LOGARITHMIC SOBOLEV INEQUALITIES IN DISCRETE PRODUCT SPACES: A PROOF BY A TRANSPORTATION COST DISTANCE

Abstract. The aim of this paper is to prove logarithmic Sobolev inequalities for measures on discrete product spaces, by proving inequalities for an appropriate Wasserstein-like distance. A logarithmic Sobolev inequality is, roughly speaking, a contractivity property of relative entropy with respect to some Markov semigroup. It is much easier to prove contractivity for a distance between measures, than for relative entropy, since for distances well known linear tools, like estimates through matrix norms, can immediately be applied. Our method shall be used to prove logarithmic Sobolev inequalities for measures satisfying a version of Dobrushin’s uniqueness condition, as well as Gibbs measures satisfying a strong mixing conditions.