IMA Lecture 10

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Problem 1

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- Caution 1: Deterministic has to be defined carefully (polynomial time construction)
- Caution 2: Here we are measuring goodness of a matrix by RIP. If we use other criteria, there are other constructions known to provide good approximation (as Anna Gilbert talked about) but they have not been shown to have RIP

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- Assume Φ is $n \times N$ and satisfies RIP of order ak with $a \geq 3$ a constant to be chosen by you
- Given any $x \in \Sigma_k$ show that the appropriate interior point method will approximated the ℓ_1 minimizer to accuracy ϵ in ℓ_1 using at most $CNn^2 \log(1/\epsilon)\|x\|_{\ell_1}$ operations

Problem 3

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- Let $\Phi(\omega)$ be the family of $n \times N$ matrices generated by the Bernouli random variable taking values $\pm 1/\sqrt{n}$ with equal probability
- Prove that for each $x \in I\!R^N$, the ℓ_1 minimization decoding x^* satisfies

$$\|x - x^*\|_{\ell_2} \leq C_0 \sigma_k(x)_{\ell_2}$$

with high probability on the draw Φ

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$$\|x - x^*\|_{\ell_1} \leq C_0 \sigma_k(x)_{\ell_1}$$
- Is it true that there is an $n \times N$ matrix Φ' such that the null spaces of Φ, Φ' have the same null space and Φ' satisfies the RIP

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- Can one remove this logarithm in a practical system?

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- Bit rate, error correction, decoding time, circuit implementation
- Rather than analyze each of these arms separately can we develop a theory which treats the entire system?