Spatiotemporal Dynamics of Optical Pulse Propagation in Multimode Fibers

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Self-Organized Instability in Multimode Fibers

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Self-Organized Instability …

- Motivation for multimode nonlinear optics
- Spatiotemporal modulation instability
- Beam self-cleaning
- Self-organized instability
  - nonlinearity, dissipation, disorder
- Implications / future directions
Pulse propagation in multimode fiber is spatiotemporally complex

4D vector field

Our job is to figure out basic processes, building blocks, and “rules”
Multimode waveguides: between 1- and 3-D

Single-mode fiber

(multimode) Step index fiber

(multimode) Graded index fiber
Why study propagation in multimode fiber now?

- Little work on multimode nonlinear pulse propagation before 2013
- Recent theoretical, computational advances e.g., transfer matrix, principal modes,…
- Relevant to multicore fibers
- Relevant to vortex beams, orbital angular momentum

Huang et al., Opt Exp 2014
Why study propagation in multimode fiber now?

- Laser / amplifier / transmission applications

- Spatial division multiplexing in telecom

- Imaging through multimode fiber/complex media

Agrell et al., J Opt 2016

Ploschner et al., Nature Photon 2015
Recent advances

- Multimode soliton formation, fission
  
  Renninger et al., Nature Commun 2013
  Wright et al., Opt Exp 2015

- Controllable spatiotemporal nonlinear processes
  
  Wright et al., Nature Photon 2015

- Spatiotemporal generation of ultrabroadband dispersive waves
  
  Wright et al., Phys Rev Lett 2015
What we are learning (so far…)

- Complex phenomena understood in terms of multimode soliton dynamics
  - Multimode Cerenkov radiation
  - Soliton fission is spatiotemporal
  - Multimode soliton self-frequency shifting
  - “Filamentation” from spatiotemporal Raman dynamics
Graded-index (GRIN) multimode fiber

\[ n^2(\rho) = n_0^2 \left[ 1 - 2\Delta \left( \frac{\rho}{R} \right)^\alpha \right], \quad \rho \leq R \]
\[ = n_0^2(1 - 2\Delta), \quad \rho > R \]
Modes of GRIN fiber
Modes of GRIN fiber

- Propagation constants equally-spaced

\[ \beta \propto n_{\text{eff}} \]

- Velocities of modes vary much less than in step-index fiber
Coupled-mode analysis

“GMMNLSE”

\[
\partial_z A_p(z, t) = i \left( \beta_0^{(p)} - \Re \left[ \beta_0^{(0)} \right] \right) A_p - \left( \beta_1^{(p)} - \Re \left[ \beta_1^{(0)} \right] \right) \frac{\partial A_p}{\partial t} + \sum_{m=2}^{3} i^{m+1} \frac{\beta_m}{m!} \partial_t^m A_p
\]

\[
+i \frac{n_2 \omega_0}{c} \left( 1 + \frac{i}{\omega_0} \partial_t \right) \sum_{l,m,n} \{ (1 - f_R) S_{plmn}^k A_l A_m A_n^* + f_R A_l S_{plmn}^R \int_{-\infty}^{t} d\tau A_m(z, t-\tau) A_n^*(z, t-\tau) h_R(\tau) \}\n\]


Coupled-mode analysis

“GMMNLSE”

\[ \partial_z A_p(z, t) = i \left( \beta_0^{(p)} - \Re \left[ \beta_0^{(0)} \right] \right) A_p - \left( \beta_1^{(p)} - \Re \left[ \beta_1^{(0)} \right] \right) \frac{\partial A_p}{\partial t} + \sum_{m=2}^{3} \frac{i^{m+1} \beta_m}{m!} \partial_t^m A_p \]

modal wavenumber mismatch  modal velocity mismatch  group velocity dispersion

\[ + i \frac{n_2 \omega_0}{c} \left( 1 + \frac{i}{\omega_0} \partial_t \right) \sum_{l,m,n} \left\{ (1 - f_R) S_{plmn}^{k} A_l A_m A_n^* + f_R A_l S_{plmn}^{R} \int_{-\infty}^{t} d\tau A_m(z, t - \tau) A_n^*(z, t - \tau) h_R(\tau) \right\} \]


Single-field model for GRIN fiber

\[
\frac{\partial A}{\partial z} = \frac{i}{2k_0} \left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) - i\beta_2 \frac{\partial^2 A}{\partial t^2} - i\frac{k_0 \Delta}{R^2} (x^2 + y^2)A + i\gamma |A|^2 A
\]

diffraction   dispersion   index profile   Kerr
Single-field model for GRIN fiber

\[
\frac{\partial A}{\partial z} = \frac{i}{2k_0} \left( \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} \right) - \frac{i}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - i \frac{k_0 \Delta}{R^2} (x^2 + y^2) A + i \gamma |A|^2 A
\]

- Gross-Pitaevskii equation
- Model of BEC
Experiments

- fs or ns pulses
- energy up to 5 µJ
- peak power kW to MW
- 1550 nm
- 1050 nm
- 532 nm
Computational challenge

- Kerr and Raman nonlinearities
- Disorder
- 55 modes
- $1 \text{ ns} \times 200 \text{ THz} = 2 \times 10^5$
What should we measure?

- Broadband space-time diagnostic does not exist

- Record overall average spectrum to compare to calculated
- Image near-field on autocorrelator
- Compute spatiotemporal autocorrelation for comparison
Spatiotemporal Modulation Instability
Spatiotemporal modulation instability

- Launch continuous wave or long pulse, normal dispersion
Spatiotemporal MI in GRIN fiber

- Periodic self-imaging plays a role
- Instability occurs for either sign of dispersion

analytic theory by Longhi, Opt Lett 2003
Matera et al., Opt Lett 1993
Space-time MI in GRIN fiber (experiment)

multimode fiber supports ~100 modes

6 m

~1 ns
125 nJ
1064 nm
Space-time MI in GRIN fiber (experiment)

- Sidebands agree with theory

Geometric parametric instability: quasi-phase-matching yields sidebands in same modes as pump

Krupa et al., Phys Rev Lett 2016
Wright et al., arXiv 2016
Beam Self-Cleaning in Multimode Fiber
Beam self-cleaning in GRIN fiber

multimode fiber supports ~100 modes

~1 ns
125 nJ
1064 nm

12 m
Beam self-cleaning in GRIN fiber

Krupa et al., arXiv 2016

multimode fiber supports ~100 modes

~1 ns
5 µJ
1064 nm
Beam self-cleaning in GRIN fiber

- $P << P_{cr}$

- Spatial coherence enhancement

- Negligible dissipation

Krupa et al., arXiv 2016
Beam self-cleaning in GRIN fiber

- Numerical simulations consistent with experiment
- Kerr nonlinearity underlies self-cleaning
- Relation to classical wave condensation?

Krupa et al., arXiv 2016
Self-cleaning of femtosecond pulsed beams

Z. Liu et al., Opt Lett 2016

60 fs
50 nJ
1035 nm

multimode fiber supports ~300 modes
1 m
- Space-time MI plays out differently from in free space

- Kerr nonlinearity underlies a new beam self-cleaning process in multimode GRIN fiber
Dissipation
- stimulated Raman scattering

Disorder
- imperfections in fiber
- short wavelength
- long fiber

- generally reduces nonlinear effects
- leads to diffusion of energy among modes
Introduce dissipation and disorder

up to $3 \mu J$

1 ns

$(P << P_{cr})$

532 nm

parabolic index profile

25-$\mu m$ core (~50 modes)

100 m

temporal profile

spatio-spectral profile

camera, TPA-AC

fiber

OSA

temporal profile

spatio-spectral profile
Beam and spectral evolution

- launch multimode beam
Beam and spectral evolution

- launch multimode beam
- fixed fiber length
- vary pulse energy
Beam and spectral evolution

- beam cleanup
Beam and spectral evolution

- beam cleanup
Beam and spectral evolution

- stimulated Raman scattering
- whole beam attracted toward fundamental mode
Beam and spectral evolution

- cascaded Raman occurs
- whole beam attracted toward fundamental mode
Beam and spectral evolution

- Field reaches an attractor: fundamental mode + background
Beam and spectral evolution

- spatiotemporal MI excited
- higher-order modes amplified across spectrum

![Image of beam and spectral evolution](image_url)
Beam and spectral evolution

- spatiotemporal MI excited
- higher-order modes amplified across spectrum
Beam and spectral evolution

- spatiotemporal MI excited
- higher-order modes amplified across spectrum
Spatiotemporal modulation instability

![Graph showing wavelength vs. intensity with STMI and Raman cascade annotations.](image_url)
Spatiotemporal modulation instability

- STMI
- Raman cascade

Graphs showing
- Wavelength (nm) vs. Intensity (arb. units)
- Shift (THz) vs. MI Order
- Sideband frequencies, spatio-spectral profiles agree with theory

Wright et al. 2016

Spatiotemporal instability

How can a nonlinear attractor be unstable?
Mode coupling

\[
\frac{da_n}{dz} = -i\beta_n a_n + \sum_{m \neq n} C_{nm}(z) a_m
\]

\[
C_{nm}(z) \propto \int \int dx dy \Delta n(x, y, z) n_o(x, y) \varphi_n^*(x, y) \varphi_m(x, y)
\]

\[
\Delta n(x, y, z) = \Delta n(x, y) \cdot f(z) \quad F(K) = \text{FT}\{f(z)\}
\]

Efficient coupling when phase-matched: \( F(K = \Delta \beta) \) is large

\[
\Delta \beta = \beta_m - \beta_n
\]
Mode structure in parabolic fiber

\[ \beta \propto n_{\text{eff}} \]

\[ n(r) \]

\[ n(r) \]
Mode coupling - nonlinearity

For nonlinearity:

$$\Delta n(x, y, z) = n_2 |E(x, y, z)|^2 = n_2 I(x, y, z)$$

$$I(x, y, z) = \left| \sum_{n=1}^{N} a_n(z) \varphi_n(x, y) e^{-i\beta_n z} \right|^2$$

$$C_{nm}(z) \propto \iint dx dy I(x, y, z)n_0(x, y)\varphi^*_n(x, y)\varphi_m(x, y)$$

$$F(K \approx \Delta \beta)$$
Mode-coupling - nonlinearity

\[ F(K) \]
Nonlinear mode coupling

- Modes in each group are coupled
- Generally, modes are unstable
- Fundamental mode is stable
Nonlinear mode coupling

\[ \beta \propto n \]

\[ n(r) \]

\[ \Delta \beta \]

\[ \beta \propto n_{eff} \]

\[ I(x, y, z) = \left| \sum_{n=1}^{N} a_n(z) \varphi_n(x, y) e^{-i\beta_n z} \right|^2 \]

so f(z) has periodic component
Nonlinear mode coupling

\[ F(K) \approx 0 \]

\[ K \approx 0 \quad K = 1/p \]
Nonlinear mode coupling

\[ \beta \propto n_{\text{eff}} \]

- Mode groups couple to each other
Mode-coupling - nonlinearity

\[ F(K) \]

- \( K \approx 0 \)
- \( K = 1/p \)
- \( K = 2/p \)
- \( K = 3/p \)
Nonlinear mode coupling

\[ I(x, y, z') = \left| \sum_{n=1}^{N} a_n(z') \varphi_n(x, y) e^{-i\beta_n z'} \right|^2 \]

\[ C_{nm}(z') \propto \int \int dx dy I(x, y, z) n_o(x, y) \varphi_n^*(x, y) \varphi_m(x, y) \]
Nonlinear mode coupling

- Bias toward lower-order modes
- Raman behaves similarly
Mode coupling - disorder

- Disorder enhances instability of higher-order modes
Mode coupling - disorder

\[ C_{nm}(z) \propto \int \int dx dy \Delta n(x, y, z) n_o(x, y) \varphi_n^*(x, y) \varphi_m(x, y) \]

Disorder

- has greatest effect on higher-order modes
- couples higher-order modes to cladding
Disorder effects on nonlinear attractor
Disorder effects on nonlinear attractor

\[ \beta \propto n_{\text{eff}} \]

Disorder
- increases intensity overlap with lower-order modes
- enhances cleanup

Confirmed in 2D numerics

So fundamental mode should be most stable…
Why is the process irreversible?

\[ \beta \propto n_{eff} \]

- Disorder randomizes intragroup coupling without removing coupling bias
- Disorder, dissipation both lose information
- Disorder + dissipation enhance clean up
- Analogy: biased random walk
What's missing?

- So far the argument only considers 2D space
- Now add coupling to time
Disorder effects on nonlinear attractor

\[ \beta \propto n \]

\[ n(r) \]

\[ \text{core} \]

\[ \beta \propto n_{\text{eff}} \]

cladding

\[ r \]
Disorder effects on nonlinear attractor

\[ \beta \propto n \]

\[ n(r) \]

\[ \text{core} \]

\[ \beta \propto n_{\text{eff}} \]

\[ r \]

\[ \text{cladding} \]
Intuition about spatiotemporal MI

\[ \beta \propto n_{\text{eff}} \]

\[ n(r) \]

\[ r \]
Mode coupling

\[
\frac{d a_n}{dz} = -i \beta_n a_n + \sum_{m \neq n} C_{nm}(z) a_m
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\[
C_{nm}(z) \propto \int \int dx dy \Delta n(x, y, z) n_o(x, y) \varphi_n^*(x, y) \varphi_m(x, y)
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Efficient coupling when phase-matched: \( F(K = \Delta \beta) \) is large

\[
\Delta \beta = \beta_m(\omega_m) - \beta_n (\omega_n)
\]
Spatio-spectral mode structure

\[ \beta \propto n_{\text{eff}} \]

\[ \omega_{n} \]

\[ \omega_{m} \]

\[ n(r) \]

\[ \tau \]
Spatio-spectral mode structure

\[ \beta \propto n_{\text{eff}} \]

\[ n(r) \]

\[ r \]

\[ \omega_n \]

\[ \omega_m \]
Intuition about spatiotemporal MI

\[
\beta \propto n_{\text{eff}}
\]
Intuition about spatiotemporal MI

\[ \beta \propto n \]

\[ \omega_n(r) \]

\[ \omega_{n_1}, \omega_{n_2}, \omega_m \]

\[ \beta \propto n_{eff} \]
Attractor is most-unstable state
What happens

- Energy from all other modes accumulates in the fundamental mode.

- Fundamental mode is
  - least affected by disorder
  - most intense
  - so is the most unstable state of the system.

- This seems to occur for almost every initial condition.
Self-organized instability

~1 ns
3 µJ
532 nm

100 m
Self-organized instability

~1 ns
3 μJ
532 nm

100 m
Self-organized instability

~1 ns

532 nm

100 m
Self-organized instability

~1 ns
3 μJ
532 nm

100 m
Self-organized instability

\[ \text{~1 ns} \]

\[ 3 \ \mu J \]

\[ 532 \text{ nm} \]

\[ 100 \text{ m} \]
Self-organized instability

- Intrinsic disorder enhances nonlinear self-organization

- Universal behavior?

<table>
<thead>
<tr>
<th>Power Level</th>
<th>Dispersion Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kW</td>
<td>Normal dispersion</td>
</tr>
<tr>
<td>10 kW</td>
<td>Anomalous dispersion</td>
</tr>
<tr>
<td>500 kW</td>
<td>Anomalous dispersion</td>
</tr>
</tbody>
</table>
Extreme events

- hydrodynamics (turbulence, hurricanes, tsunamis)
- geosciences (earthquakes, floods, landslides)
- environment and climate sciences (forest fires, competition of species)
- economics (financial markets)
- social sciences (distributions of populations)
- power grid and computer networks (black-outs)

unpredictable black swans (self-organized criticality) OR coherent structures (dragon king)
Possible Future Directions
Classical wave condensation

Wave turbulence theory
- random optical waves can “thermalize”

- initial *incoherent* field self-organizes to form large coherent structure
- equipartition of energy in higher-order modes
- analogous to Bose-Einstein Condensate

- 2D and 3D: no condensation

- 2D + waveguide: condensation predicted theoretically

Aschieri et al., Phys Rev A 2011
Optical turbulence

- Optical wave turbulence studied in 1D systems
- True turbulence requires 3D
Does network topology underlie process?

Multimode fibers are small-world networks

- Coupling is primarily between nearest neighbors
- Presence of “shortcut” links can lead to a strong-coupling transition, many-mode self-organization

Need to understand many-mode nonlinear interactions

- Mode-dependent gain and loss
- Mode-dependent, longitudinally-varying disorder

A small-world network
Strogatz, Nature 2001
3D nonlinear wave propagation (polychromatic) with disorder is only rigorously described by full-field models – very expensive.

Analytic results so far only in the limit of zero and strong disorder.

Huge success of transfer matrix models for linear propagation – extend to weak nonlinear regime?

Rigorous integration of disorder into the coupled NLSEs is non-trivial.

Disorder is:
- Linear (monochromatic coupling)
- Local (waves at same time and place)
- Dissipative through radiation mode interactions?
- Introduces temporal scattering?
Nonlinear wave propagation

3D soliton and conical wave

# of modes

Dissipation

1D DS

1D soliton

Disorder
Nonlinear wave propagation

Disorder of modes

1D soliton

3D soliton and conical wave

Incoherent 3D soliton and conical wave

1D soliton

Incoherent 1D soliton

1D incoherent DS

Disorder

1D DS

Dissipation

3D DS

# of modes

dispersive

MMF

SMF
A multimode fiber laser is a new environment for nonlinear waves. It adds

- spatially-dependent gain, saturable absorption
- spatial and spectral filtering
Multimode fiber lasers can have much higher energy than single-mode fiber lasers

- Larger mode area
- Modal dispersion

\[ E \sim \sum \text{dispersion} \]
Nonlinear wave propagation in multimode fiber

- Spatiotemporal modulation instability
- Beam self-cleaning
- Nonlinearity, dissipation, disorder cooperate to yield instability
- Model system for controlled studies of critical phenomena

*Wright et al., Nature Photon 2016*