Highway traffic smoothing via trajectory control of connected and automated vehicles

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Stop-and-Go Traffic — Freeway
Stop-and-Go Traffic – Arterial

- Stop-and-go waves
Impacts of Stop-and-Go Traffic

- Traffic congestion in US
  - 42 hours of delay per car commuter
  - Costs $960 per auto commuter

Tampa: 11th most congested cities
http://mobility.tamu.edu/ums/report/
Energy Consumption & Emissions

- In US
  - Congestion wastes 3.1 billion gallons of fuel /year
  - 70% petroleum fuel consumption
  - 30% greenhouse gas emission

Beijing, China

Mexico City, Mexico
Traffic Safety

- In US
  - 2,200,000 injuries
  - 33,000 fatalities
Why Stop-and-Go

- Limitations of human drivers
  - Disconnected
  - Uncooperative
  - Unpredictable
  - Slow
  - Erroneous
  - …
Connected Vehicles

- Vehicle connection = Information sharing
Automated Vehicles

- Human drivers → Robot drivers
Cure: Connection + Automation

- Connected automated vehicles (CAVs)
- Enable trajectory-level vehicle control and coordination
- The fundamental highway traffic problem
  - Past – accommodating human drivers
  - Future - designing robot drivers
Objectives of This Study

- Efficient and parsimonious algorithm to smooth a stream of CAVs along a road
- Applicable to various road facilities
Infrastructure

- Single lane highway segment \([0, L]\)
- Fixed signal timing \(G, R, G, \ldots\) at location \(L\)
Entry Boundary Condition

- Indexed by $n = 1, 2, \ldots, N$
- Entry time $t_n^-$, speed $v_n^-$, known a priori
Physical Bounds

- Trajectory $p_n(t)$
- Speed $\dot{p}_n(t) \in [0, \bar{v}]$, acc. $\ddot{p}_n(t) \in [a, \bar{a}]$
Exit Boundary Constraint

- Exit during green time:
  \[ \text{mod}(p_n^{-1}(L), G + R) \leq G \]
Vehicle Following Safety

- Two consecutive vehicles $n-1$ and $n$
- Shadow trajectory $p_{n-1}^s(t) = p_{n-1}(t + \tau) - s$
- Reaction time $\tau$
- Safety spacing $s$
- **Safety constraint:**
  \[ p_n(t) \leq p_{n-1}^s(t) \]
Travel Time MOE

\[ T := \sum_{n \in \mathcal{N}} \left( p_n^{-1}(L) - t_n^- \right) / N, \]
Fuel Consumption MOE

- E.g., VT-micro, CMEM, MOVES

\[ E := \sum_{n=1}^{N} \int_{t_n^-}^{p_{n}^{-1}(L)} e\left(p_{n}(t), \dot{p}_{n}(t), \ddot{p}_{n}(t)\right) dt / N \]
Safety MOE

- Surrogate measure – Inverse Time-To-Collision (iTTC)

\[ S := \sum_{n=1}^{N} \int_{t_n^-}^{p_{n-1}(L)} \left( h^{iTTC} - \frac{\dot{p}_n(t) - \dot{p}_{n-1}(t)}{p_{n-1}(t) - p_n(t) - l} \right) dt / N \]
Trajectory Optimization (TO)

\[
\min_{\{p_n(t)\}} M(\{p_n(t)\}) := \alpha T + \beta E + \gamma S
\]

subject to

- Infinite dimension
- High nonlinearity
- Differential equations
- Non-convexity
- Vehicle interactions

\[
p_n(t^-) = 0; \quad \forall n \text{ (entry)}
\]

\[
\dot{p}_n(t^-) = \nu_n^- \quad \forall n \text{ (entry)}
\]

\[
0 \leq \dot{p}_n(t) \leq \nu; \quad \forall n, t \text{ (kinematics)}
\]

\[
a \leq \ddot{p}_n(t) \leq \ddot{a}, \quad \forall n, t \text{ (kinematics)}
\]

\[
\text{mod}(p_n^{-1}(L), G + R) \leq G, \quad \forall n \text{ (exit)}
\]

\[
p_n(t) \leq p_{n-1}(t + \tau) - s, \quad \forall n \neq 1 \text{ (safety)}
\]
New Thoughts

- Dimensionality reduction
  - Trajectory $\rightarrow$ A small number of sections
  - Each section with a constant acceleration $\rightarrow$ Analytical parabola

- Parsimony
  - Trajectory smoothness $\rightarrow$ Optimality of all MOEs
  - A few variables on acceleration levels $\rightarrow$ Trajectory smoothness
Shooting Heuristic (SH) Outcome

- A small number of analytical sections
- four variables: \( \bar{a}^f, \bar{a}^b \in [0, \bar{a}], a^f, a^b \in [0, a] \)
Forward Shooting Process \((n = 1)\)

- Accelerate with rate \(\bar{a}^f\) up to speed \(\bar{v}\)
- 1\textsuperscript{st} variable: forward acc. \(\bar{a}^f \in [0, \bar{a}]\)
Forward Shooting Process ($n = 1$)

- Then maintain speed $\bar{v}$ all the way
- Hit the red light?
Backward Shooting Process \((n = 1)\)

- Shift the section above location \(L\) rightwards to the next green phase
Backward Shooting Process \((n = 1)\)

- Back up with acceleration \(\bar{a}^b\) down
- 2\(^{nd}\) variable: backward acc. \(\bar{a}^b \in [0, \bar{a}]\)
Backward Shooting Process \((n = 1)\)

- Merge with deceleration \(a^b\)
- 3\textsuperscript{rd} variable: backward dec. \(a^b \in [0, \bar{a}]\)
Backward Shooting Process \((n = 1)\)

- Merge the forward and backward trajectories
- Obtain a feasible trajectory \(p_1\)
Forward Shooting Process \( (n > 1) \)

- The same till blocked by \( p_{n-1}^s \) (\( p_{n-1} \)'s shadow)
- Pause at a proper place
Forward Shooting Process \((n > 1)\)

- Merge into \(p_{n-1}^s\) with deceleration \(a_f^\) 
- 4\textsuperscript{th} variable: forward dec. \(a_f^\in [0, a]\)
Forward Shooting Process \((n > 1)\)

- Then exactly follow \(p_{n-1}^s\)
Backward Shooting Process \((n > 1)\)

- The same as that for \(n = 1\)
Shooting Heuristic (SH) Outcome

- A *small* number of *analytical* sections
- *four* variables: $\bar{a}^f, \bar{a}^b \in [0, \bar{a}], \underline{a}^f, \underline{a}^b \in [0, a]$
Gradient – Based Algorithm

Initialization

Acceleration values $a^f, \bar{a}^f, a^b, \bar{a}^b$

Update

Search an improvement gradient

Shooting heuristic (SH)

Trajectory set $P^{SH}(a^f, \bar{a}^f, a^b, \bar{a}^b)$

Are terminal criteria met?

Evaluation MOEs $M(P^{SH})$

No

Yes

Return $P^{SH}$
Benchmark (Top) vs. SH (Bottom)

time (sec): 30.00

space (m)

0 500 1000

v (m/s)

0 10 20

space (m)

0 500 1000

v (m/s)

0 10 20
## Benchmark vs. SH

<table>
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<th>$C$(s)</th>
<th>$L$(m)</th>
<th>$f^s$</th>
<th>$\Delta T$</th>
<th>$\Delta E$</th>
<th>$\Delta S$</th>
<th>$\Delta M$</th>
<th>Solution Time</th>
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<td>1500</td>
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<td>32.78%</td>
<td>66.36%</td>
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<td>67.06%</td>
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Quadratic Time Geography (QTG)

- Speed limits \([0, \bar{v}]\) + acceleration limits \([a, \bar{a}]\),
- Quadratic space-time cone & prism
- Feasible region of a trajectory for TO
Feasibility

- **Theorem**: if the signal is ignored (or always green), $SH$ (with $a^f = a^b = \bar{a}, a^f = a^b = a$) is feasible if and only if the original problem is feasible

- **Theorem**: if $L \geq \bar{v}^2 / (2\bar{a})$, $SH$ is feasible if and only if the subset of solutions to the original problem with all exit speeds of $\bar{v}$ is feasible

- **Theorem**: if $L \geq \bar{v}^2 / \bar{a} + \bar{v}^2 / (-2\bar{a}) + s(N -$
Optimality – Time & Throughput

• **Theorem**: When $L \geq \bar{v}^2 / (2\bar{a})$, if SH yields a feasible solution, then this solution always achieves the theoretically minimum travel time,

$$T := \sum_{n \in N} \left( t^+_n - t^-_n \right) / N$$

$$t^+_n := \begin{cases} 
G \left( \bar{p}^{-1}_{0v_n \bar{t}_n} (L) \right), & \forall n = 1. \\
G \left( \max \left\{ \bar{p}^{-1}_{0v_n \bar{t}_n} (L), \frac{t^+_n + s}{\bar{v}} + \tau \right\} \right) & \forall n = 2, \ldots, N. 
\end{cases}$$

It also achieves theoretically maximum throughput

$$\bar{R} := N / (t^+_n - t^-_n)$$
Optimality – Energy

- Based on the optimal control (Minimum Pontryagin Principle & Bang-Bang Control), for certain simple objective functions (e.g., acceleration magnitude), trajectories adapted from the SH results are the optimal.
- Numerical comparisons show that the heuristic solutions are clos to exact solutions
**Connection to Classic Traffic Models**

- Earlier finding: Cellular automata = Kinematic wave = Newell’s car following model (Daganzo 2006)
- **Theorem:** As $\bar{a} \to \infty$ and $\bar{a} \to -\infty$, $SH =$ Newell’s model
Insights Into Queue Back Spill

- Slower acceleration may not necessarily yield longer queue!
Trajectory Control + Signal Timing

\[
\min_{R_1, R_2} Z := \sum_i (D_i + wF_i)
\]

Delay  Fuel consumption
Marginal Fuel Consumption

- Homogenous Setting
- Marginal fuel consumption (MFC) = Fuel consumption difference before and after signal-caused delay
- Numerical results show MFC is well approximated by a linear function of the red time

\[ R^2 = 0.9916 \]
Trajectory Control + Signal Timing

\[
\min_{R_1, R_2} Z := \sum_i (D_i + wF_i)
\]

\[
= \frac{R_1^2 \gamma_2 + R_2^2 \gamma_1}{2 \gamma_1 \gamma_2 (R_1 + R_2 - L)} + \frac{w}{(R_1 + R_2 - L)} \sum_{i=1}^{2} \beta \left( \frac{R_i^2}{2} + \frac{R_i l_i}{\bar{v}} \right)
\]

Delay

Marginal fuel consumption
Freeway Speed Harmonization in Mixed Traffic

1) Prediction problem
2) Shooting heuristic problem

Sensors

Queuing

Queue dissipation time

Space

Time

Traffic Sensor $M$

Traffic Sensor $m$

Traffic Sensor 1

$p(t)$

$c(t)$

CAV

HV
Field Tests

FHWA Turner Fairbank Testbed

Chang’an University Test Track, China
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References:
Li, X., Ghiasi, A. and Xu, Z. “A piecewise trajectory optimization model for connected automated vehicles: Exact optimization algorithm and queue propagation analysis” under review
Xu, Z., Wang, Y., Li, X., Zhao, X., “An Exact Model for Trajectory Optimization and Comparison with Shooting Heuristics” working paper