Wave Propagation in Photonic Graphene/Rogue Waves

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• Introduction/background
• Optical lattices with honeycomb (HC)–hexagonal structure: ‘Photonic Graphene’
• Photonic Graphene is a photonic analogue of the 2d material graphene
• A NL discrete system governs wave dynamics in HC lattices
• In HC ‘bulk’ lattices find: conical–elliptical–straight-line diffraction: continuous 2d NL Dirac system
Introduction – con’t

Edge waves

- Theory: linear and NL traveling localized edge waves
- Certain edge waves can propagate with no backscatter – ‘Topological Floquet Insulator’ – topological state
- Envelope of NL edge mode satisfies nonlinear Schrödinger (NLS) eq–solitons inherit topology
- Bounded domains:
  - Rectangular domain: transmission/reflection of edge pulses
  - Study the effects of linear and NL pulse propagation
- Conclusions
- Later–outline investigations of Rogue Waves in elliptically birefringent fibers
Periodic Optical Lattice

From Maxwell’s eq. find lattice NLS eq (normalized form)

\[ i\psi_z + \nabla^2 \psi - V(r)\psi + \sigma |\psi|^2 \psi = 0, \quad \sigma \ll 1 \]

\( V(r) \) periodic pot’l: \( V(r + mv_1 + nv_2) = V(r), \quad m, n \in \mathbb{Z} \)

where \( v_1, v_2 \) are the lattice vectors, \( \sigma \) const

Consider non-simple honeycomb lattice
Honeycomb Lattices

Honeycomb (HC) lattices also arise in the study of the 2d material Graphene

Material Graphene: ultra thin carbon material
First demonstrated exp’t 2004; nobel prize 2010
Graphene exhibits important properties physically and mathematically

Here we study: Photonic Graphene – photonic analogue of graphene

Segev’s group: ('07–), MJA, Zhu, Curtis, Ma ('09–), Fefferman, Weinstein ('12–), Z. Chen group (2013–) ...
Linear problem: \( \psi(r, z) = \varphi(r)e^{-i\mu z} \) with \( |\varphi| \ll 1 \):

\[
(\nabla^2 - V(r) + \mu)\varphi = 0
\]

\( V(r) \) is a 2-d periodic potential with lattice vectors: \( \mathbf{v}_1, \mathbf{v}_2 \)

Bloch theory:

\[
\varphi(r; k) = e^{ik \cdot r} U(r; k),
\]

\( U(r; k) \) is periodic in \( r \) and \( \varphi(\cdot, k) \), \( \mu(k) \) are periodic in \( k \) with ‘dual’ lattice vectors: \( k_1, k_2 \)

Dispersion relation: \( \mu = \mu(k) \)
TB Limit

In gen’l cannot determine the relevant e-fcns analytically. But in asymptotic limit $|V| \gg 1$: the tight binding (TB) limit, can approx. the potential and e-fcn’s;

Potential approximated by

$$V(r) \approx \sum_{\mathbf{v}} V_j(r - \mathbf{v}), \quad j = A, B$$

where $V_j(r), \quad j = A, B$ denotes the approximating potential with minima at site $S_v$

There are associated localized functions near the potential minima ($V_j(r)$) – termed ‘orbitals’

TB approx used widely in physics to study lattice systems: linear HC lattices (‘Graphene’): Wallace 1947
Honeycomb (HC) Lattices

Typical honeycomb (HC) lattice $V(r)$ from:

$$V(r) = V_0 \left| e^{ik_0 b_1 \cdot r} + \eta e^{ik_0 b_2 \cdot r} + \eta e^{ik_0 b_3 \cdot r} \right|^2$$

where $b_1 = (0, 1), b_2 = (-\frac{\sqrt{3}}{2}, -\frac{1}{2}); b_3 = (\frac{\sqrt{3}}{2}, -\frac{1}{2})$

$V_0, k_0, \eta$ const. $V_0 > 0$ is the lattice intensity;
$\eta$ corresponds to geometric lattice deformation (strain);
$\eta = 1$ standard HC lattice
Typical HC–Intensity Plot

HC lattice in physical space: below ($\eta = 1$): intensity plot; local minima form a HC lattice
Typical HC–Dispersion Surface

Typical HC dispersion relation $\mu(k)$ first two bands below:

Note: 1st, 2nd bands can touch at certain isolated points, called Dirac points

Dirac points form sites of a hexagonal HC lattice in the $k$ plane
For $|V| >> 1$

$$\psi \sim \sum_{\mathbf{v}} a_\mathbf{v}(z) \phi_A(\mathbf{r} - \mathbf{v}) e^{i\mathbf{k} \cdot \mathbf{v}} + \sum_{\mathbf{v}} b_\mathbf{v}(z) \phi_B(\mathbf{r} - \mathbf{v}) e^{i\mathbf{k} \cdot \mathbf{v}}$$

where the sum is over all lattice sites: $\mathbf{v}$

$$(\nabla^2 - V_j(\mathbf{r})) \phi_j(\mathbf{r}) = -E \phi_j(\mathbf{r}); \quad j = A, B$$

Substitute $\psi$ into lattice NLS eq., multiply $\phi_j(\mathbf{r} - \mathbf{p}) e^{-i\mathbf{k} \cdot \mathbf{p}}; \quad j = A, B$ and integrate
Discrete HC System – With Deformation

Find discrete system

\[ i \frac{d a_p}{dZ} + \mathcal{L}^- b_p + \tilde{\sigma} |a_p|^2 a_p = 0 \]
\[ i \frac{d b_p}{dZ} + \mathcal{L}^+ a_p + \tilde{\sigma} |b_p|^2 b_p = 0 \]

\[ \mathcal{L}^- b_p = b_p + \rho(b_{p-v_1} e^{-i k \cdot v_1} + b_{p-v_2} e^{-i k \cdot v_2}) \]
\[ \mathcal{L}^+ a_p = a_p + \rho(a_{p+v_1} e^{i k \cdot v_1} + a_{p+v_2} e^{i k \cdot v_2}) \]

where NL: \( \sigma \sim \varepsilon \tilde{\sigma}, Z = \varepsilon z, |\varepsilon| \ll 1 \)

\( \rho: \) deformation parameter \( (\rho = \rho(\eta)) \)
Continuous NL Dirac System

When $a_v$ and $b_v$ vary slowly with respect to $v$; at the Dirac point $k = K$ find deformed NL Dirac (NLD) system

\[
i \partial_{\tilde{Z}} a + (\partial X + i \zeta \partial Y)b + \tilde{\sigma}_e |a|^2 a = 0
\]
\[
i \partial_{\tilde{Z}} b + (-\partial X + i \zeta \partial Y)a + \tilde{\sigma}_e |b|^2 b = 0
\]

where
\[
\zeta = \sqrt{\frac{4\rho^2 - 1}{3}}, \ X = \nu x, \ Y = \nu y, \ \tilde{Z} = \nu Z, \ \tilde{\sigma}_e = \nu \tilde{\sigma}, \ |\nu| \ll 1, \ (\varepsilon \sim \nu)
\]

When $\rho = 1, \zeta = 1$ conical diffraction
$1/2 < \rho < 1$: elliptical diffraction
$\rho \to 1/2$: straight line diffraction

When $\tilde{\sigma}_e = 0$ above system reduces to (deformed) 2+1d wave eq. (MJA, S. Nixon, Y. Zhu 2009, 2010)
Conical Diffraction

Below: simulations of lattice NLS and NLD: $\rho = 1$
Top Fig. lattice NLS
Bottom Fig. NLD system (‘a’ envelope)
IC: $a$ is a unit Gaussian and $b = 0$

Thus NLD system yields conical diffraction seen in lattice NLS eq; exp’ts Segev group (’07)
Elliptical Diffraction–NLD

NLD: The rings in the conical diffraction are elliptic if $\rho \neq 1$, where the ratio of axes is
$$\zeta = \sqrt{\frac{4\rho^2-1}{3}}.$$  
Below 2 elliptic rings when (a) $\rho = 0.8$  (b) $\rho = 0.6$
Deformation—con’t

• Critical behavior when $\rho \sim 1/2$
• $|2\rho - 1| \ll 1$ find approximate ‘straight-line’ diffraction
• Various small parameters – different balances lead to different equations: $\beta = 2\rho - 1, \varepsilon, \nu$
• Find numerous new nonlocal nonlinear equations

MJA, Y. Zhu 2013
\[ |2\rho - 1| = |\beta| \ll 1, \quad \nu^2 \ll 1, |\varepsilon| \ll 1 \quad \beta \gg \nu^2 \]

\[ a \sim \nu F, \quad \theta = x - z; \quad Z = \nu^2 z : \]

\[ \partial_\theta \left( \partial_Z F - \sigma i|F|^2 F \right) + \partial^2_\gamma F = 0 \quad \text{NLSKZ} \]

This nonlocal eq is similar in spirit to ‘KZ’ eq. Khokhlov & Zabolotskaya 1969:

\[ \partial_\theta (\partial_t u + u \partial_\theta u) + \partial^2_\gamma u = 0 \quad \text{KZ} \]
Longitudinally Varying Waveguides

Normalized lattice NLS

\[ i \partial_z \psi = -\nabla^2 \psi + V(r)\psi - \gamma |\psi|^2 \psi \]

Introduce longitudinally varying waveguides (Rechtsman et al '13)

\[ x' = x - f_1(z), \ y' = y - f_2(z), \ z' = z \]

\( f(z) = (f_1(z), f_2(z)) \): ‘path function’; transform with

\[ \psi \rightarrow \psi \exp \left[ i \int_0^z |A_p(\xi)|^2 d\xi \right] \text{ with } A_p(z) = -f'(z) \]

find lattice NLS with a pseudo-field: \( A_p(z) \):

\[ i \partial_z \psi = -(\nabla + i A_p(z))^2 \psi + V(r)\psi - \gamma |\psi|^2 \psi \]
TB with Pseudo-Field: Discrete System

TB approx with pseudo-field $A_p = A_p(z)$ ($\psi \rightarrow a_{mn}, b_{mn}$) yields:

$$i \frac{d a_{mn}(z)}{dz} + e^{i \mathbf{d} \cdot A_p} (L - b)_{mn} + \sigma |a_{mn}|^2 a_{mn} = 0$$

$$i \frac{d b_{mn}(z)}{dz} + e^{-i \mathbf{d} \cdot A_p} (L + a)_{mn} + \sigma |b_{mn}|^2 b_{mn} = 0$$

where

$$(L - b)_{mn} = b_{mn} + \rho (b_{m-1, n-1} e^{-i \theta_1} + b_{m+1, n-1} e^{-i \theta_2})$$

$$(L + a)_{mn} = a_{mn} + \rho (a_{m+1, n+1} e^{i \theta_1} + a_{m-1, n+1} e^{i \theta_2})$$

and $\theta_j = (\mathbf{d} - \mathbf{v}_j) \cdot A_p$, $j = 1, 2$, $\rho$ deformation, $\mathbf{d}$ is a vector between adj. horiz. sites, above use $m, n$ row, column format.
Zig-Zag, Arm Chair Edges

Zig-Zag (ZZ): Left Right; Armchair: Top, Bottom
Analysis Discrete System: ZZ

Let: \( a_{mn} = a_n e^{im\omega}, b_{mn} = b_n e^{im\omega} \) find

\[
\begin{aligned}
i \partial_z a_n + e^{id \cdot A_p} (b_n + \rho \gamma^* b_{n-1}) + \sigma |a_n|^2 a_n &= 0 \\
i \partial_z b_n + e^{-id \cdot A_p} (a_n + \rho \gamma a_{n+1}) + \sigma |b_n|^2 b_n &= 0
\end{aligned}
\]

where \( \gamma = \gamma(\omega, \rho, A_p) \)

Take: \( A_p(z) \) periodic & rapidly varying:

\( A_p = A_p(\zeta), \quad \zeta = \frac{z}{\varepsilon}, |\varepsilon| \ll 1; \)

e.g. \( A_p = \kappa(\sin \zeta, -\cos \zeta) \): ‘helical waveguides’

Expt’s: Rechts et al (2013); asymptotic thry: MJA, Curtis, Ma (2014)
Edge Modes: ZZ

Multiple scales:
\( a_n = a_n(z, \zeta); \ b_n = b_n(z, \zeta); \ \zeta = \frac{z}{\varepsilon}; \ \partial_z = \frac{1}{\varepsilon} \partial_\zeta + \partial_z \)

Expand \( a_n, b_n \) in powers of \( \varepsilon \);

Apply BCs (ZZ) find **Edge Modes (ZZ)** (exp decay):

\[
\begin{align*}
    a_n & \sim 0, \quad b_n \sim C(Z, \omega) r^n, \quad |r| = |r(\omega, \rho, A_p)| < 1
\end{align*}
\]

Linear problem (first order):

\[
C(Z, \omega) = C_0 \exp \left( -i \alpha(\omega) Z \right), \ Z = \varepsilon z
\]

\( C_0 \) const. \( \alpha(\omega) \equiv \alpha(\omega, \rho; A_p) \in \mathbb{R} \): ‘edge dispersion relation’
(Floquet coef): obtain explicit formulae;
Linear Problem–Edge Dispersion Relations

Dispersion relations (helical waveguides): thin curves are ‘bulk’ modes; lines in the gap are edge modes:

\[ \alpha \omega = 0 \]

Left Fig: ‘Topological Floquet Insulator’ (Rechts et al ’13)
Right Fig: allows left and right going waves

In general: number of intersections: \( I \) with \( \alpha = 0 \) \( I = 0, 1, 2 \)
\( 0 \leq \omega < \pi \); left fig \( I = 1 \) (topological)
right fig \( I = 2 \) (nontopological)
Nonlinear Edge Wave Envelope Evolution Eq

Discrete edge mode: \( a_{mn} \sim 0 \)

\[
b_{mn} \sim C(Z, y)e^{i\omega_0 m r^n}, \quad |r| < 1
\]

where slowly varying (\(|\nu| \ll 1\)) edge mode envelope \( C \) satisfies:

\[
i \partial_Z C = \alpha_0 C - i\alpha'_0 \nu C_y - \frac{\alpha''_0}{2} \nu^2 C_{yy} + \frac{i\alpha'''_0}{6} \nu^3 C_{yyy} - \alpha_{nl,0} |C|^2 C + \cdots
\]

where \( \alpha_0 = \alpha(\omega_0) \) etc.

May transform to standard NLS-type eq: ‘maximal balance’

If \( A_p = 0 \) then \( \alpha = 0 \): stationary mode
Nonlinear Edge Wave Evolution–NLS

Say $\alpha''_0 \neq 0$ transform envelope eq:

$$i\partial_Z C = \alpha_0 C - i\alpha'_0 \nu C_y - \frac{\alpha''_0}{2} \nu^2 C_{yy} - \alpha_{nl,0} |C|^2 C + \cdots$$

Introduce:

$$Y = y - \nu \alpha'_0 Z, \quad \tilde{Z} = \nu^2 Z, \quad \tilde{C}(Y, \tilde{Z}) = C(Z, y) e^{i\alpha_0 Z}$$

$$i\partial_{\tilde{Z}} \tilde{C} + \frac{\alpha''_0}{2} \tilde{C}_{YY} + \sigma_{\text{eff}} |\tilde{C}|^2 \tilde{C} = 0$$

where $\sigma_{\text{eff}} = \frac{\alpha_{nl,0}}{\nu^2} = O(1)$; Std NLS equation
Typical Linear Edge Wave Evolution–Fig

Left: Discrete; Right Linear Schröd (LS) eq
Fig (a-b): $\rho = 1$: ‘Topological Floquet Insulator’: $\mathcal{I} = 1$
Fig. (c-d): $\rho = 0.4$: at $\alpha'_0 = 0; \alpha''_0 \neq 0$: $\mathcal{I} = 2$ (nontopological)
Nonlinear Edge Wave Evolution–Figs

Fig: $\rho = 1$ Solitons

NL problem inherits topology: $\mathcal{I} = 1$
Bounded Graphene: Zig-Zag, Arm Chair Edges

Zig-Zag (ZZ): Left Right; Armchair: Top, Bottom
Bounded Photonic Graphene (PG): Rectangle

Robust Transmission

Click: Robust Transmission–Combined

\[ \rho = 1.0: \text{ Topological Floquet Insulator} \]
Bounded Photonic Graphene (PG): Rectangle

Robust Reflection

Click: Robust Reflection–Combined

\[ \rho = 0.4: \] Not a Topological Floquet Insulator
Soliton Interactions

For certain parameters, NL, two solitons can exhibit robust interactions: two different choices of parameters and soliton speeds

Topological case

\[ |b(z)| \text{ at the edge} \]
Mode Propagation–Linear

Linear propagation $\rho = 1$ : topological case; different points on the dispersion curve

Left: Linear $\omega = \pi/2$  
Right: Linear $\omega = 7\pi/12$
Mode Propagation–NL

NL propagation $\rho = 1$: topological case; different points on the dispersion curve

Left: NL $\omega = \pi/2$  
Right: NL $\omega = 7\pi/12$: NLS eq
Bdded Rg’ns: Triangle, Hexagon

Considered topological case: $\rho = 1$

Left Triangle (T)  Right: Hex (H): ZZ edges
\[ \rho = 1: \text{8 loops: Top: Linear } (\omega = \pi/2), \text{ Bottom: NL } (\omega = 7\pi/12) \]

Blue: largest perimeter (x); Green: medium (+); Red: smallest (0)

Variance measures the width of the profile
Hexagon: Variance & Mode: Linear – NL

\[ \rho = 1: \text{8 loops: Top: Linear (} \omega = \frac{\pi}{2}, \text{ Bottom: NL (} \omega = \frac{7\pi}{12}) \]

Blue: largest perimeter (x); Green: medium (+); Red: smallest (0)
Conclusion

HC photonic lattices: Photonic Graphene

- In tight binding (TB) limit find/study discrete NL system governing Bloch wave envelope
- Bulk lattices: conical, elliptical diffraction, straight-line diffraction
- Straight-line case: novel nonlocal NL eq

Ref: MJA, S. Nixon, Y. Zhu ’09; MJA, Y. Zhu ’10 -‘13
Conclusion: HC Edge States

- With rapidly varying helical waveguides construct asymptotic edge wave theory in TB limit
- Find linear and NL traveling waves: MJA, C. Curtis, Y-P Ma, 2014
- Certain linear edge modes can propagate with no backscatter: topological modes
- Envelope of NL edge modes satisfy NLS equation: solitons inherit topology
- Bounded Domain–Rectangle: exhibits robust transmission & reflection; states can also be stationary MJA , Y-P Ma, 2015
Conclusion: HC Edge States

- Bounded domains: Rectangle (Rect): ZZ and Arm Chair Edges; Triangle (T): ZZ, Hexagon (H): ZZ; topological: $\rho = 1$

- Linear modes over moderate-long distances are strongly affected by dispersion; integrity of pulses deteriorate

- When NL balances dispersion: NLS regime, pulses maintain variance and integrity: MJA, Y-P Ma, 2016
Outline – Rogue Waves

- Introduction
- Coupled NLS (CNLS) equations important in water waves
- CNLS equations arise in NLO: elliptically birefringent fibers
- CNLS: modulational instability (MI)
- Rogue waves: pdfs
- Structure of rogue waves
- Conclusion

Introduction

• Until recent years studies of rogue/freak waves primarily studied in water waves
• Seamen have told of the occurrence of huge waves in the sea
• A rogue wave is considered as one which is substantially larger than the average background wave – e.g. factor 3
• Many different mechanisms advanced: focusing of currents, winds and more recently: NL phenomena
• Nonlinear effects given more support with measurement of rogue waves in optical fibers: Soli et al, 2007
• Key eq in fiber optics is the NLS eq – which exhibits modulational instability (MI)
First measured rogue wave: Draupner platform in North Sea: Jan 1, 1995: peak elevation 18.5 m (61ft)
Most studies in WW deal with $1 + 1$d systems: simplest NL deep water model: $1 + 1$d NLS eq

In $2 + 1$d water waves interact at an angle

2015: MJA and T. Horikis found CNLS eq in WW: this arises from rogue condition (angle) depends on group velocities

Studied: MI, pdfs of rogue events and structure of rogue waves
Elliptically Birefringent Fibers

NL waves in an elliptically birefringent fiber is governed by following normalized eq

\[ i \frac{\partial u}{\partial z} + \frac{d_1}{2} \frac{\partial^2 u}{\partial t^2} + (|u|^2 + g|v|^2)u = 0 \]

\[ i \frac{\partial v}{\partial z} + \frac{d_2}{2} \frac{\partial^2 v}{\partial t^2} + (g|u|^2 + |v|^2)v = 0 \]

Study: focusing: \( d_1 = d_2 = 1 \); semi-focusing: \( d_1 = -1, d_2 = 1 \)

defocusing: \( d_1 = d_2 = -1 \)

\[ g = g(\theta) = \frac{2 + 2 \sin^2 \theta}{2 + \cos^2 \theta}; \quad \theta = \text{ellipticity angle} \]

\[ \frac{2}{3} (\theta = 0^\circ) \leq g \leq 2 (\theta = 90^\circ) \]
Modulational Instability

In CLS eq substitute

\[ u(t, z) = [u_0 + u_1(t, z)]e^{i\theta_1 z}, \ v(t, z) = [v_0 + v_1(t, z)]e^{i\theta_2 z} \]

where

\[ u_0 e^{i\theta_1 z} = u_0 e^{i(u_0^2 + g v_0^2)z}, \ v_0 e^{i\theta_2 z} = v_0 e^{i(g u_0^2 + v_0^2)z} \]

assume \( u_1, v_1 \sim e^{i(kt - \omega z)} \)

Find quartic eq for \( \omega(k) \); instability from \( \text{Im} \ \omega \); max growth rate: \( \text{Im}\{\omega_{\text{max}}\} \) corresponds to \( k_{\text{max}} \)

\( k_c \) is the max value of the wave numbers, centered around \( k_{\text{max}} \) for which the system is unstable
Modulational Instability—con’t

Left: Maximum growth rate vs. angle $\theta$. Right: Critical wave number, $k_c$, vs. angle $\theta$

Focusing: MI growth rate largest as is $k_c$; larger than scalar NLS (sNLS)

Semi-focusing: has larger growth rates and wider range of unstable k’s than sNLS eq
Modulational Instability—con’t

Left: Maximum growth rate vs. angle $\theta$. Right: Critical wave number, $k_c$, vs. angle $\theta$

Defocusing: can have MI (some angles)
Might suggest CNLS would exhibit some rogue events in defocusing case
Integrate eq numerically; use ICs –wide gaussian

\[ u(t, 0) = v(t, 0) = e^{-t^2/2\sigma^2}, \quad \sigma = 30 \]

with 10% additive noise; \(10^5\) simulations;

Measured relative size:

\[ \eta(t, z) = \sqrt{\frac{|u(t, z)|^2 + |v(t, z)|^2}{\max\{|u(t, 0)|^2 + |v(t, 0)|^2\}}} \]

Computed typical PDFs
The PDFs for largest growth rate: $\theta = 90^\circ$ and sNLS

Focusing CNLS and sNLS have many rogue events
Semi-focusing CNLS: fewer rogue events
Defocusing CNLS: negligible number of rogue events
Mean value of the PDFs (left) and mean value of the max 10% events (right) vs angle $\theta$

Focusing has larger mean and max rogue wave values
sNLS equation has larger mean and max rogue wave values than semi-focusing/defocusing at $35^\circ < \theta < 90^\circ$
PDFs–con’t

Mean value of the PDFs (left) and mean value of the max 10% events (right) vs angle $\theta$

Larger MI growth rates do not imply more numerous rogue waves
Agrees with the results in deep water waves
Typical Rogue Event–Focusing

Focusing CNLS largest rogue waves $\Rightarrow u \approx v$:

**Figure:** Typical max wave amplitude for the focusing case at $\theta = 90^\circ$ at $z = z_*$
Focusing CNLS rogue waves: Find largest rogue event approx by $u \approx v$: scalar NLS

$$i \frac{\partial u}{\partial z} + \frac{d}{2} \frac{\partial^2 u}{\partial t^2} + (g + 1)|u|^2 u = 0$$

Semi focusing CNLS $d_1 = -1; \; d_2 = 1$: Find largest rogue events approx by $u = 0, \; v \neq 0 \Rightarrow$: scalar NLS for $v$ with $g = 0$
Known solutions of sNLS;
i) Peregrine rational soliton:

\[ u(t, z) = u_0 \left[ 1 - \frac{2d[1 + 2i(g + 1)u_0^2z]}{d/2 + 2d(g + 1)^2u_0^2z^2 + 2(g + 1)u_0^2t^2} \right] e^{i(gu_0^2)z} \]

ii) Standard soliton:

\[ u(t, z) = u_0 \text{sech} \left( \sqrt{\frac{g + 1}{d}} u_0 t \right) e^{i[(g+1)u_0^2/2]z} \]
Rogue Event –Zoom In

Focusing CNLS largest rogue waves: Find $u = v$: scalar NLS: Zoom in:

\[ |u| \quad \text{numerical NLS soliton} \quad \text{Peregrine} \]

\[ |v| \quad \text{numerical NLS soliton} \quad \text{Peregrine} \]

Figure: Typical max wave amplitude for the focusing case when $\theta = 90^\circ$; the numerical value is fitted against the Peregrine and single soliton sol’ns of sNLS
Approx Rogue Event–con’t

Semi-focusing CNLS largest rogue waves: Find $u \sim 0$, $v \neq 0$: scalar NLS; Zoom in:

Typical max wave amplitude for the semi-focusing case at $\theta = 90^\circ$; The numerical value is fitted against the Peregrine and single soliton sol’ns of sNLS

Remark: For WW the rogue event approx by solitary waves –no rational solution known
Conclusion

- Studied rogue events in CNLS eq: elliptically polarized birefringent fibers:
- In focusing case there are a larger number of rogue events as compared to all others
  This correlates with the increase in growth rate and size of MI region
- Larger MI growth rates do not always lead to many rogue events: i.e. the semi-focusing/defocusing cases
- Rogue wave in focusing CNLS system $u \sim v$ satisfies sNLS; Peregrine soliton is a good approx to rogue event
- Semi-focusing CNLS system: $u \sim 0$, $v$ satisfies sNLS; standard soliton is a good approx to rogue event