Quantification and Management of Uncertainty in Model-Based Vehicle Design

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Complexity of Vehicle Design

<table>
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<th>Vehicle NVH</th>
<th>Safety</th>
<th>Body Structure (NVH &amp; Durability)</th>
<th>Chassis &amp; Full Vehicle Durability</th>
<th>Vehicle Dynamics</th>
<th>TASE* &amp; Climate Control</th>
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<tr>
<td>- Idle Tactile (V)</td>
<td>- FRONT IMPACT (V) - New FMVSS 208 - NCAP - OOP - IIHS Offset</td>
<td>- Trimmed Body Principal Modes (V)</td>
<td>- Chassis NVH - Frame Principal Modes - Frame Static Stiffness - BIP Principal Modes (S) - PM at Body Attach. Loc.(S) - LP6 for Body Attachments (S) - Static Stiffness for Body - Attachment Locations (S) - Body SDS/WCR/FMVSS (S/C) - Hood (S) - Decklid (S) - Doors (S) - Trailer Tow (C) - Dash/Cowl fatigue (C)</td>
<td>- Vehicle Dynamics (V) - Steering - Handling - Ride - Braking</td>
<td>- Aerodynamics CFD Analysis (V) - Heat Management (V) - Coolant Flow Simulations (S) - Vehicle Level Climate Control (V) - Front End Air Flow - Front End Openings - System Level Climate Control (S) - A/C Performance - Heater Performance</td>
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<tr>
<td>- Idle Acoustic (V)</td>
<td>- SIDE IMPACT (V) - 33.5 mph FMVSS214 - LINCAP</td>
<td>- Trimmed Body Static Stiffness (V)</td>
<td>- Chassis Durability - Front Suspension - Rear Suspension</td>
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<td>- Driveline Unbalance Tactile (V)</td>
<td>- Rear Impact (V) - 35 mph RMB - 50 mph C/C Inline - 50 mph C/C Side - 50 mph C/C 50% Offset</td>
<td>- Trimming Body Principal Modes (V)</td>
<td>- Frame and Mounting - System</td>
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<td>- Driveline Unbalance</td>
<td>- Roof Crash (S)</td>
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<td>- Rough Road Tactile (V)</td>
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<td>- Impact Harshness Tactile</td>
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<td>- Wind Noise</td>
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*TASE: Thermal Aerodynamics System Engineering
Simulation Process Complexity

- System
  - Define
  - Virtual Test
  - Test
  - Optimization

- Sub-System
  - Design
  - Virtual Integration
  - Integrate
  - Optimization

- Component

- Model
  - Implement

Time

Model-System Integration
Sources of Uncertainty

**SOURCES OF UNCERTAINTY THAT AFFECT MODEL PREDICTION**

- **Model Bias**
  - Due to lack of knowledge, missing underlying physics

- **Parameter Uncertainty**
  - Due to naturally fixed but unknown model parameters

- **Interpolation Uncertainty**
  - Having to predict the response where no existing data is available

- **Experimental Variability**
  - Noise of the test data

**DESIGN UNDER UNCERTAINTY**

- To achieve a design that is insensitive to uncertainties


Uncertainty quantification provides critical information about “how much we could be wrong” in the modeling process
Procedure of Information Fusion

Information Sources

- Low-Fidelity Model
- Intermediate-Fidelity Model
- High-Fidelity Model / Physical Tests

Training Data

- Low-Fidelity Data
- Intermediate-Fidelity Data
- High-Fidelity Data

Validation Data

Construct a new model

Validate the model

Satisfied?

Yes

Design

No

Additional Sampling

Model Refinement
Typical Challenging Scenarios

1. **Surrogate Modeling (Metamodelling)**
   - Have a sophisticated and yet expensive model
   - Build a surrogate model to replace the original model

2. **Regression Analysis**
   - Have test data but no model
   - Construct a model based on the test data

3. **Model Bias Correction**
   - Have test data and a “not so accurate” model
   - Update the model and correct its bias

4. **(Multi-)Model Fusion**
   - Have test data and several different models
   - Integrate the information from tests and models

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Response Surface Modeling (RSM)
Example: Vehicle Design of Frontal Impact

Collaborators
Ford Passive Safety Department

Design Variables
1. rail_inner
2. rail_outer_f
3. rail_outer_r
4. rail_reinf_f
5. rail_reinf_r
6. subframe_arm
7. subframe_frt
8. shot_gun

Responses: Weight, Chest G & Crush Distance

Find $\mu_{x_1}, \ldots, \mu_{x_8}$, $\min E[\text{Weight}]$, s.t. $\Pr \{\text{Chest G} \leq T_1\} \geq \alpha_1\%$, $\Pr \{\text{Crush Distance} \leq T_2\} \geq \alpha_2\%$

- Jiang, Chen, Fu, Yang, “Reliability-Based Design Optimization with Model Bias and Data Uncertainty”, SAE Int. J. Mater. Manf., 6(3), 2013

Training data
- 64 x 3 FE Simulations
- 64 RSM Simulations

Validation data
- 15 x 3 Interpolation FE Simulations
- 25 x 3 Extrapolation FE Simulations
Gaussian Process Modeling for Model Bias Correction

Formulation

\[ y^e(x) = y^m(x) + \delta(x) + \varepsilon \]

Basic Idea

- Treat both the original model and the bias function as GPs
- The test response would also be a GP (by definition)
- Use MLE to estimate the parameters of the GPs.
Situations with Multiple Models

“High-fidelity” physics-based CAE model

“Intermediate-fidelity” physics-based CAE model

“Intermediate-fidelity” surrogate model

“Low-fidelity” simplified handbook equations

\[
\begin{align*}
2m_c v_1^2 &= \frac{1}{2} (2m_c + m_t) v_2^2 + E_{\text{structural}} \\
2m_c v_1^2 &= \frac{1}{2} (2m_c + m_t) \left( \frac{2m_c v_1}{2m_c + m_t} \right)^2 + E_{\text{structural}} \\
E_{\text{structural}} &= 2m_c v_1^2 - \frac{2m_c^2 v_1^2}{2m_c + m_t} \\
E_{\text{structural}} &= 2m_c v_1^2 \left( 1 - \frac{m_c}{2m_c + m_t} \right)
\end{align*}
\]

Multidisciplinary Design Collaboration

Multidisciplinary Design Optimization (MDO) Collaboration Across Systems
A Multidisciplinary System

System Quantities of Interest (QOIs): $y_{sys}$

Discipline 1

Discipline 2

$x_1$ $x_2$ $x_5$

Disciplinary outputs $y_1$ $y_2$

$u_{12}$ $u_{21}$
**OBJECTIVE**

- To improve the **global** modeling capability of a multidisciplinary system such that the prediction uncertainty of system QOIs is acceptable over the input space.

**Resources**: Experiments and/or simulations

**SEQUENTIAL DECISION MAKING**

- **Where** in the input space of a multidisciplinary system shall we allocate more resources?

- **To what** disciplinary response(s) shall we allocate more resources?

- **Which** type of resource shall we allocate, experiments or simulations?

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- **Jiang et al.**, DETC2015-47302, DAC Top 10 Best Paper
DISCIPLINARY UNCERTAINTY QUANTIFICATION

- $u_i^e(x_i, x_s, u_{i.i}^e) = \hat{u}_i^e(x_i, x_s, u_{i.i}^e) + Z_{ui.i}(x_i, x_s, u_{i.i}^e)$
- $y_i^e(x_i, x_s, u_{i.i}^e) = \hat{y}_i^e(x_i, x_s, u_{i.i}^e) + Z_{yi.i}(x_i, x_s, u_{i.i}^e)$

1ST-ORDER TAYLOR SERIES APPROXIMATION

- $u_i^e(x_i, x_s, u_{i.i}^e) \approx \hat{u}_i^e(x_i, x_s, \mu_{u.i})$
  $+ \sum_{j=1, j \neq i}^{ND} \frac{\partial \hat{u}_i^e}{\partial u_{j.i}^e} (u_{j.i}^e - \mu_{u.j}) + Z_{ui.i}(x_i, x_s, u_{i.i}^e)$
- $y_i^e(x_i, x_s, u_{i.i}^e) \approx \hat{y}_i^e(x_i, x_s, \mu_{u.i})$
  $+ \sum_{j=1, j \neq i}^{ND} \frac{\partial \hat{y}_i^e}{\partial u_{j.i}^e} (u_{j.i}^e - \mu_{u.j}) + Z_{yi.i}(x_i, x_s, u_{i.i}^e)$

A MATRIX FORM

- $A(u^e - \mu_u) \approx Z_u, \quad y^e - \mu_y \approx BA^{-1}Z_u + Z_y$
- $\mu_{u.i} \approx \hat{u}_i^e(x_i, x_s, \mu_{u.i}), \quad \mu_{y.i} \approx \hat{y}_i^e(x_i, x_s, \mu_{u.i})$
- $\Sigma_u \approx (A^{-1})\Sigma_{zu}(A^{-1})^T, \quad \Sigma_y \approx (BA^{-1})\Sigma_{zu}(BA^{-1})^T + \Sigma_{zy}$.

- Jiang et al., DETC 2014-34708
Multidisciplinary Statistical Sensitivity Analysis

VARIANCE-BASED SENSITIVITY INDICES

\[
\text{MSI}(Z_i) = \frac{\text{Var}_{Z_i} \left( \mathbb{E}_{Z_{-i}} (y|Z_i) \right)}{\text{Var}(y)}
\]

\[
\text{TSI}(Z_i) = 1 - \frac{\text{Var}_{Z_{-i}} \left( \mathbb{E}_{Z_i} (y|Z_{-i}) \right)}{\text{Var}(y)}
\]

RELATIVE-ENTROPY-BASED SENSITIVITY INDICES

\[
\text{MSI}(Z_i) = -E_{Z_{-i}} \left[ \int_{-\infty}^{\infty} f_{y|Z_{-i}}(y|Z_{-i}) \log \frac{f_{y|Z_{-i}}(y|Z_{-i})}{f_Y(y)} \, dy \right]
\]

\[
\text{TSI}(Z_i) = E_{Z_i} \left[ \int_{-\infty}^{\infty} f_{y|Z_i}(y|Z_i) \log \frac{f_{y|Z_i}(y|Z_i)}{f_Y(y)} \, dy \right]
\]

- Jiang et al., AIAA 2014-2870
- Jiang et al., AIAA J., 2015 (accepted)
AFTER SELECTING LOCATIONS AND RESPONSES...

**Decision made in previous steps:**
To allocate resources to selected $N_L$ locations for response $L$, and $N_{L'}$ locations for response $L'$, etc.

**Suggest an affordable resource allocation plan**
e.g., conducting experiments at $N_{Le}$ locations and simulations at $(N_L - N_{Le})$ locations for response $L$; similarly for $L'$, etc.

**Monte Carlo loop**
- Generate hypothetical data
- Update the emulators
- Evaluate the reduced uncertainty of $y_{sys}$

**Evaluate the "expected" reduced uncertainty of $y_{sys}$**

**Is the reduced uncertainty of $y_{sys}$ acceptable?**
- Yes: **Use this plan**
- No: **Suggest another resource allocation plan**
Conclusions

- Multidisciplinary Design
- Six Sigma
- Quality Process
- Design under Uncertainty
- Uncertainty Quantification
- Uncertainty Propagation
- Dynamic Design Process
- Design Process
- Performance Uncertainty
Thank You!

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