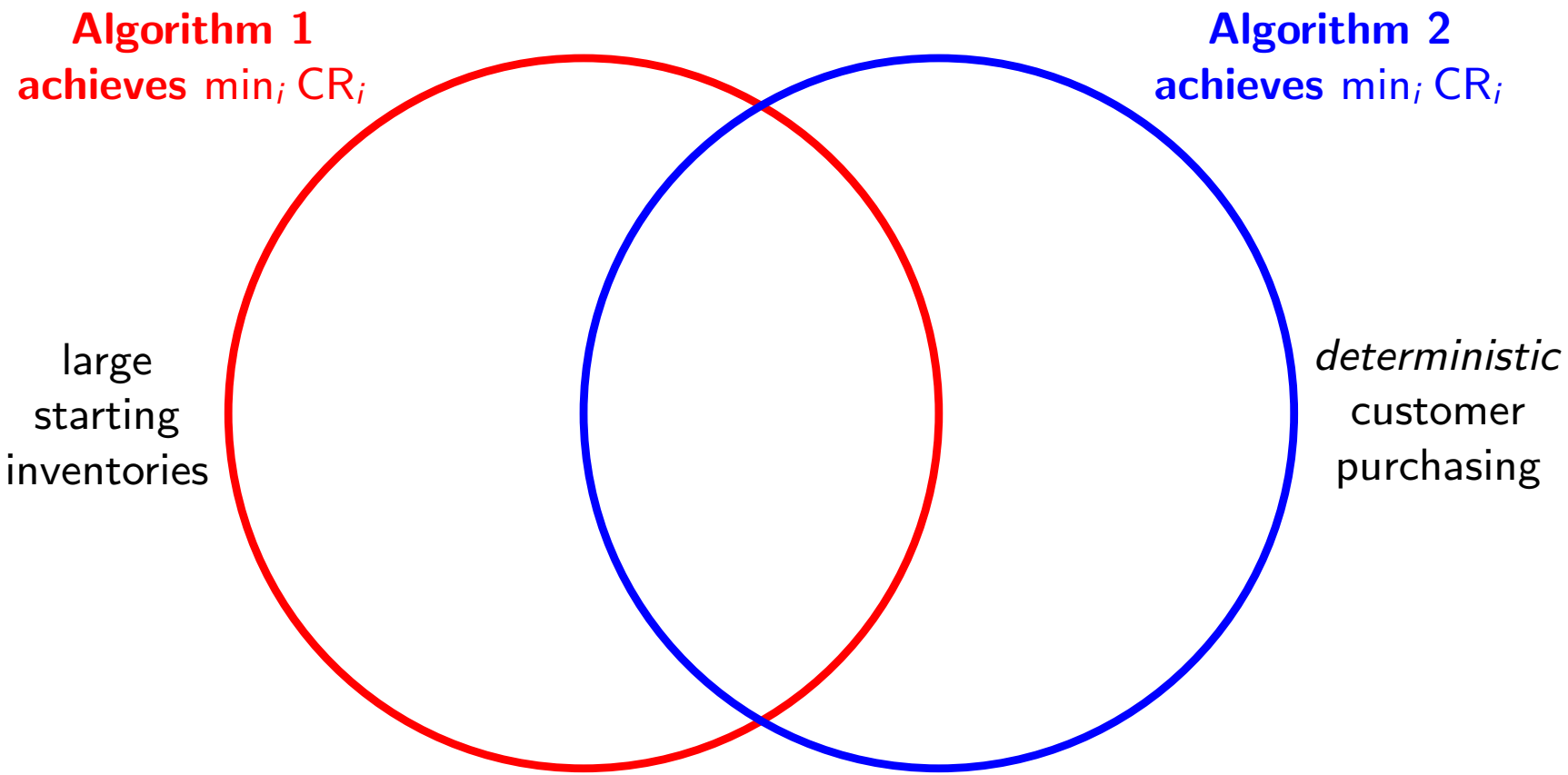

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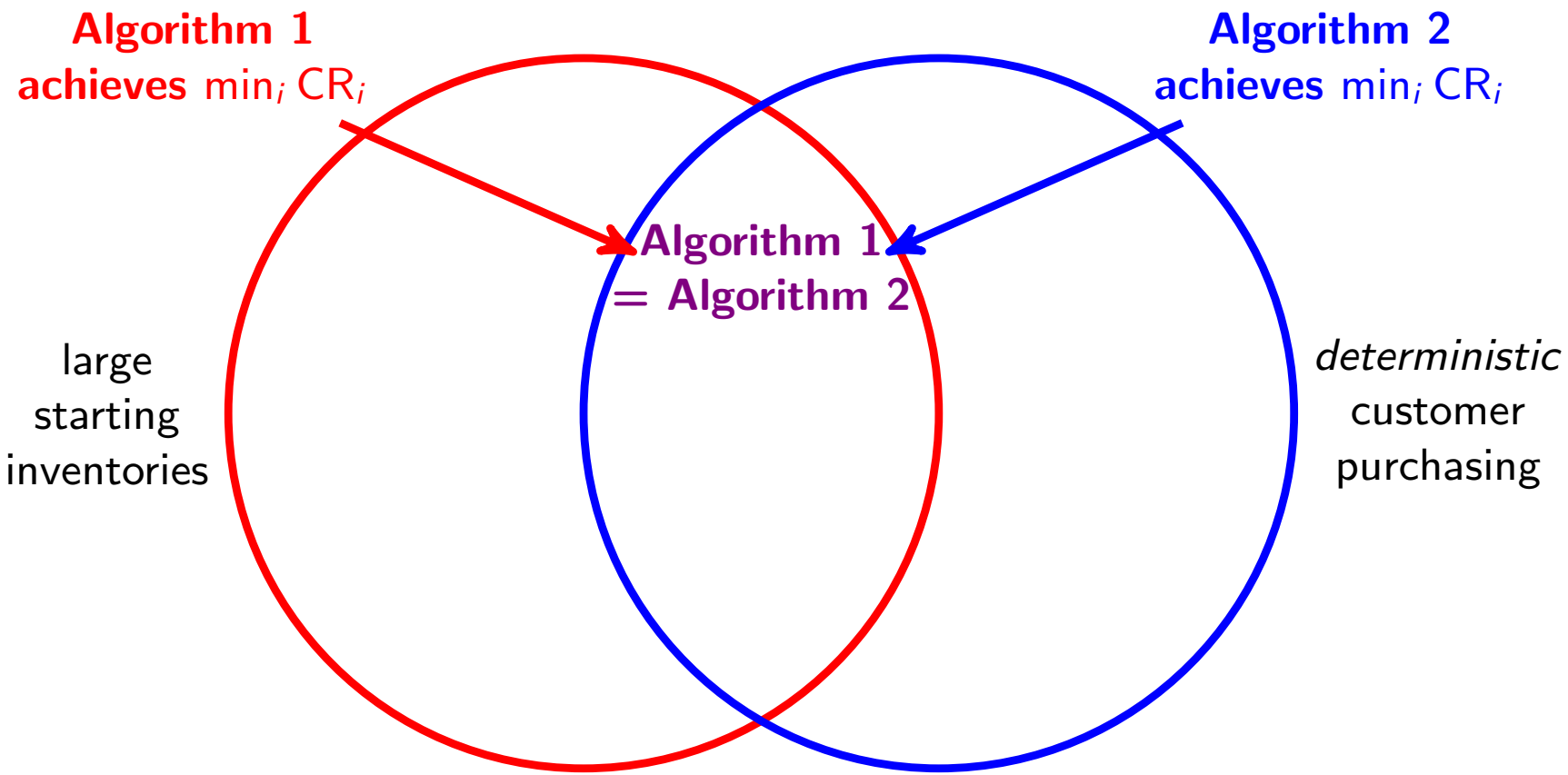

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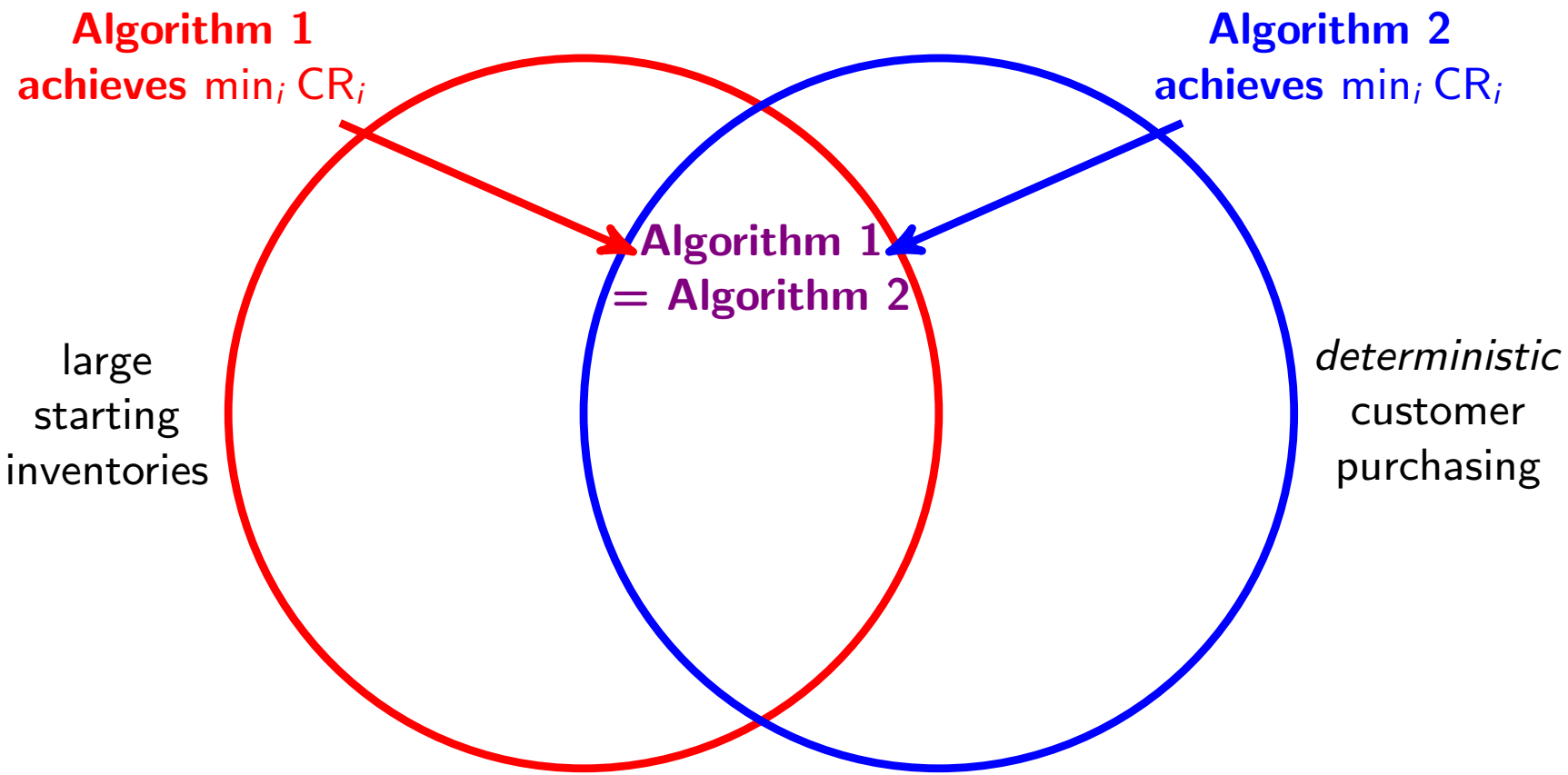

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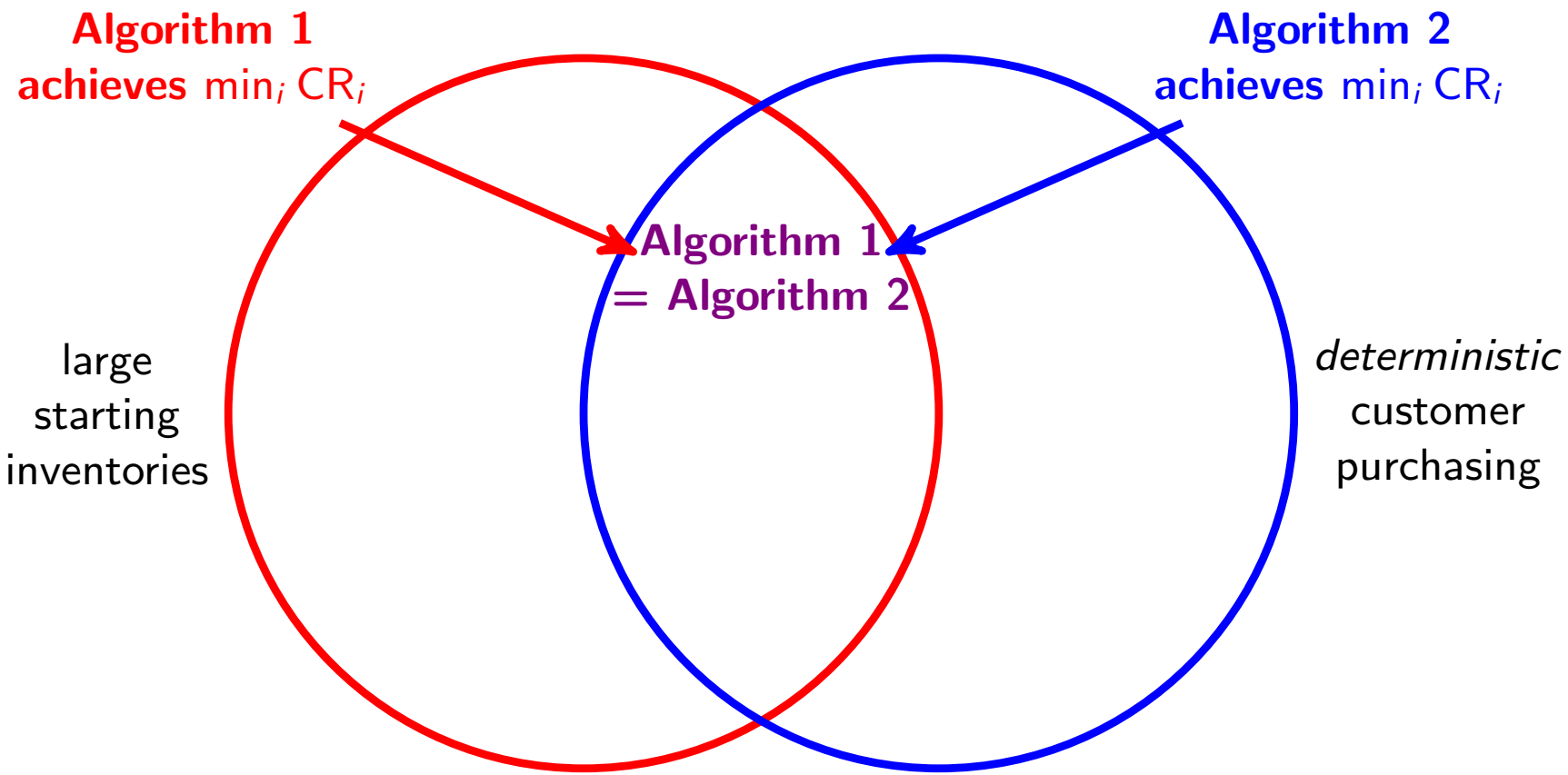

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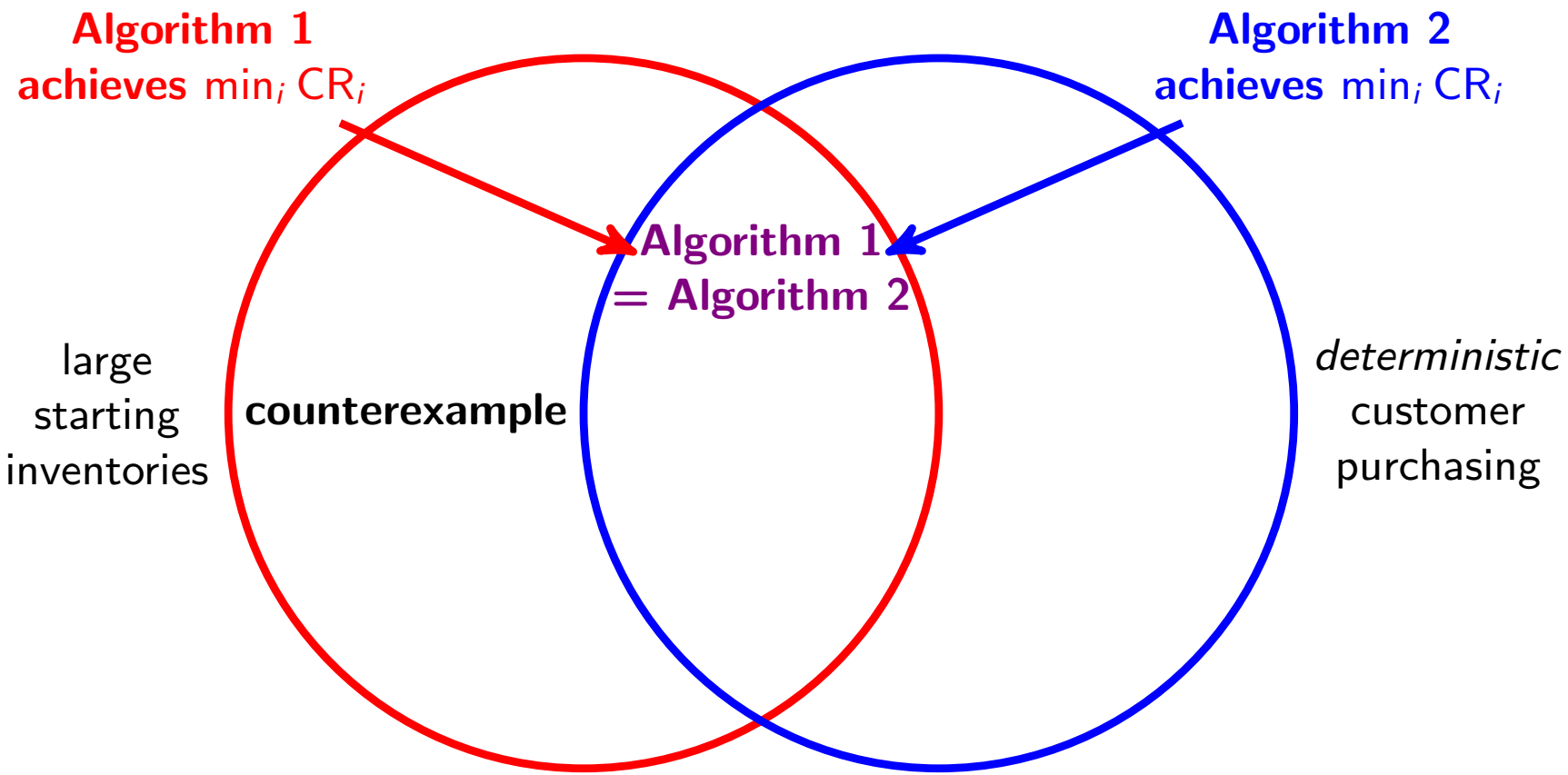

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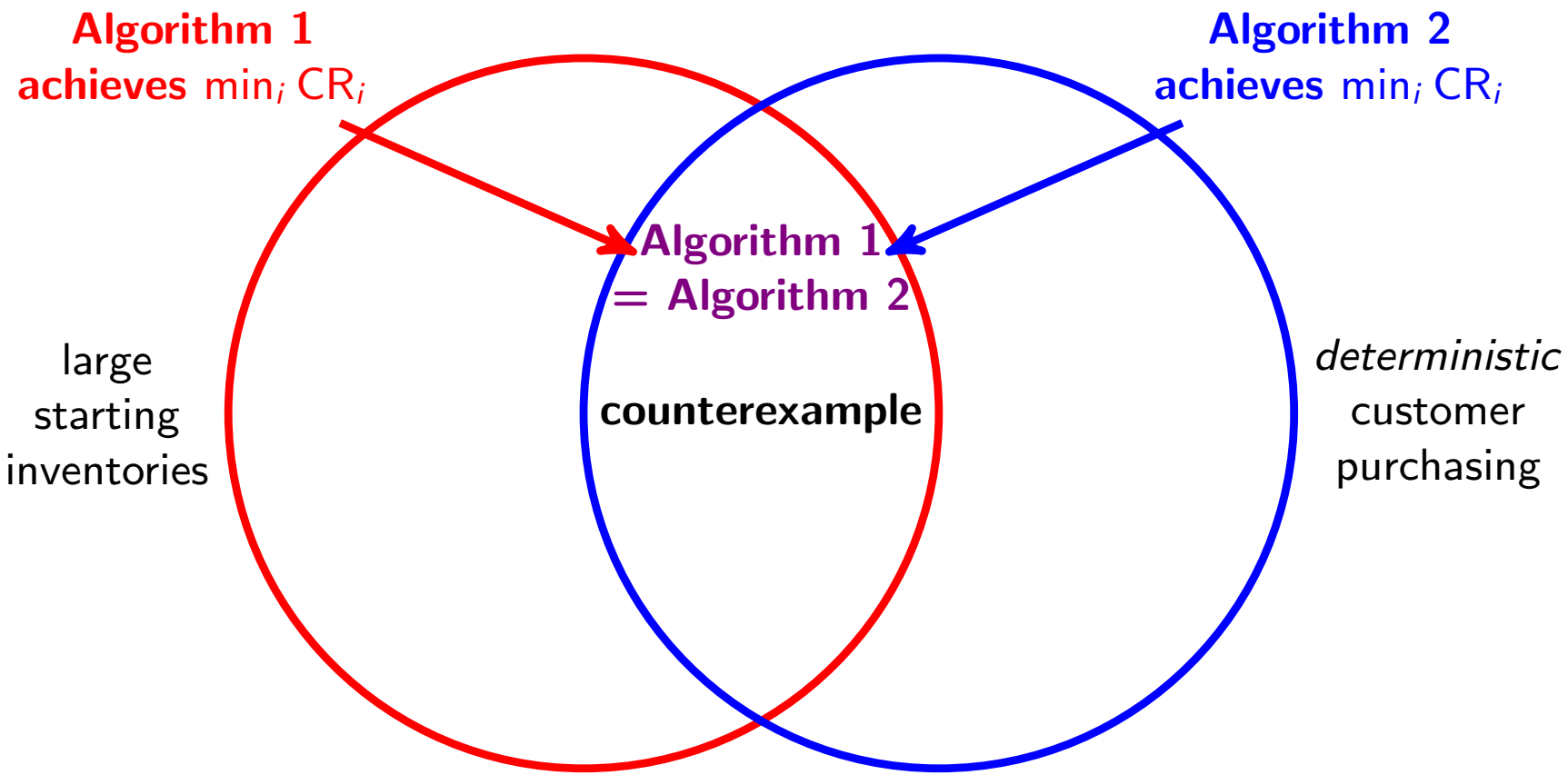

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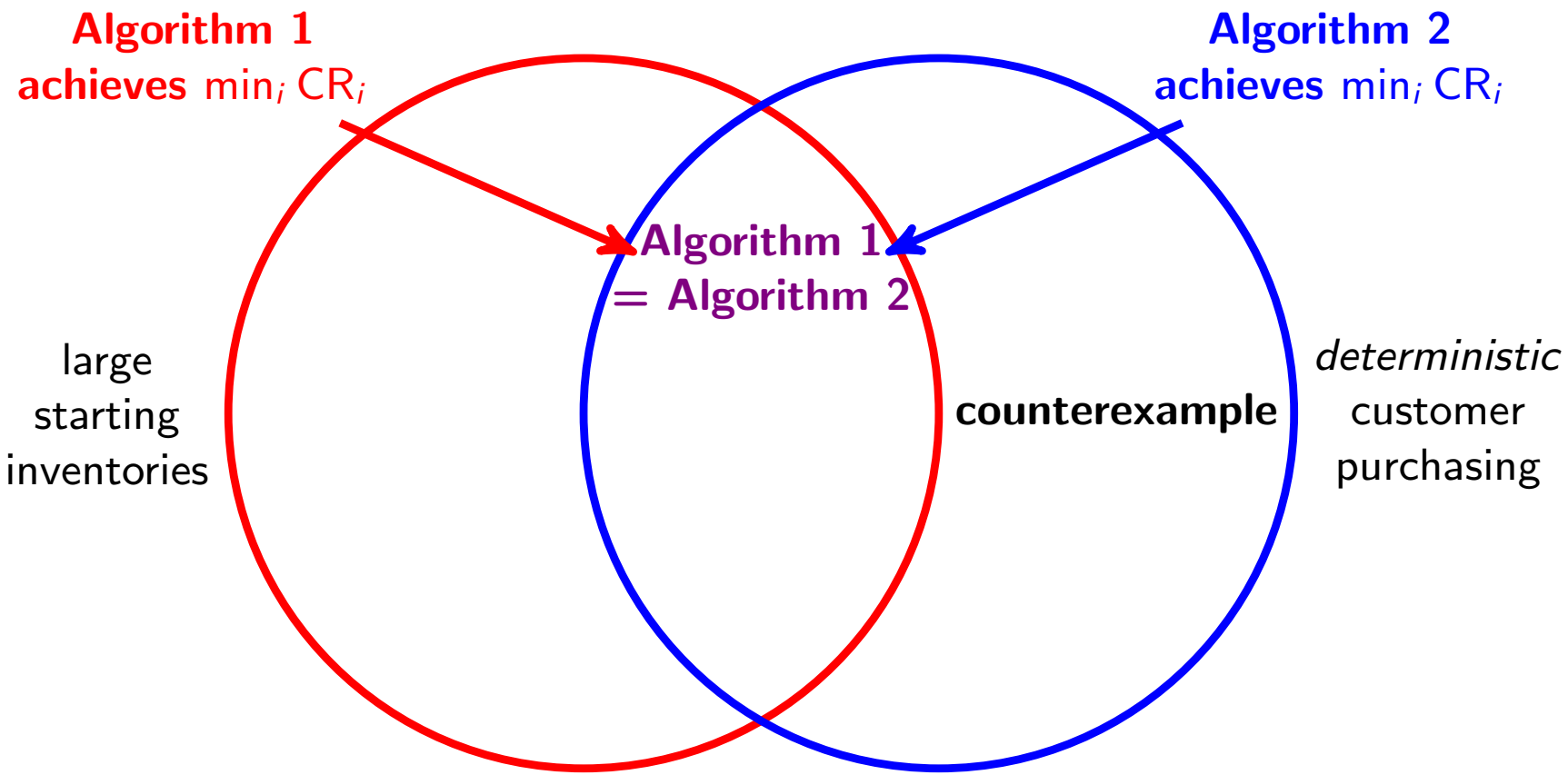

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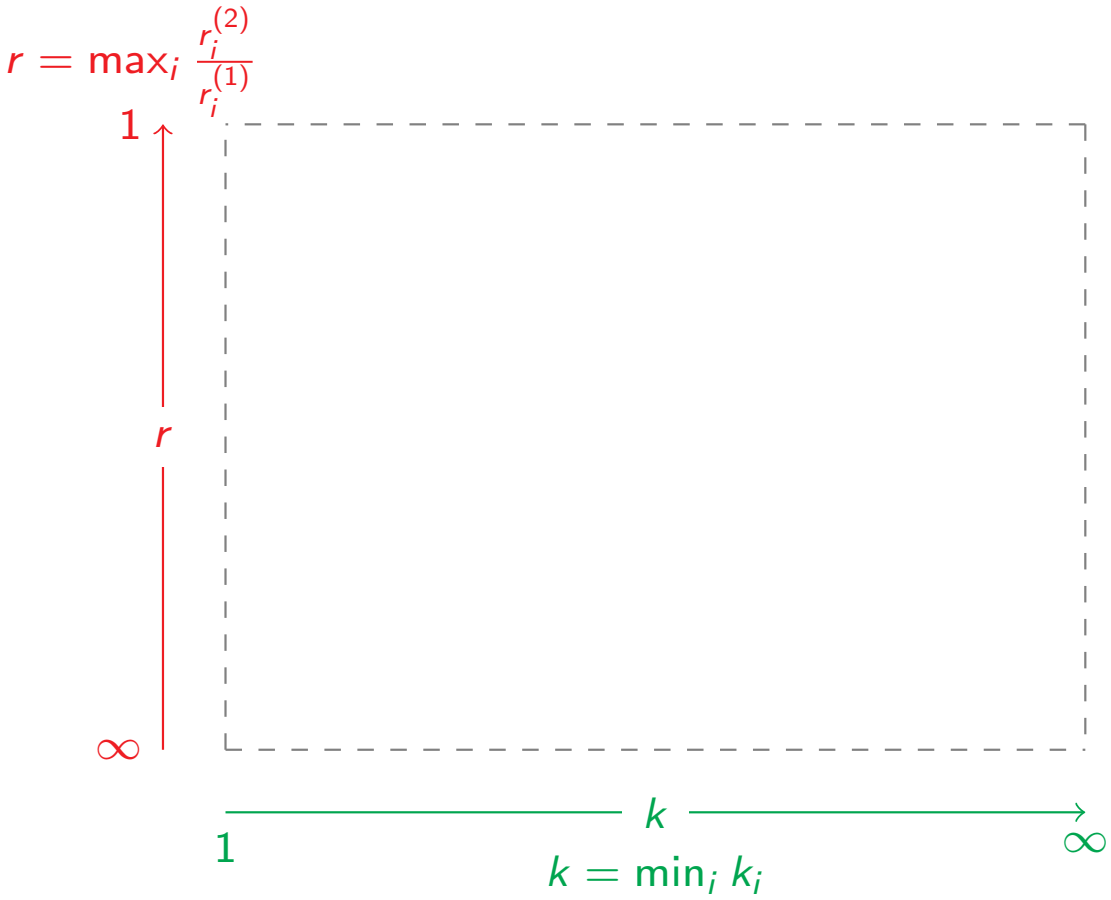
Illustration of Competitive Ratios with Two Prices per Item

General Stochastic Setting



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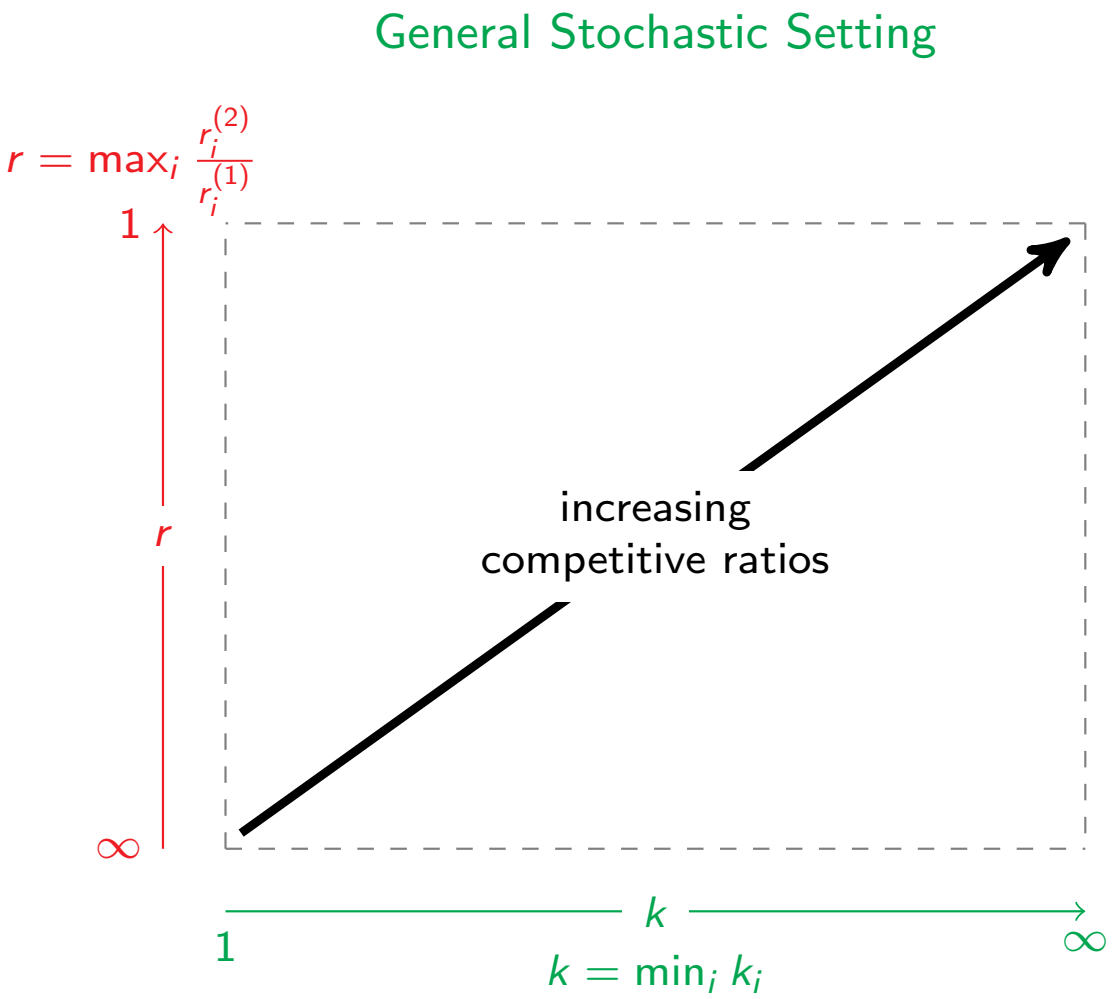

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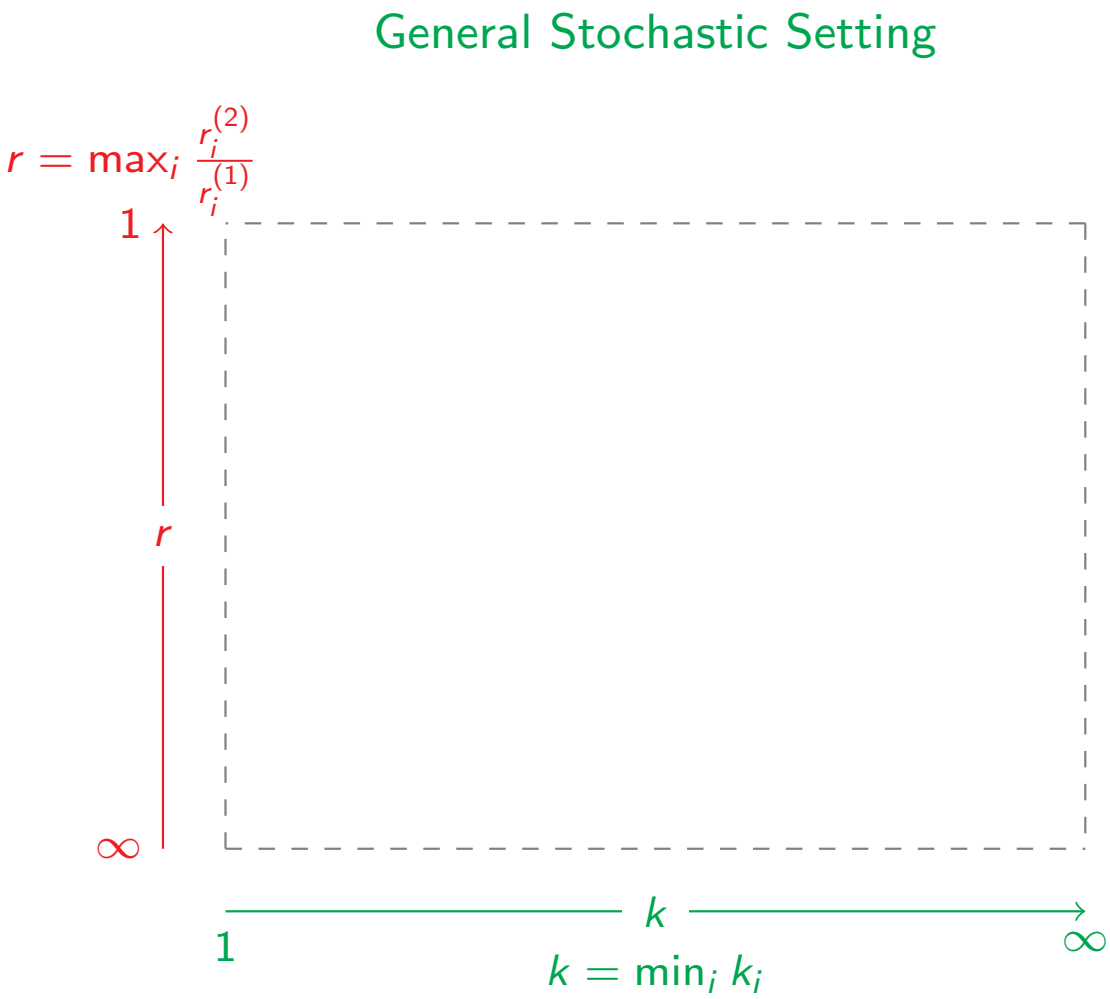

tight results

non-tight results

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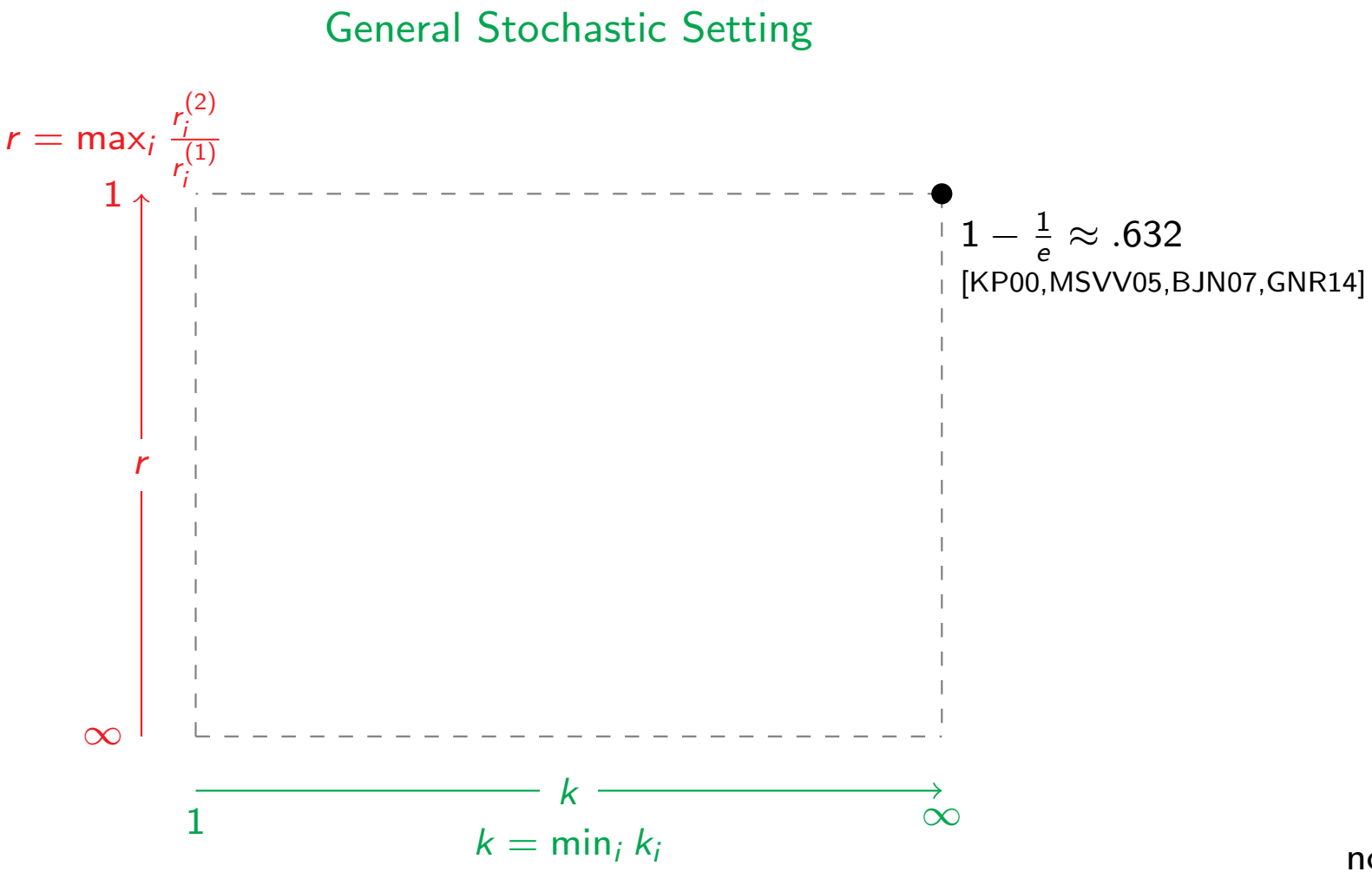

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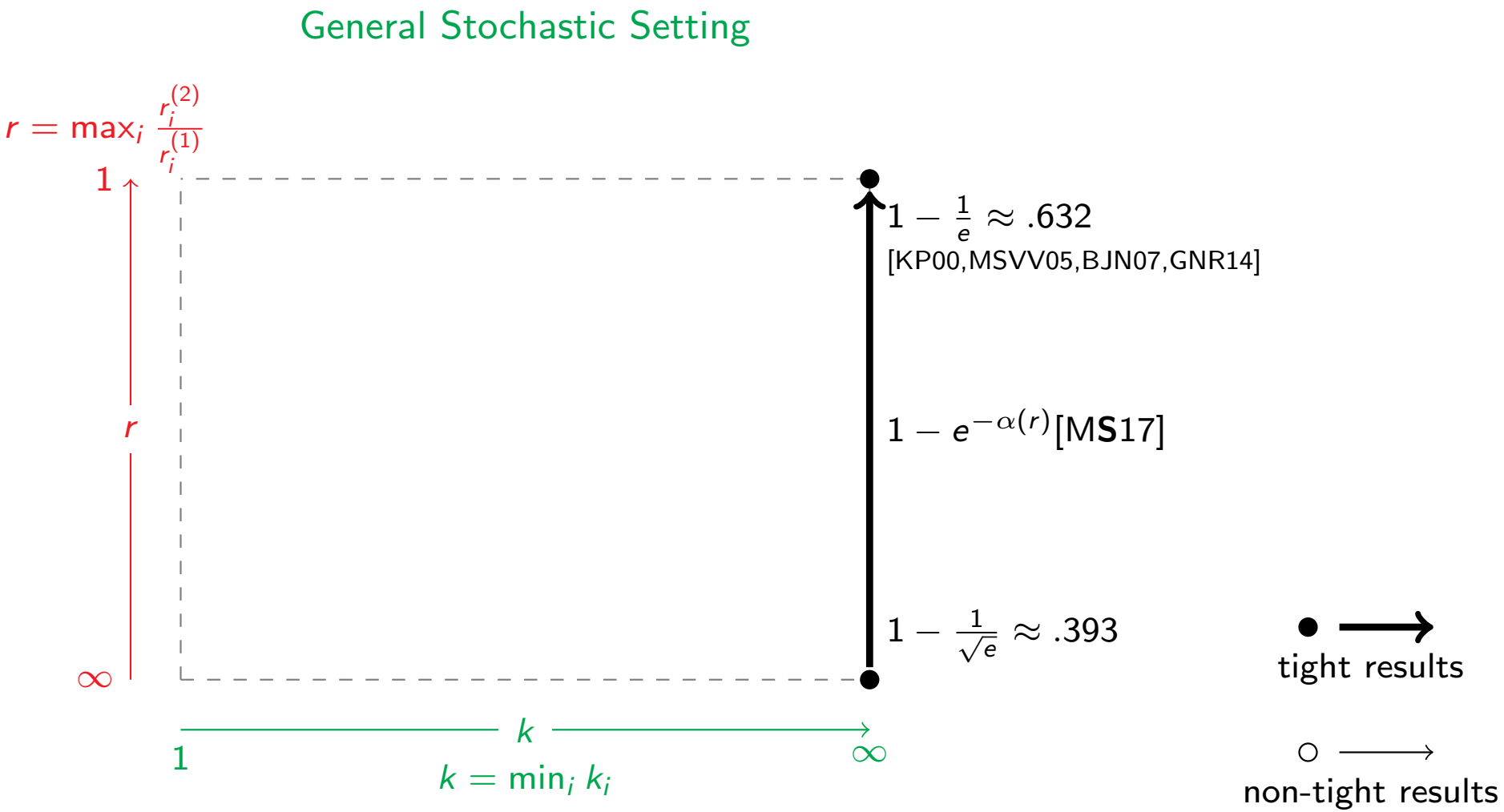

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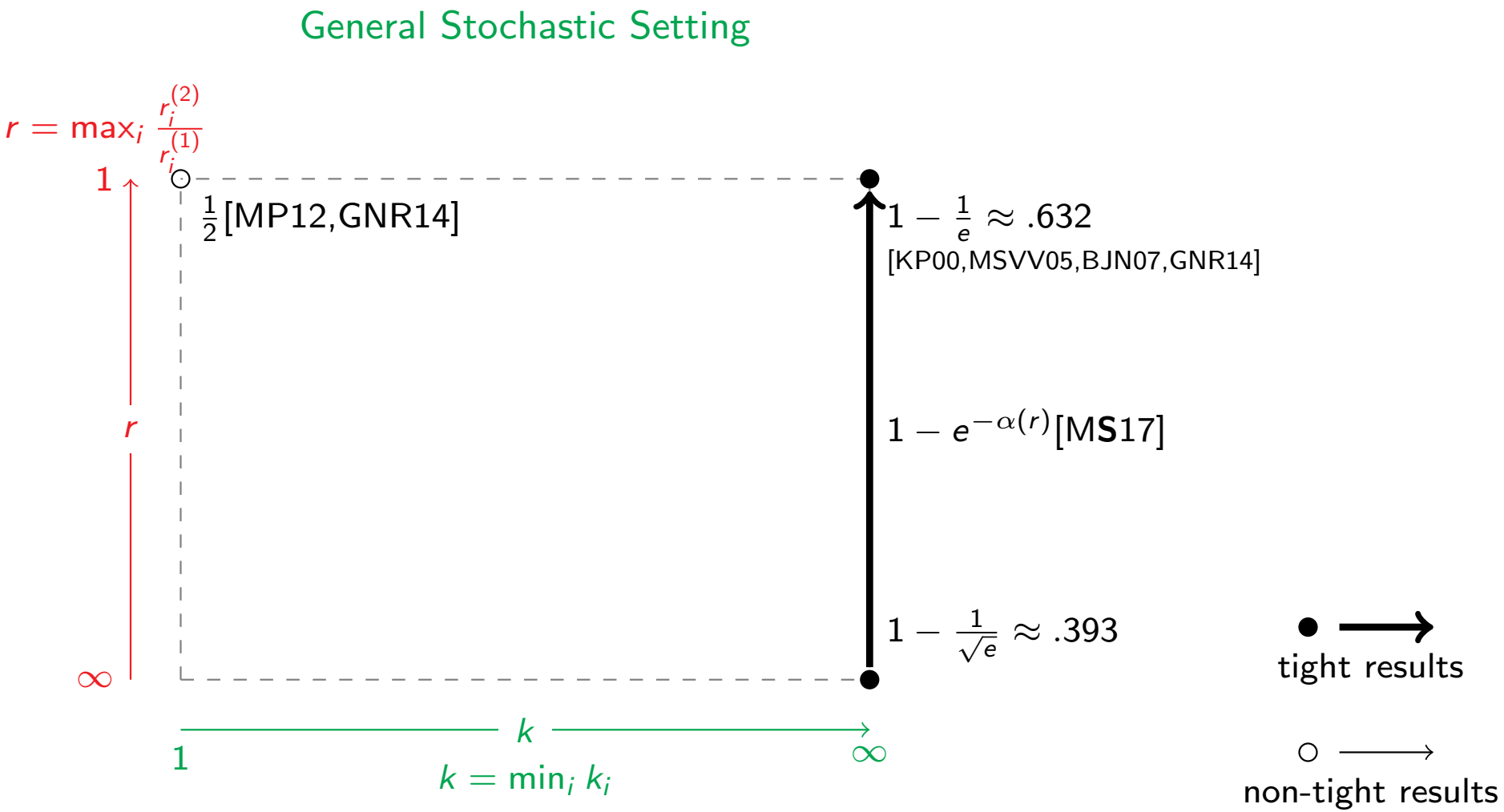

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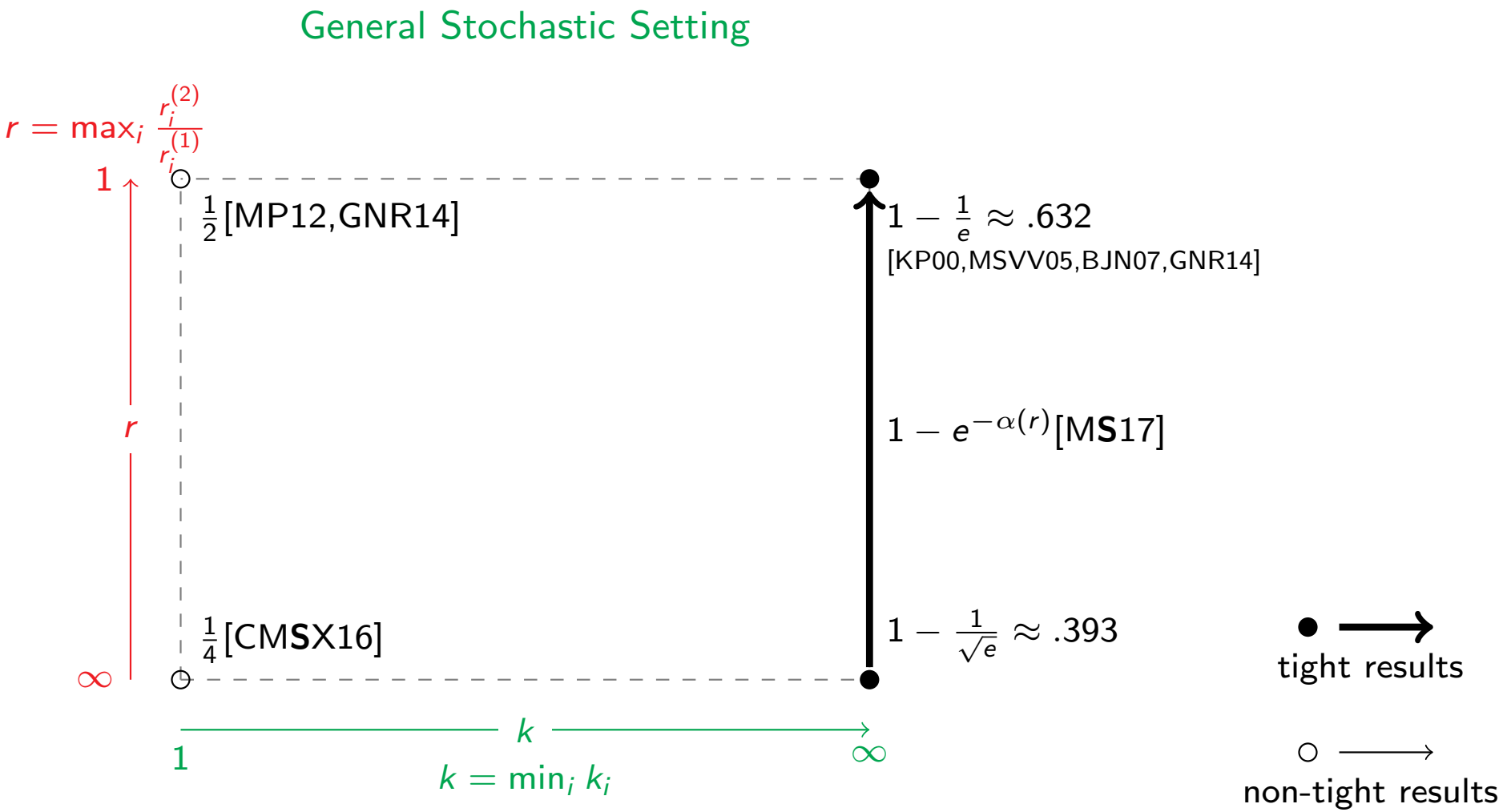

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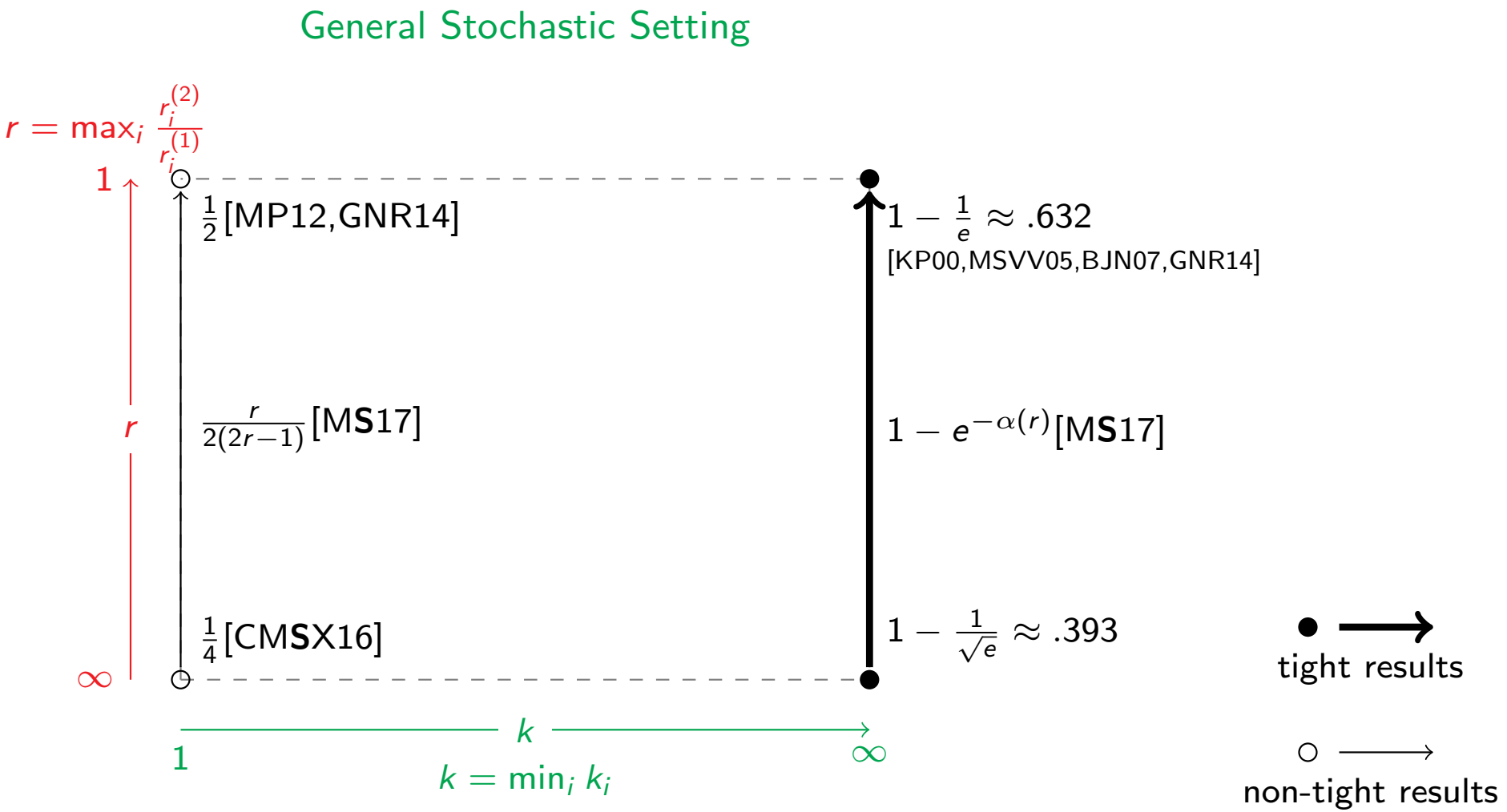

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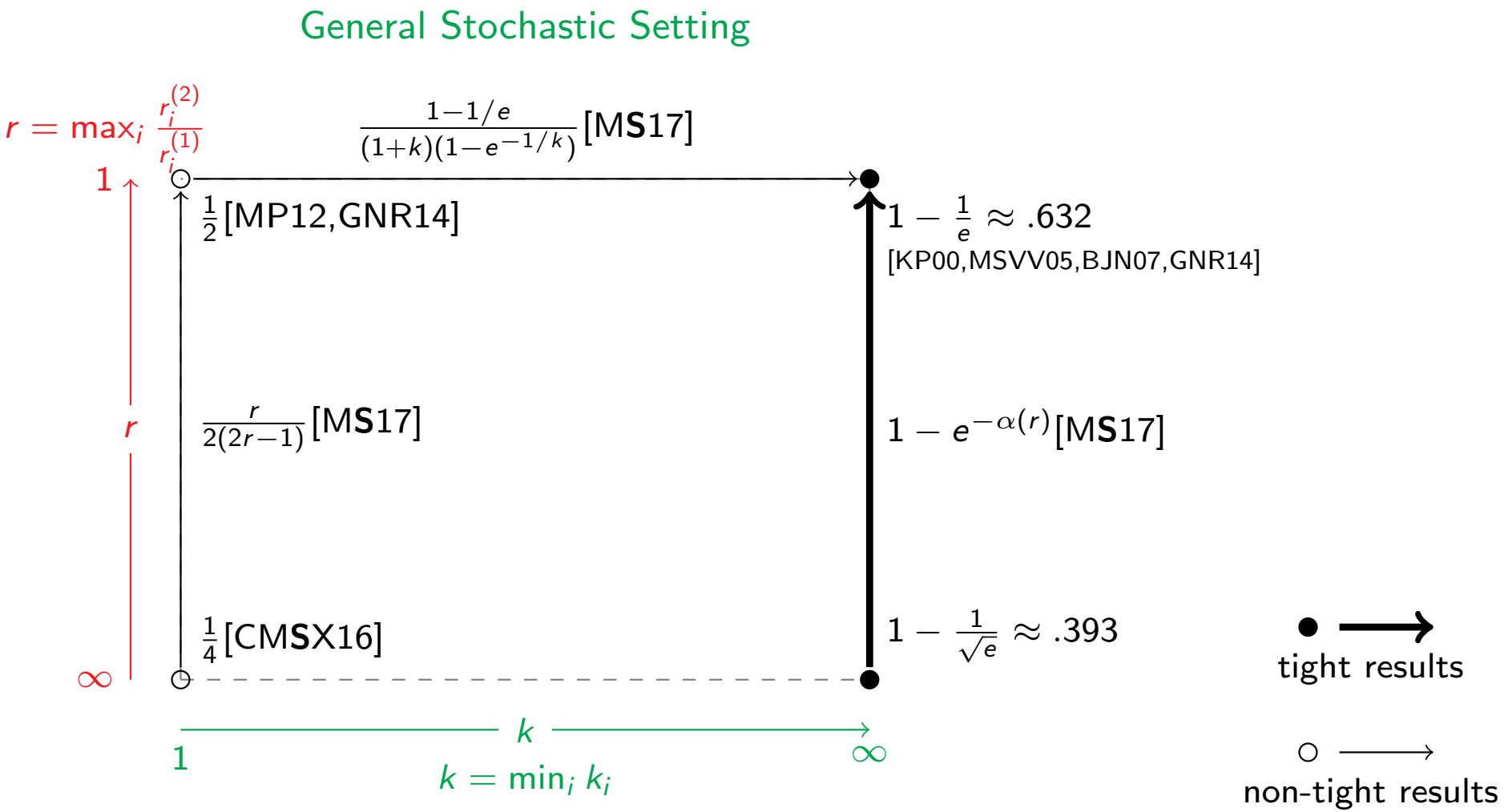

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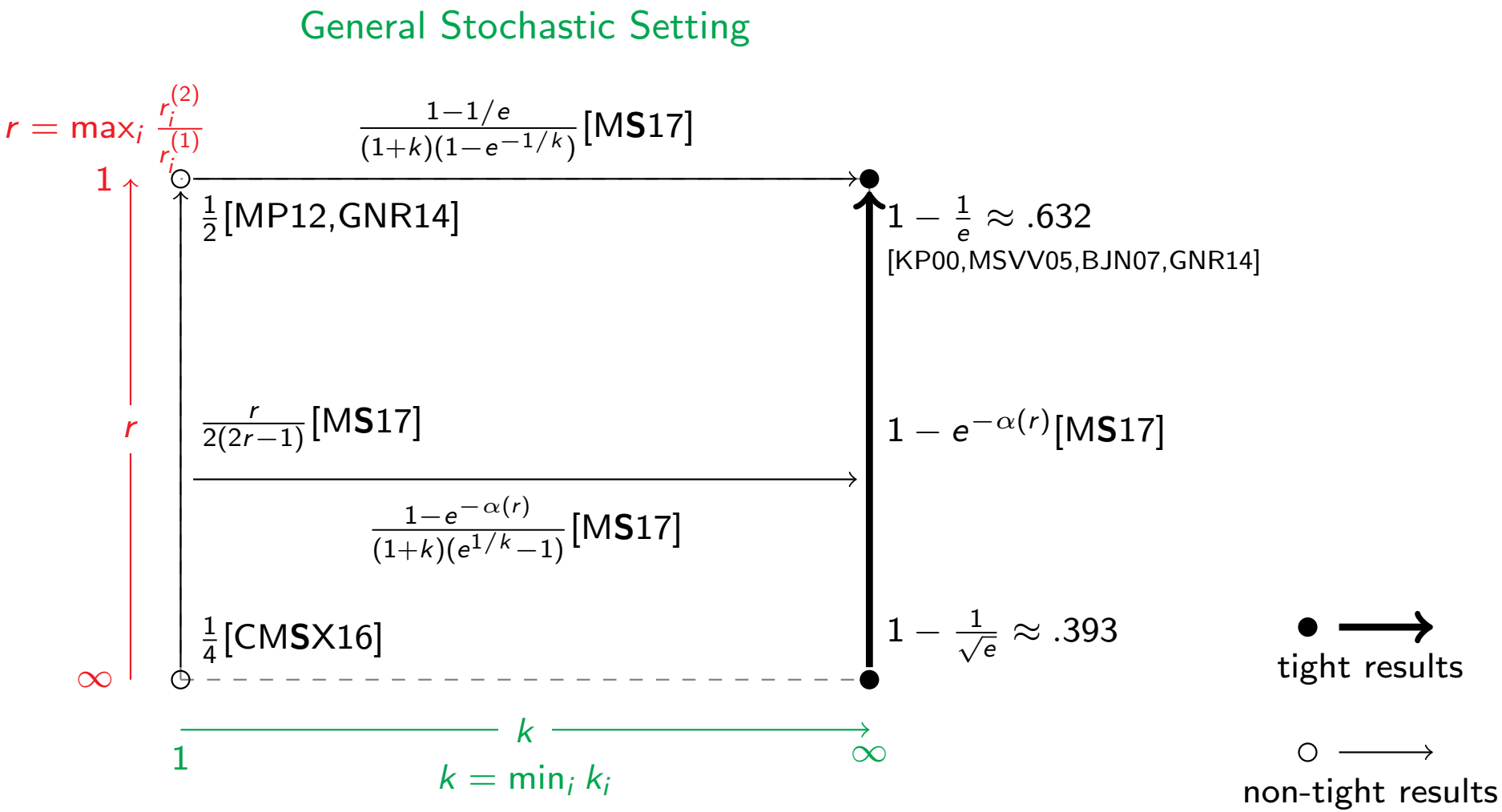

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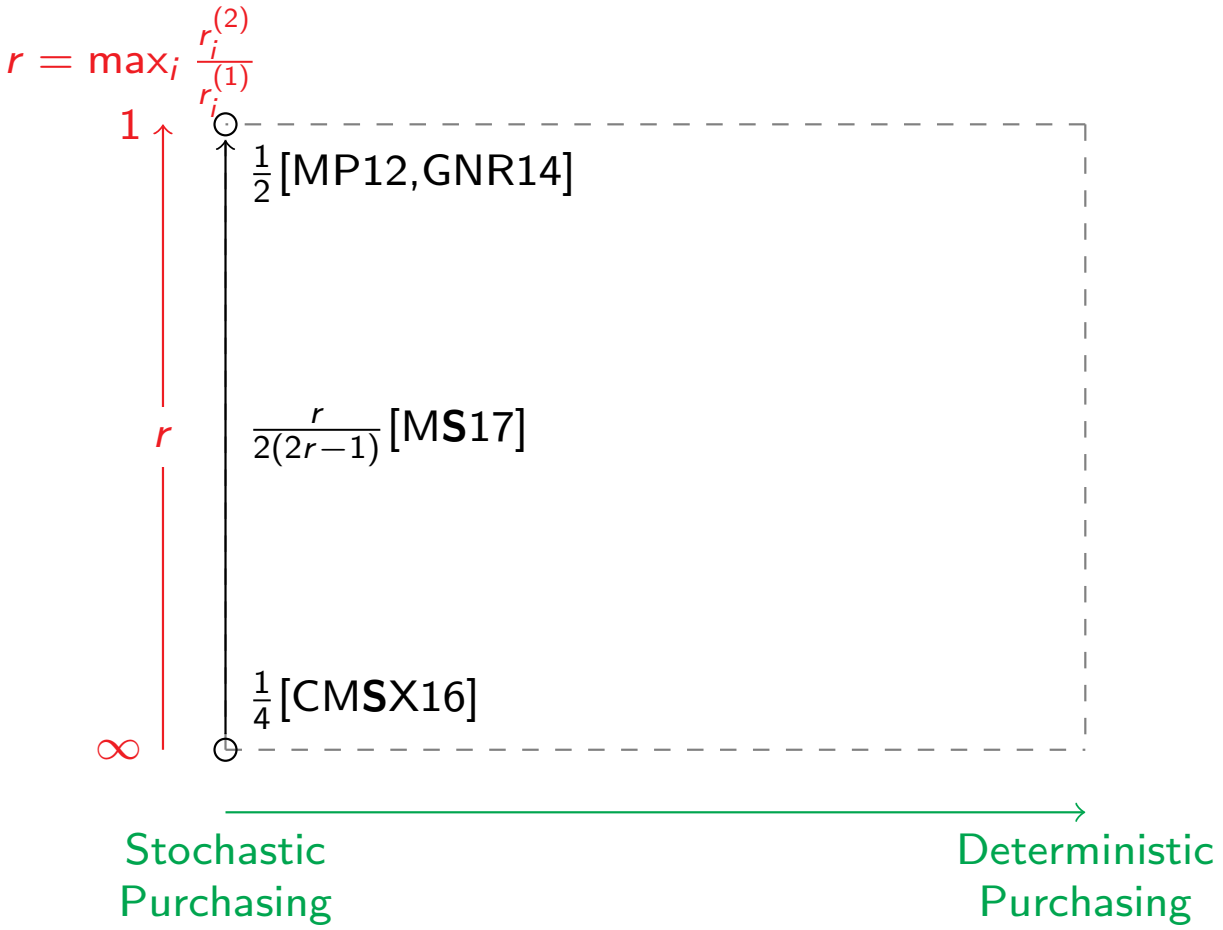

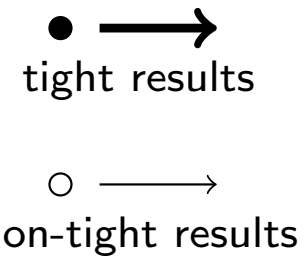

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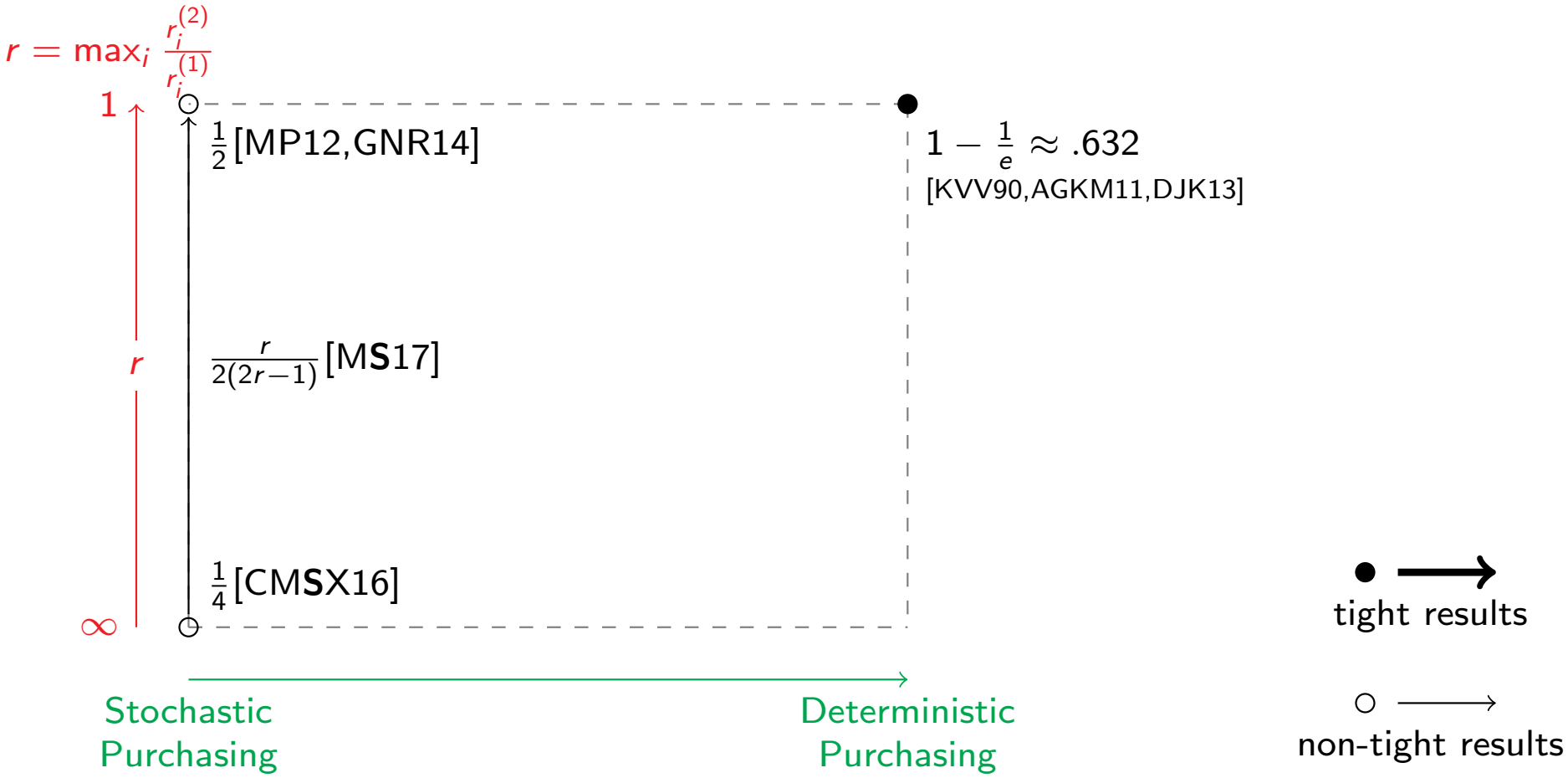

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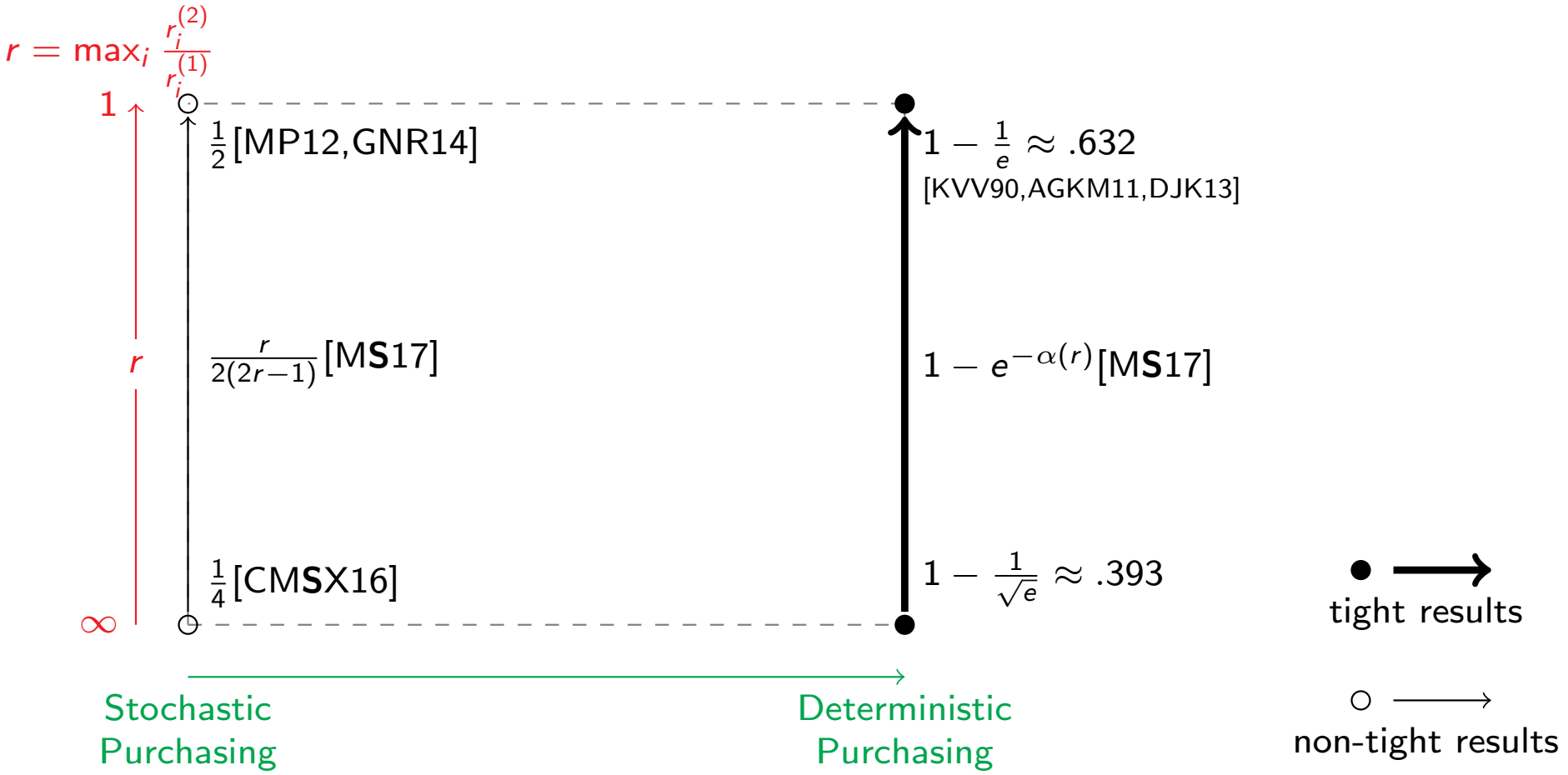

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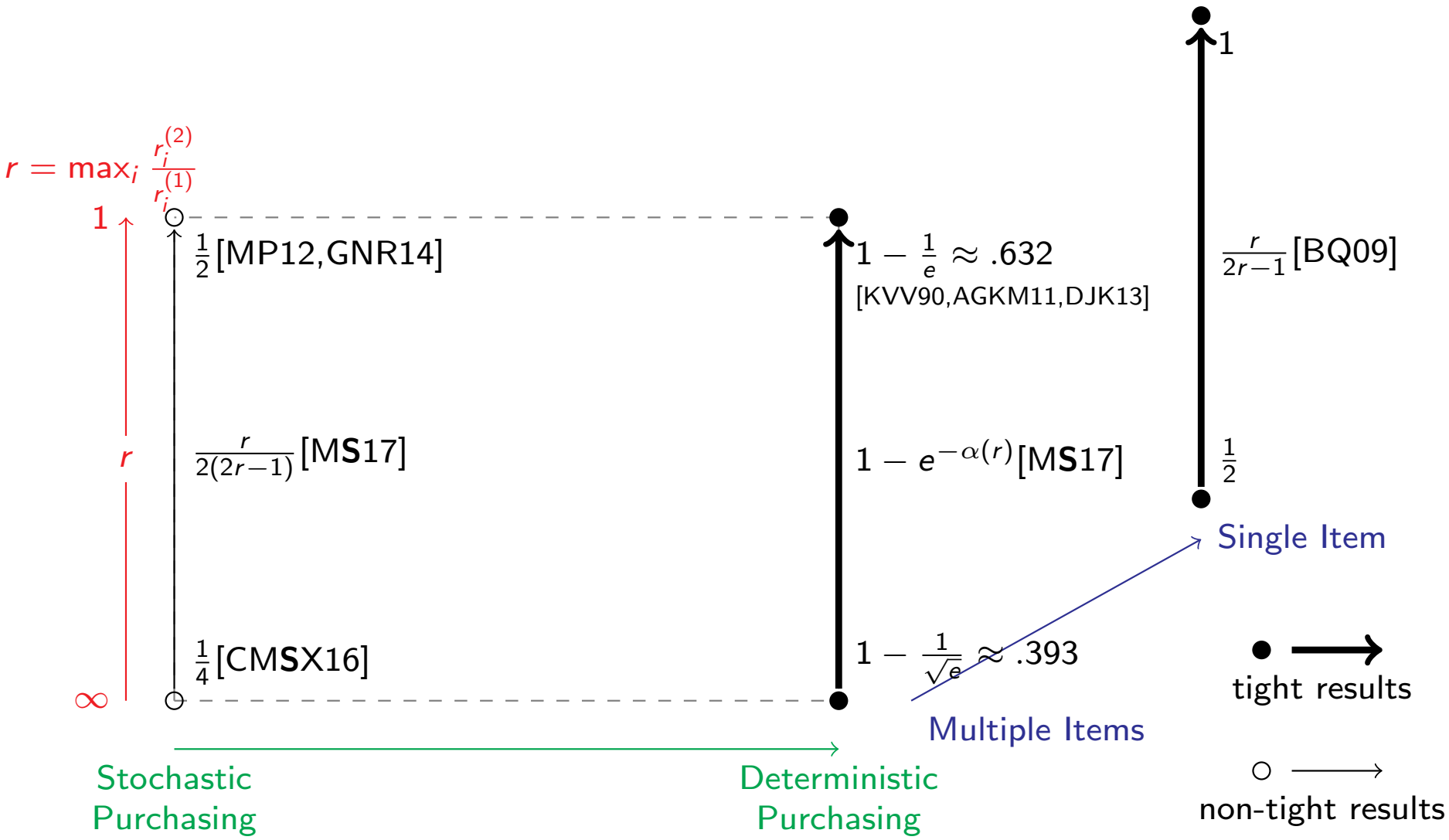

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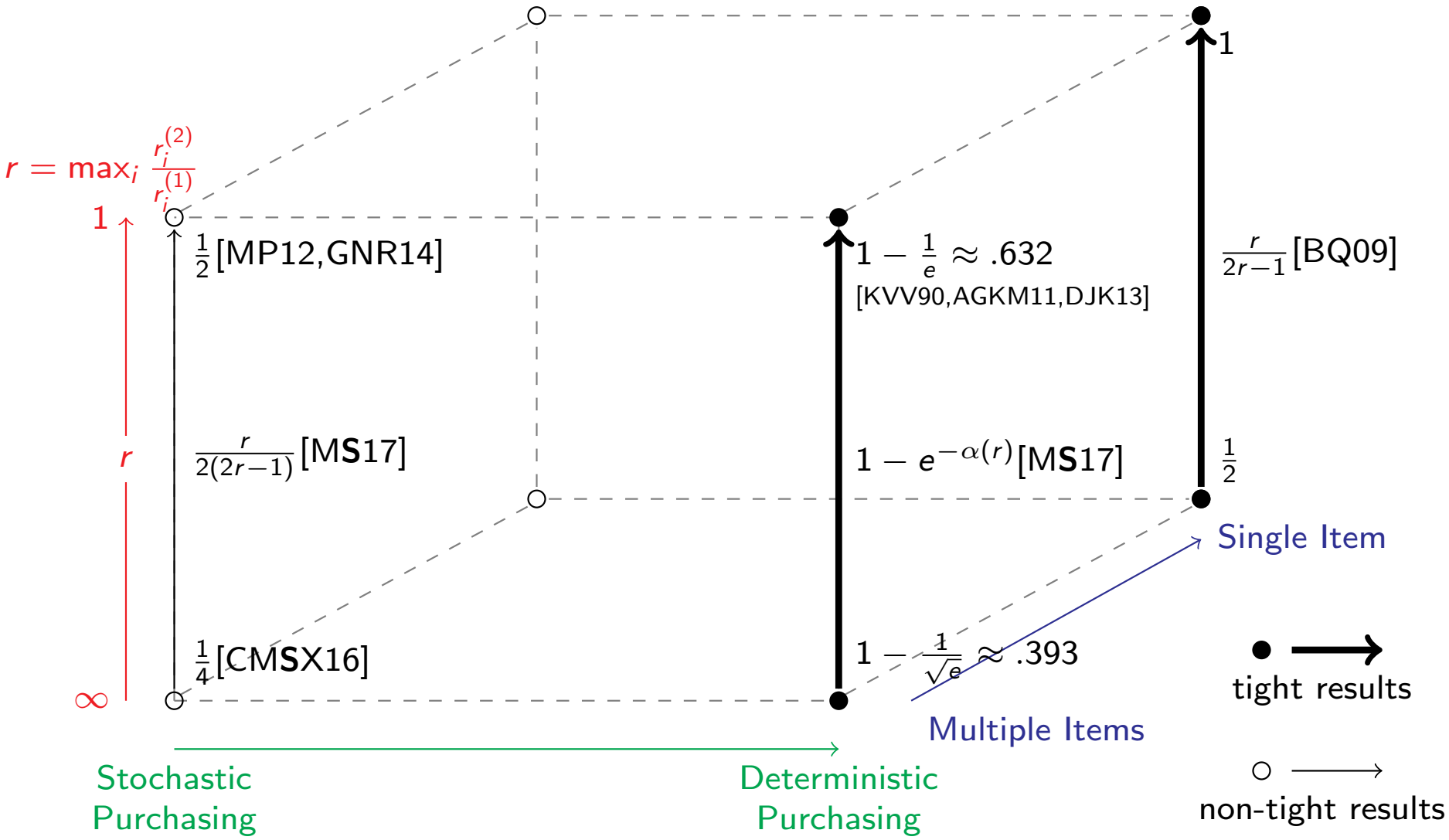

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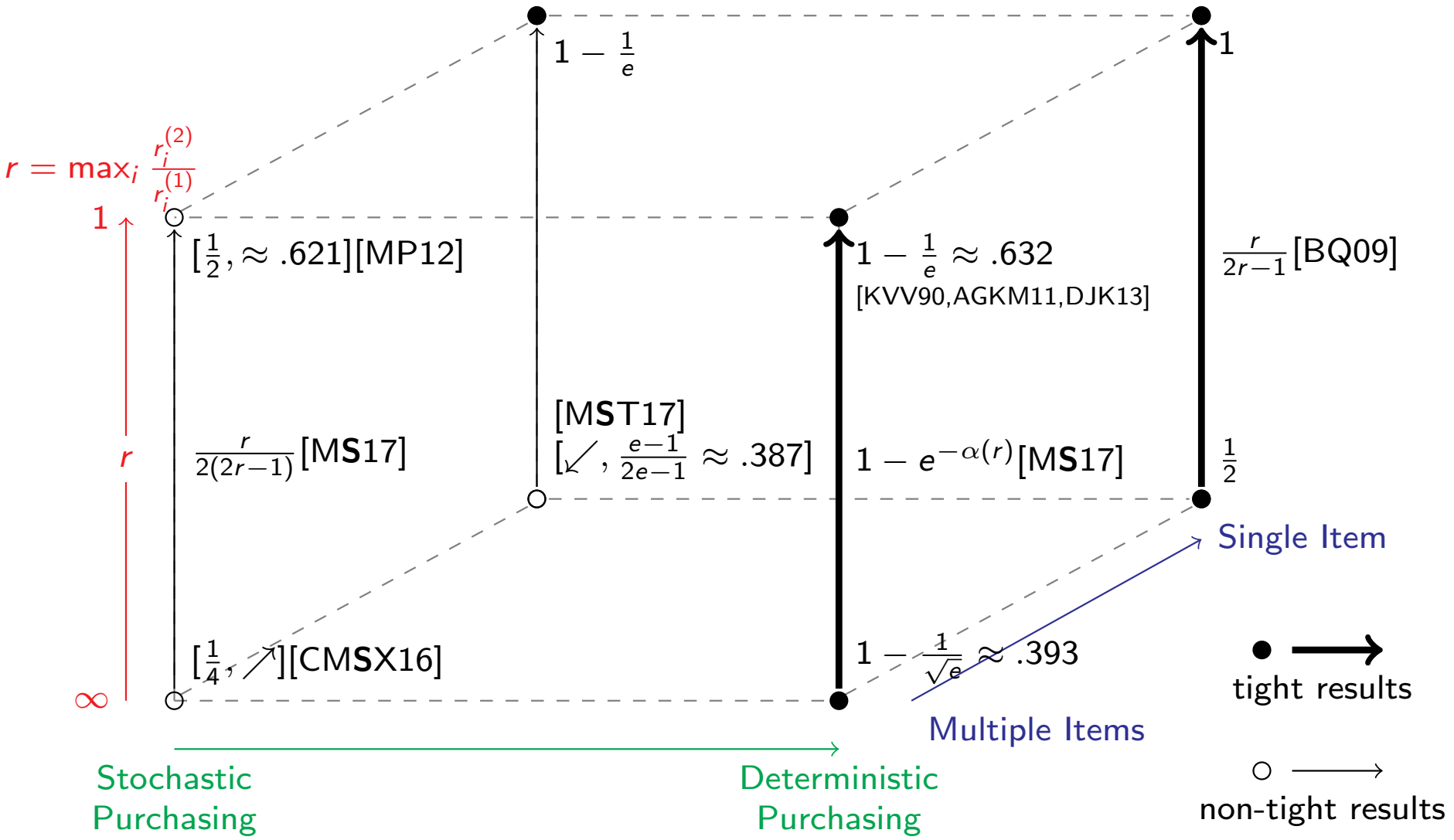

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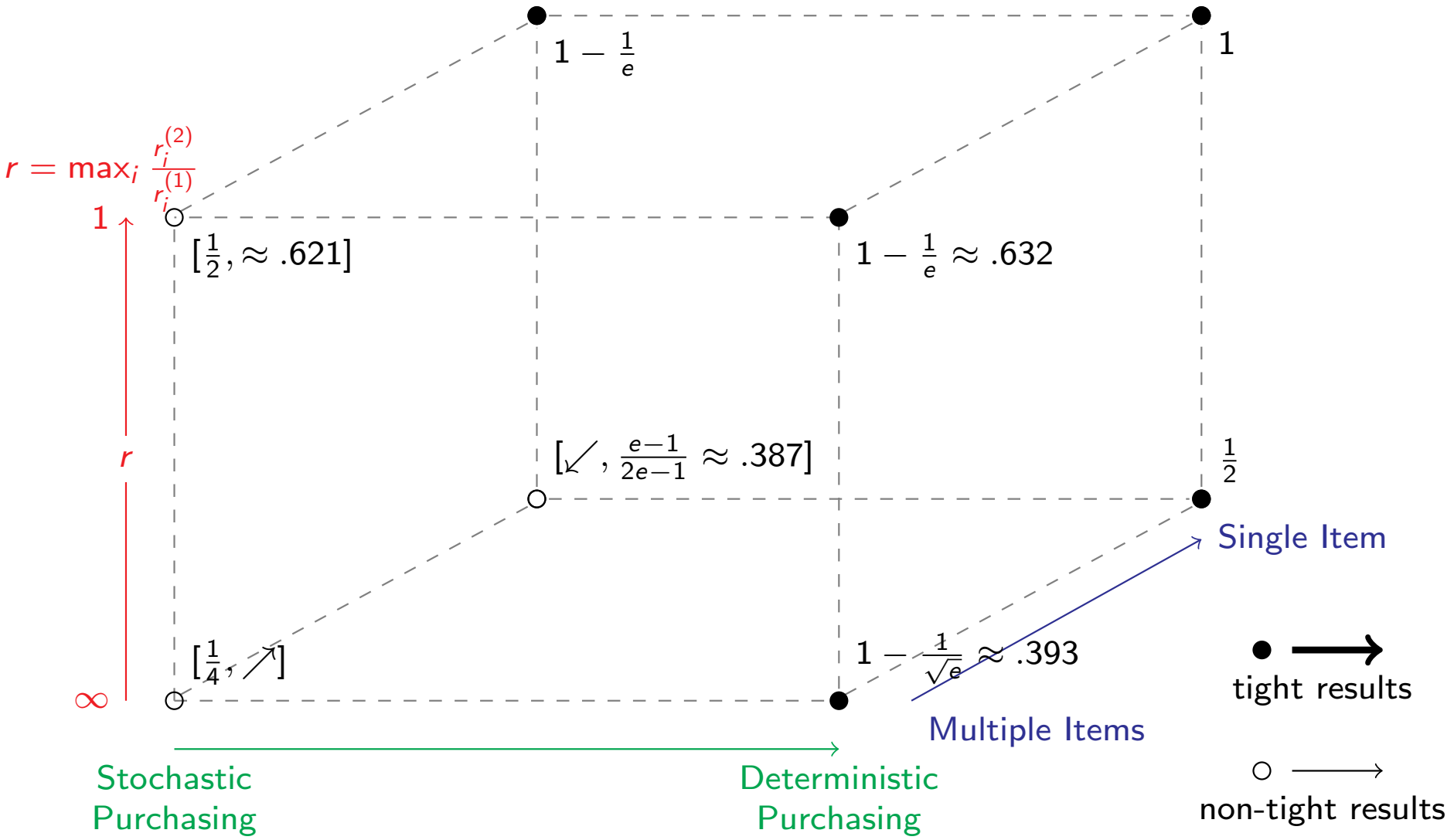

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- on average, 1340 total arrivals per check-in date (scaled by 10)

Algorithms Compared

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- **Hybrid Algorithms**
  - follow the forecast-dependent algorithm during each time step,
  - however, if the decision prescribed is suboptimal in the worst-case beyond a certain threshold,
  - then it uses the decision of the forecast-independent algorithm instead.

Results

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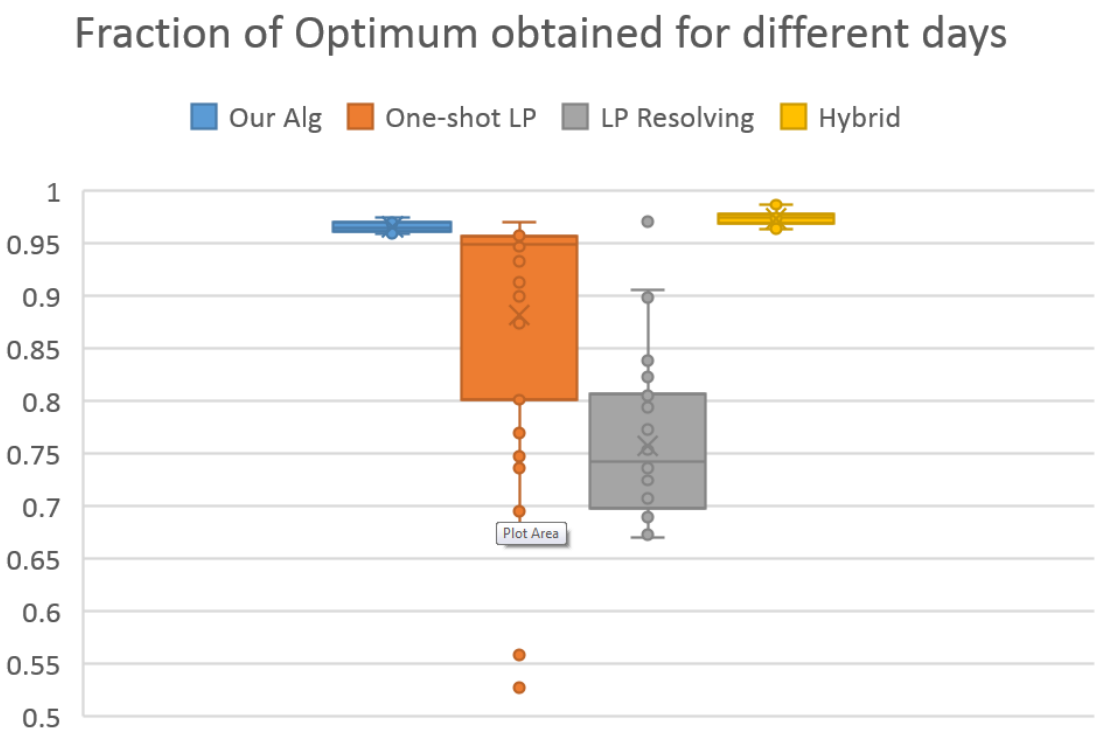
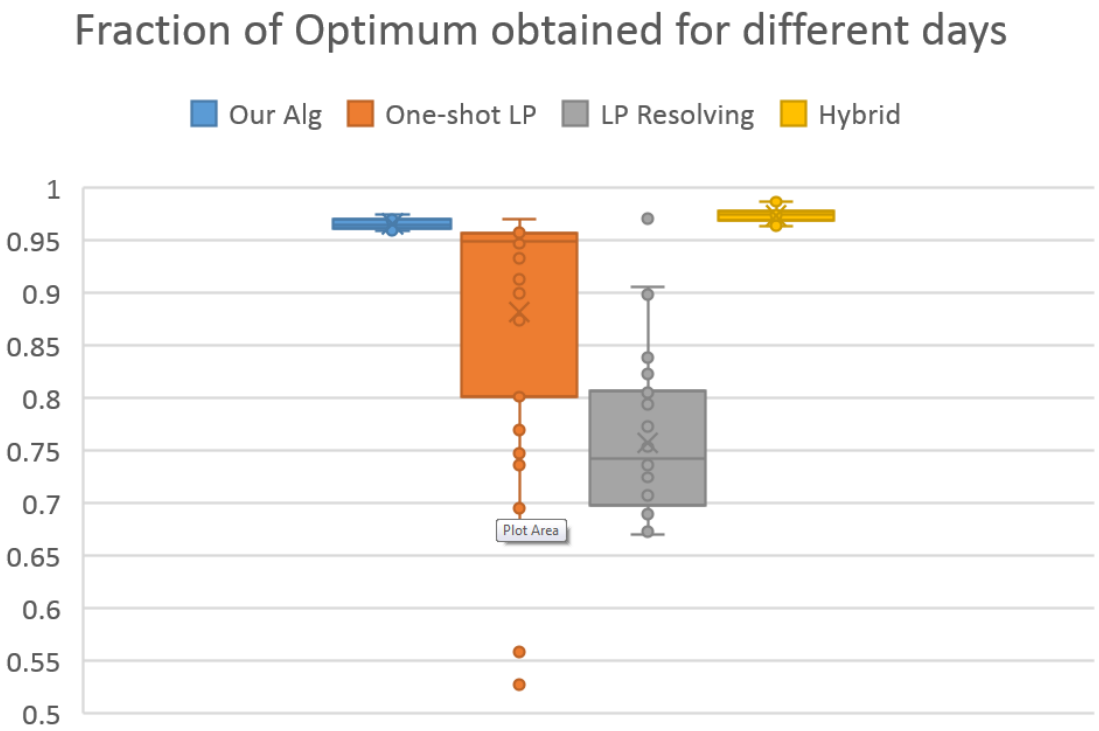

Fraction of Optimum obtained for different days
Our Alg
One-shot LP
LP Resolving
Hybrid
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
Plot Area

Results



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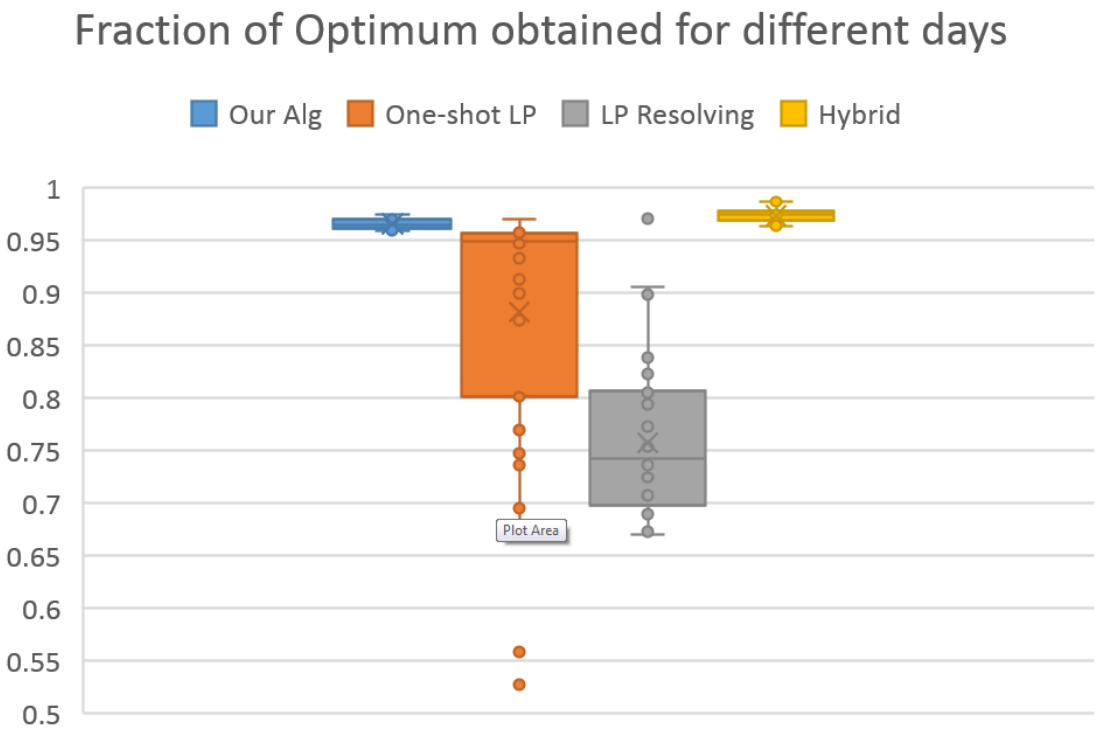

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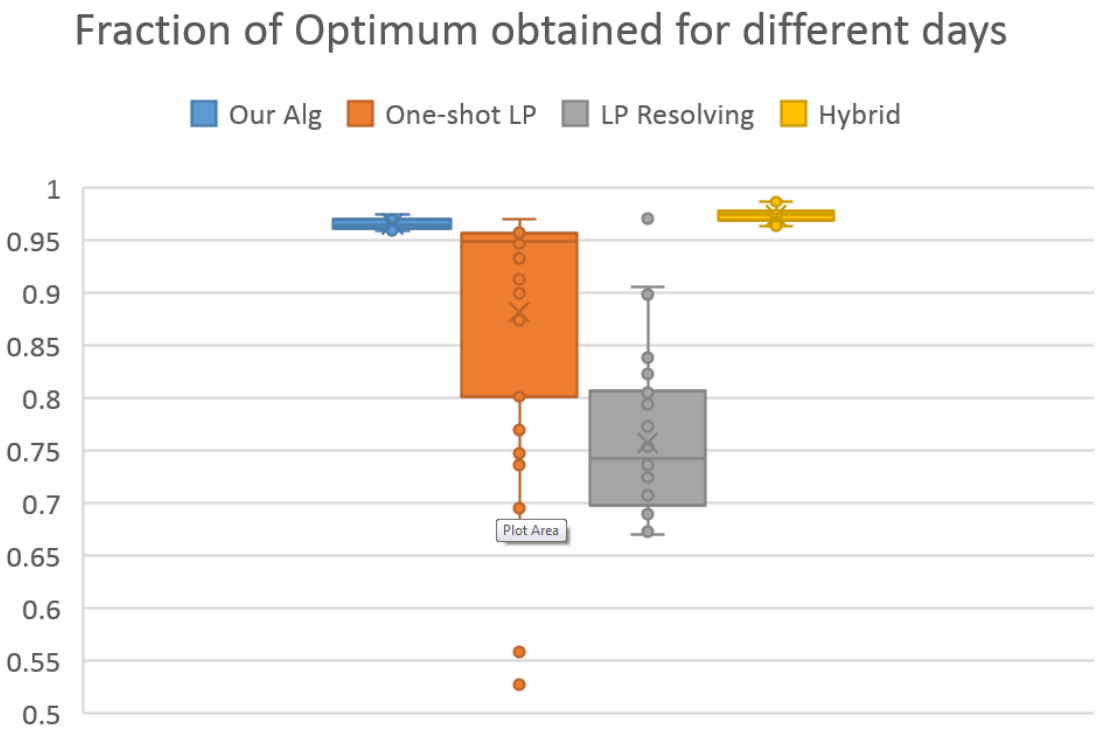

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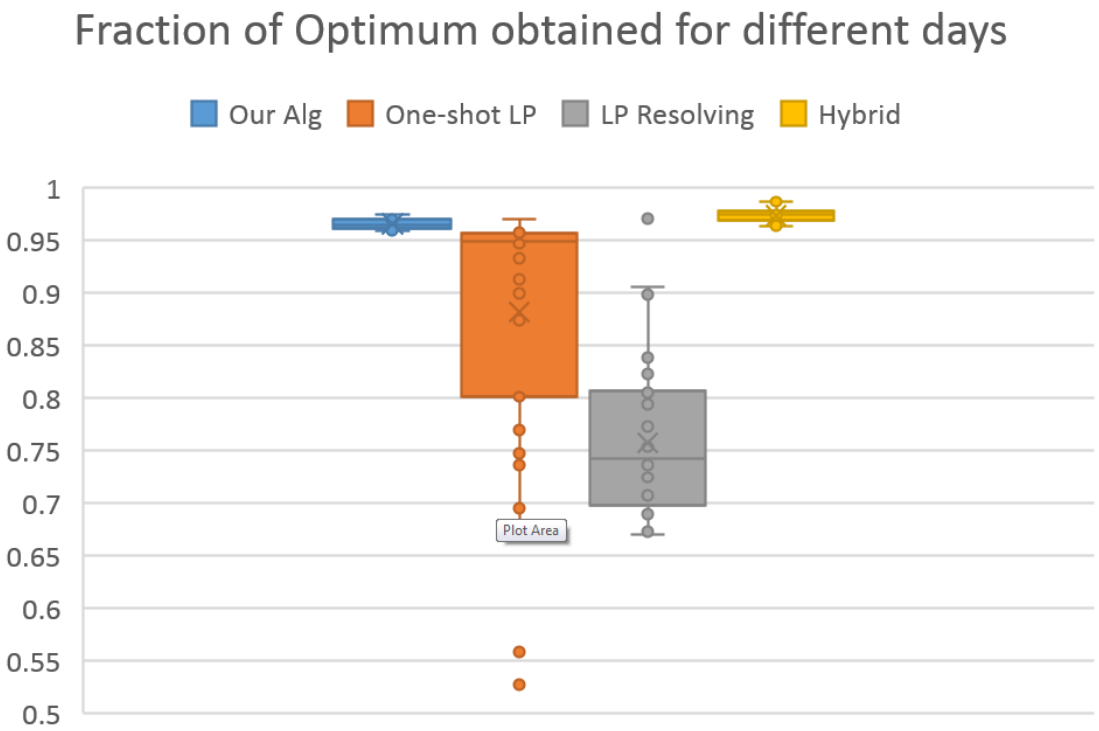

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- the Hybrid of our algorithm and LP Resolving performs even better
- the forecast-dependent algorithms exhibit a lot more variance
- we also tested additional forecast-independent algorithms

Results—Greater Fare Differentiation

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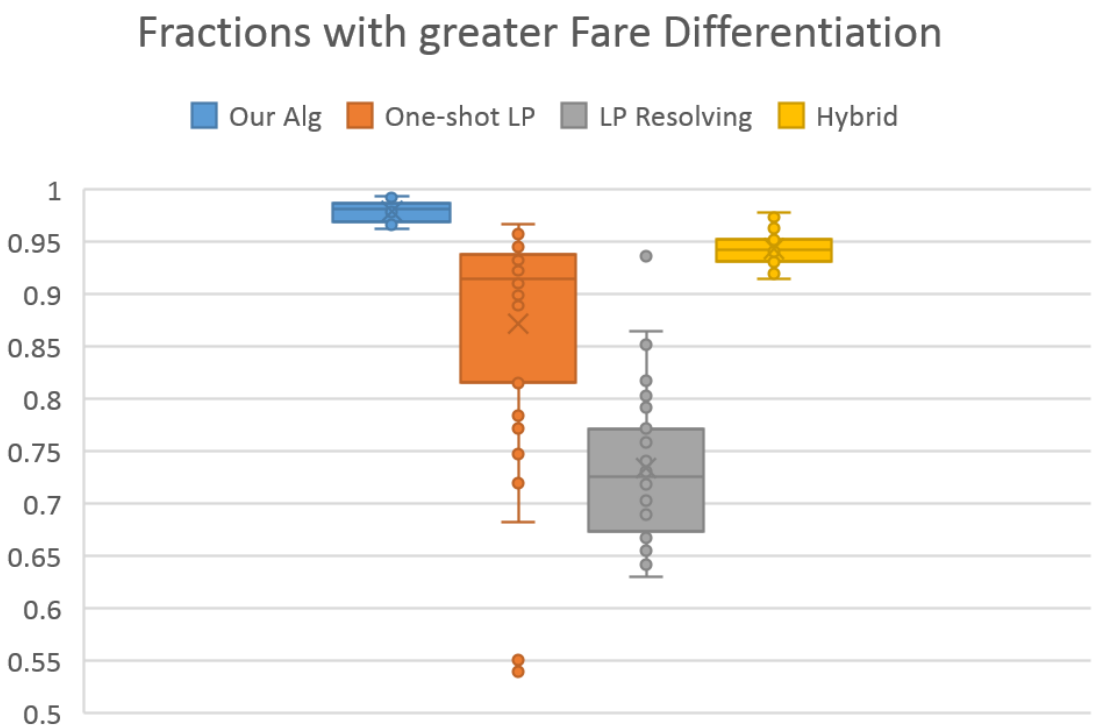
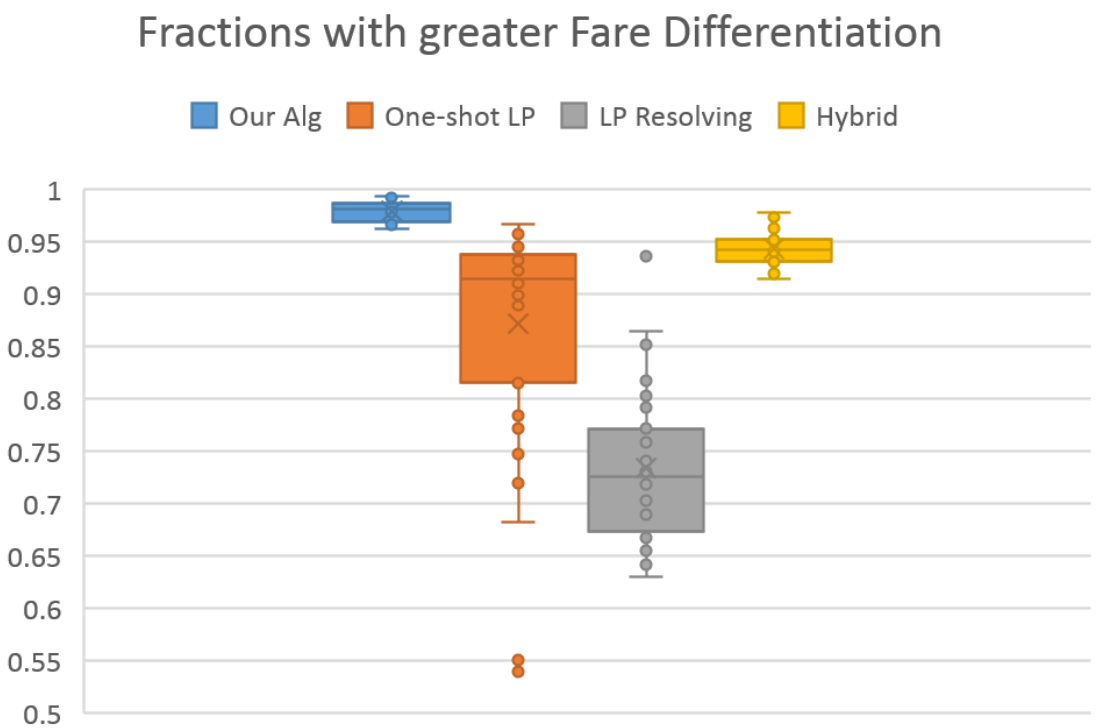


Results—Greater Fare Differentiation



- our algorithm performs better than the Hybrid in this situation

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Complements to Competitive Ratio Analysis:

- Heuristics when the stochastic process generating future arrivals is given (Chan/Farias '09, Ciocan/Farias '12, Chen/Farias '13, Jasin/Kumar '12, Gallego et al. '15)
- Improved algorithms/bounds assuming the arrivals appear in a random order (Kesselheim et al. '13)
- Analyze difference instead of ratio (Reiman/Wang '08, Badanidiyuru/Kleinberg/Slivkins '13, Ferreira/S.-L./Wang '16, Cheung/S.-L. '16)

Thanks!

Email

David Simchi-Levi: dslevi@mit.edu

Papers

1. *Tight Weight-dependent Competitive Ratios for Online Edge-weighted Matching, with Application to Revenue Management*, with **Will Ma** (available on SSRN, 2017)
2. *Dynamic Recommendation at Checkout under Inventory Constraint*, with **Xi Chen**, **Will Ma**, and **Linwei Xin** (available on SSRN, 2016)