Data Reduction in Viscoelastic Turbulent Channel Flows

Antony N. Beris

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IMA ANNUAL PROGRAM YEAR WORKSHOP

Flowing Complex Fluids: Fluid Mechanics-Interaction of Microstructure and Flow
DNS: Governing Equations

**Continuity equation**

\[ \nabla^+ \cdot \nu = 0 \]

**Momentum equation**

\[
\frac{\partial \nu}{\partial t^+} = \nu \cdot \nabla^+ \nu - \nabla^+ p_p + \beta_0 \nabla^{+2} \nu + \left( 1 - \beta_0 \right) \nabla^+ \cdot \tau + \frac{1}{\text{Re}_{\tau_0}} e_x
\]

**WALL SCALES**

- Friction velocity: \( \tau_w = \rho U_{\tau}^2 \)
- Length scale: \( \nu_0/U_{\tau} \)
- Time scale: \( \nu_0/U_{\tau}^2 \)

**Constitutive model**

\[
\frac{\partial c}{\partial t^+} + \nabla^+ c = -\tau^+ - \alpha \tau^+ \cdot \tau^+
\]

\[ \tau^+ = \frac{\left( f \left( c \right) c - I \right)}{\text{We}_{\tau_0}} \]

\[ f \left( c \right) = \frac{L^2}{L^2 - \text{Trace}(c)} \]

\( \alpha = 0, L \text{ finite}: \text{FENE-P} \)

\( \alpha = L^{-2}, f = 1: \text{GIESEKUS} \)
Viscoelastic stress is given in terms of the conformation tensor, \( c = \langle R R \rangle \):

\[ R: \text{end-to-end distance} \]

**Critical Factor: Extensional Viscosity Behavior at Large Extensional Rates:**

\[
\frac{\bar{\eta}}{\eta_0} \sim 2L^2 (1 - \beta) \sim \frac{2}{\alpha} (1 - \beta)
\]
• Periodic boundary conditions in ‘x’ and ‘z’ directions
• No slip boundary conditions in ‘y’ direction
Typical Channel Flow Results*

Mean velocity profiles

\[ \text{We}_r = 100, \ L=60, \ \beta_0 = 0.8 \]
\[ \text{DR} \sim 63\%-65\% \]

\[ \text{We}_r = 50, \ L=60, \ \beta_0 = 0.8 \]
\[ \text{DR} \sim 49\% \]

\[ \text{We}_r = 50, \ L=30, \ \beta_0 = 0.9 \]
\[ \text{DR} \sim 30\% \]

Newtonian, \[ \text{Re}_r = 125, 180 \]

\[ \text{Re}_r = 180, 395, 590 \]

* *Housiadas, Beris & Handler, Phys. Fluids 17, 035106 (2005)
**Motivation/Objectives**

- A significant effect: Large Scale Structures seem to be significantly enhanced by viscoelasticity (see above figures).
- Can we exploit this in order to lead to make better use of the simulation data?
- Principal Orthogonal Decomposition, or Karhunen-Loeve (K-L) analysis has been traditionally used as a tool to study the large scale structures. The key question is: Can be also used in data reduction and low dimensional models?
Concept of Coherent Structures:
Structures associated with localized regions of concentrated vorticity, and persist for times that are long compared to smallest local scales of the flow.

Investigative Techniques:

- Flow visualization
Outline

1. Introduction

2. Karhunen-Loeve (K-L) Decomposition

3. Time-Evolution K-L Analysis of Coherent Structures

4. Flow Statistics based on K-L Reconstructions

5. Extended K-L and Ways to Approach Data Reduction in Viscoelastic Turbulence

6. Summary
Karhunen-Loeve (POD) Analysis

Maximize \( \lambda = \langle |(u, \Psi)|^2 \rangle \)

\( u = u^n_i (y, t) \quad \Psi = \Psi^n_i (y) \)

Under constraint \( (\Psi, \Psi) = 1 \)

This gives the eigenvalue problem:

\[
\int_{-1}^{1} R_{ij}(y, y'; n, m) \Psi_j(y') dy' = \lambda \Psi_i(y)
\]

\[
\langle u^n_i (y), u^n_j (y') \rangle
\]

\( \nabla \cdot \Psi = 0, \quad \Psi = 0 \) on the walls

The eigenfunctions can then be used to represent the original velocity field in an optimally convergent series expansion

\[
u^n_i (y, t) = \sum_k a_k(t) \Psi^n_k (y)
\]

First \( N \) K-L modes capture more energy than do first \( N \) modes of any other orthogonal decomposition.
Low wavenumber eigenmodes strengthen,
High wavenumber eigenmodes weaken,
with viscoelasticity
Almost an order of magnitude decrease in the number of modes required to capture 90% of the flow energy as $\text{We}_{\tau_0}$ increases from 0 to 50.

Flow Visualization

About 40% of kinetic energy captured in both cases

Newtonian: 100 most energetic modes

Viscoelastic: 9 most energetic modes

K-L analysis is more effective in the viscoelastic flow case
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Computation of Time-Dependent K-L coefficients

Collection of velocity realizations from DNS

K-L decomposition

Spatial eigenfunctions: $\Psi^{n,m}(y)$

$$a^{n,m,q}(t) = \left(u^{n,m}, \Psi^{n,m,q}\right)$$

Investigate $a^{n,m,q}(t)$ to gain dynamic information about coherent structures
Fluctuation Energy and Representational Entropy

\[ FE(t) = \sum_k d_k a_k(t) \bar{a}_k(t) \]

\[ p_k(t) = \frac{d_k a_k(t) \bar{a}_k(t)}{FE(t)} \]

\[ RE(t) = -\sum_k p_k(t) \log_e(p_k(t)) \]

- FE represents activity in coherent structures
- RE is proportional to number of active modes

Time-Series of Individual Modes

Newtonian

Viscoelastic

• Intermittency and chaotic nature of turbulence dynamics
• More pronounced activity of modes associated with viscoelasticity
• Activities lasting for longer times in viscoelastic turbulent flow

*Samanta et. al., J. Turbulence, 9 (41), Pages: 1-25 (2008)
Cross Correlation among Modes

\[ \hat{\sigma}_i = \sqrt{\left( \langle a_i^2 \rangle - \langle a_i \rangle^2 \right)} \]

\[ \hat{\rho}_{lm}(\text{stagger}) = \frac{\sum_{n=1}^{N} (a_i^n - \langle a_i \rangle) \times (a_m^{n+\text{stagger}} - \langle a_m \rangle)}{N \times \hat{\sigma}_i \times \hat{\sigma}_m} \]

Stationary frame of reference
Cross correlation functions for (2,1,1)

Moving frame of reference
Cross correlation functions for (2,1,1)

\[ a_i^n = a^{(k_x, j_z, q)}(t_n) \]

\[ a_i^n = a^{(k_x, j_z, q)}(t_n) \times e^{i(2\pi f k_x) t_n} \]

• Moving frame of reference is the appropriate choice to study correlations
Advective Velocity of Flow Structures

- For fixed $f$, multiple resonances occur at different $\Delta t^+$ values
- Multiple interactions between modes involving different structures occurring at similar distances from the wall

*Samanta et. al., *J. Turbulence*, 9 (41), Pages: 1-25 (2008)
Pre-conditioned KL projection coefficient: 
\[ a_i^n = a^{(k_x,j_z,q)}_{xy}(t_n) \times e^{i(2\pi f k_x) t_n} \]

Standard Deviation: 
\[ \hat{\sigma}_i = \sqrt{\left( \langle a_i^2 \rangle - \langle a_i \rangle^2 \right)} \]

Cross-correlation coefficient: 
\[ \hat{\rho}_{lm}^{(\text{stagger})} = \frac{\sum_{n=1}^{N} (a_i^n - \langle a_i \rangle) \times (a_m^{n+\text{stagger}} - \langle a_m \rangle)}{N \times \hat{\sigma}_i \times \hat{\sigma}_m} \]

*Samanta et. al., *J. Turbulence*, 9 (41), Pages: 1-25 (2008)
K-L Time-Evolution Analysis: Conclusions

• Moving frame of reference is the appropriate choice to study the structural correlations of the turbulent flow.

• A clear increase in time scales of the dynamic events with viscoelasticity is observed.

• The analysis of mode–mode cross-correlations can be more systematically performed in a two dimensional parameter space allowing for variable moving velocity reference frames.

• Such analysis can reveal independent information on the existence of coherent structures in which multiple K–L modes participate, which are convected at close to the average flow velocity: Can it be used for data reduction?
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Mean Streamwise Velocity Profiles

• Viscoelasticity broadens the buffer layer
Turbulence Intensity

- Top 832 K-L modes gives a good representation of velocity statistics

Polymer Statistics from K-L Reconstructions

• Appreciable reduction in the average trace of conformation tensor

Missing Scales: Small Scales

Extensive analysis of PDFs appears in a Phys. Fluids in press article (2009)
Flow Statistics from K-L Reconstructions: Conclusions

• High energy K-L modes of velocities (i.e. sufficient enough to capture more than 90% of fluctuating kinetic energy on an average) have been found adequate to provide a good estimate for low-order statistics of velocity fluctuations such as the r.m.s. of velocity fluctuations and the average $xy$-component of Reynolds stress.

• Trace of conformation tensor from K-L reconstructions falls short of DNS results.

• Small size active turbulent scales seems to be important for the proper evaluation of conformation trace statistics.

• How can we introduce small scale information in the analysis?
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Extended Karhunen-Loeve Analysis

2 Approaches:

1. Modify the objective criterion to also include a portion of the pseudodissipation rate $\varphi$ in the quantity to be maximized:

$$\varphi = \nu \sum_{i=1}^{3} \sum_{j=1}^{3} \left\langle \left( \frac{\partial u_i}{\partial x_j} \right)^2 \right\rangle$$

2. Perform Karhunen-Loeve analysis for the vorticity field instead for the velocity using the enstrophy $\zeta$ (square of vorticity fluctuations) as the new objective function:

$$\zeta = \sum_{i=1}^{3} \sum_{j=1}^{3} \left\langle \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)^2 \right\rangle$$

Then the velocity is recovered through post-processing
Extended K-L Analysis

- Inner product calculations in full spectral space
  \[
  \left( \hat{u}, \phi \right)^{n,m} = \left( \hat{u}^T \right)^{n,m} \cdot (B)^{n,m} \cdot (\phi)^{n,m}
  \]

- K-L constrained maximization problem can be formulated in a generic sense to allow optimization of various flow measures
  \[
  I \equiv \frac{1}{N} \sum_{k=1}^{N} \left| \left( \hat{u}_k, \phi \right)^{n,m} \right|^2 + (\lambda (\phi, \phi)^{n,m} - 1)
  \]
  \[
  \left( A \right)^{n,m} \cdot (\phi)^{n,m,q} = \lambda^{n,m,q} \cdot (B)^{n,m} \cdot (\phi)^{n,m,q}
  \]
  \[
  \left( A \right)^{n,m} = \left( B^T \right)^m \cdot (R)^{n,m} \cdot (B)
  \]

- Keeping the same information matrix, i.e. the velocity correlation information, \((R)^{n,m}\) and \((B)^{n,m}\) can be suitably modified to adopt to new K-L objective function

- Adopted \((B)^{n,m}\) to optimize fluctuating kinetic energy and pseudodissipation \((\phi \text{K-L})\)
Smaller Scales Result in Slower Convergence

- Adopted K-L method to optimize fluctuating kinetic energy and pseudodissipation ($\phi$ K-L)
- Implemented K-L decomposition of vorticity field ($\zeta$ K-L)
- $\phi$ K-L shows faster convergence than K-L when then contributions of the discarded modes are not accounted for
- Contributions from discarded modes can be accounted for via rescaling as

$$\left(\frac{\text{Cumulative fraction}}{\text{rescaled}}\right) = \left(\frac{\text{Cumulative fraction}}{1 + (\text{loss estimate})}\right)$$
In both $\zeta$ K-L and $\phi$ K-L, the objective measures have large contribution from the shear component of velocity gradient ($\partial u/\partial y$).

It is suspected that objective measures based on velocity gradients are dominated by small scale structures which, by their very nature require way too many modes to be represented well.
3rd Approach: Parameter Rescaling

- Rescaling of Weissenberg number can be done to compensate for lost information (small-scales of turbulence) during reconstruction of conformation field.

- This rescaling can be based on a measure of extension in the flow, \( \dot{E}(y) = \frac{|d \cdot \omega|}{\|\omega\|} \) (*).

(*) Sureshkumar et al., Physics of Fluids, 9, 743-755, 1997
• A simple rescaling of Weissenberg number can offer quite adequate compensation for the lost extensional features of the flow

• Overestimation of the average trace values at the walls is attributed to the application of the compensation uniformly in the entire channel width
Extended K-L and New Data Reduction Approach: Conclusions

• A general K-L decomposition algorithm in application to 3D complex data has been developed where new objective functions can be used.

• Velocity gradient based K-L objective functions fail to capture velocity or conformation statistics any better than standard K-L.

• A compensation by way of rescaled value of Weissenberg number together with a standard K-L modes decomposition provides the best way to approach data reduction in viscoelastic turbulent channel flows.
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1. Introduction
2. Karhunen-Loeve (K-L) Decomposition
3. Time-Evolution K-L Analysis of Coherent Structures
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5. Probability Density Functions of Turbulent Quantities
6. Extended K-L and Ways to Approach Data Reduction in Viscoelastic Turbulence
7. Summary
Summary

• Time-dependence study of K-L modes reveals useful information about the modification of dynamics of coherent structures in turbulence imparted by viscoelasticity.

• Velocity statistics from K-L reconstructed velocity fields agree well with DNS results.

• Suppression of small scale turbulent structures by K-L data reduction prohibits proper evolution of the conformation field.

• PDFs of velocity gradients provide evidence on enhanced intermittency in the modification of small scale turbulence by viscoelasticity.

• Selected set of standard K-L modes (in the order of few thousands) with a compensation through rescaling of Weissenberg number provides the best way to approach data reduction in viscoelastic turbulent flow: can this lead also to low-dimensional models?
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