RADAR imaging from multiply scattered waves

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Imaging from Wave Propagation

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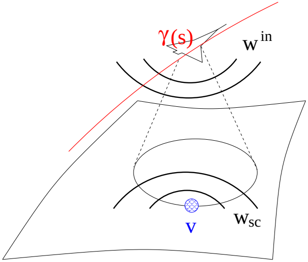
Outline

1. Introduction
2. Traditional SAR
3. Geometric Structure of Λ
4. Scattering from Environment

Introduction

- Traditional SAR cannot distinguish scatterers on left of flight path from those on right, unless we operate in side-scan mode (beam-form to one side)
- Problem is compounded in a waveguide situation, yielding many more artifacts from multiple scattering between walls and target (Cirencester, 2005, Cheney & Nolan)
- Even in side-scan mode, latter artifacts persist
- In this talk, we consider a single wall
- In side-scan mode, we show that we can rid ourselves of both types of artifacts at once!

SAR - Low Directivity ... for now



- Emit low-frequency (30-90 Mhz) radio waves from antenna
- Goal: Construct image of the ground from scattered waves

SAR

- Treat Electric Field as though its components satisfy a scalar wave equation:

$$\left(\frac{1}{c^2(x)} \partial_t^2 - \nabla^2 \right) u(t,x) = 0,$$

where $c(x)$ is the wave speed of the field

- Linearize about constant background (air):

$$c^{-2}(x) - c_0^{-2} := V(x_1, x_2)\delta(x_3)$$

where c is speed of wave propagation

SAR

- Suppose antenna is flown on a flight path

$$\Gamma := \{\gamma(s) : \; s \in (s_{\min}, s_{\max}) \}$$

- Possible to show (as suggested by earlier figure): signal recorded by antenna at location $\gamma(s)$ at time t is approximately

$$d(s,t) = \int d\omega dx \; e^{-i\omega(t-2|(x,0)-\gamma(s)|/c_0)} \, W(x,s,t,\omega)V(x)$$

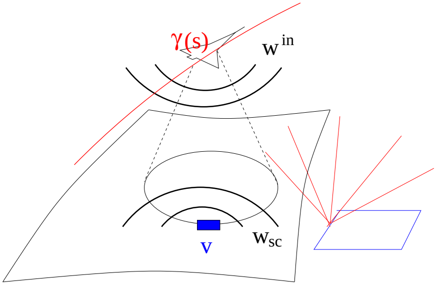
where W is a weighting function that incorporates source radiation pattern, bandwidth, etc

- This defines a scattering operator:

$$\mathcal{F} : V \mapsto d$$

SAR

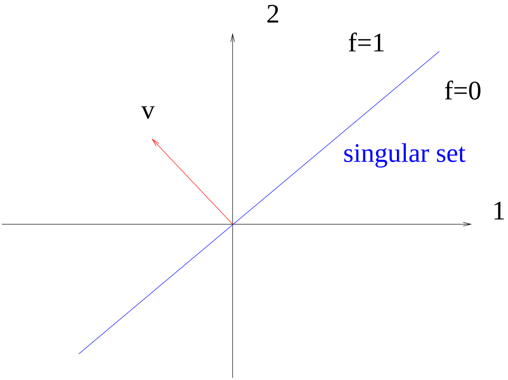
- V has singularities at boundaries of objects on ground - e.g. walls of buildings



- These singularities will be mapped to singularities in the data

Singularities, with directions

E.G. $f(x) = H(v \cdot x)$ has a singularity at each point on line perpendicular to v through origin in the direction v



SAR

- Define
$$X \equiv R^2, \quad Y := (s_{\min}, s_{\max}) \times (0, T)$$
$$\Rightarrow \mathcal{F} : \mathcal{E}'(X) \to \mathcal{E}'(Y)$$
- $\mathcal{F}$ is an oscillatory integral operator with homogeneous phase and non-vanishing gradient
- Amplitude of $\mathcal{F}$ behaves as an approximate symbol (decays in ω when differentiated w.r.t. ω)
- Thus $\mathcal{F}$ is a Fourier integral operator (FIO)

SAR

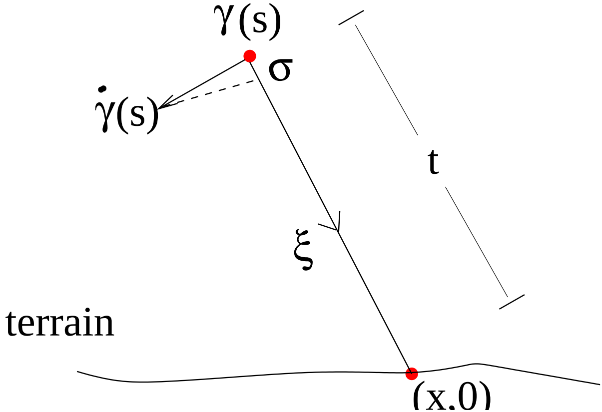
- Recall phase of $\mathcal{F}$:

$$\phi(s,t,x,\omega) := -i\omega(t - 2|(x,0) - \gamma(s)|/c_0)$$

- According to theory of FIOs, singularities in V at a point $x \in X$ in a direction ξ produce singularities in the data d at (s,t) in a direction (σ,τ) whenever they belong to the set relation (hat means unit vector):

$$\begin{aligned}\Lambda \quad = \quad & \{\ ((s,t,\sigma,\tau),(x,\xi))\ | \\ & t = 2|\gamma(s) - (x,0)|/c_0, \\ & \sigma = 2\tau(\widehat{\gamma(s) - x}) \cdot \dot{\gamma}(s)/c_0, \\ & \xi = 2\tau(\widehat{(x,0) - \gamma}(s))_H/c_0 \\ & W(x,s,t,\omega) \neq 0\ \}\end{aligned}$$

SAR - Graphical Illustration of Wavefront Relation Λ



Lagrangian Submanifolds

- Lagrangian submanifolds can be reduced (locally) to one like the following model form ...
- Think of a particle at x_0 on a hypersurface $\mathcal{H}$, with momentum p_0 at $t = 0$
- At a later time $t > 0$, Hamiltonian* mechanics tells us that the particle will now be located at $(x_1, p_1) = (C_1(x_0, p_0), C_2(x_0, p_0))$, where C is a canonical transformation (that ensures conservation of energy)
- The set of such pairs $\{(x_0, p_0), (x_1, p_1)\}$ is the canonical Lagrangian submanifold

* *This year is Hamilton's bi-centenary*

SAR

- Λ is a Lagrangian submanifold with the properties

$$\pi : \Lambda \to T^*Y, \quad \rho : \Lambda \to T^*X$$

are *local* diffeomorphisms *except* at points in

$$\Sigma := \{ ((s,t,\sigma,\tau),(x,\xi)) \in \Lambda \mid$$

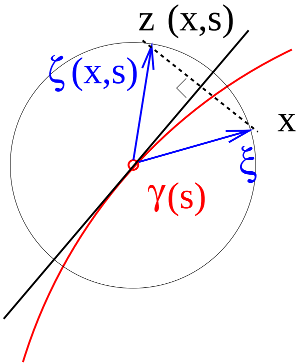
horizontal component of $(x-\gamma(s))$ and $\dot{\gamma}(s)$ are co-linear $\}$

Artifact Analysis

- Imaging methods often consist of application of a (weighted) adjoint scattering operator $\mathcal{F}^*$ to the data
- $\mathcal{F}^* : \mathcal{E}'(Y) \to \mathcal{E}'(X)$
- Singularities in data are mapped to singularities in resulting *image* by Λ^* – the transpose relation of Λ
- In summary, our proposed image transforms the singularities of V via the composite relation $\Lambda^* \circ \Lambda$
- Goal: arrange $\Lambda^* \circ \Lambda \subseteq I$ (identity relation), so that we recover the visible singularities of the model V

Artifact Analysis

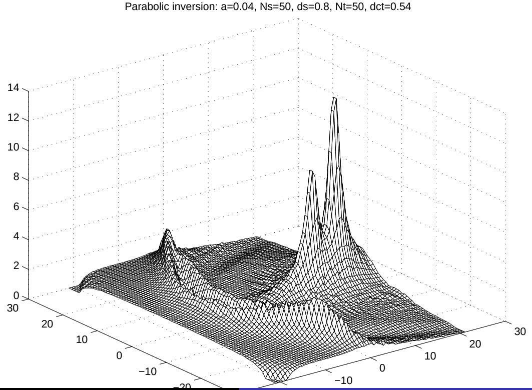
- Problem: π is not injective, in fact it is a 2:1 map



- So a singularity (x, ξ) could be correctly imaged along with artifact $(z(x, s), \zeta(x, s))$ artifact

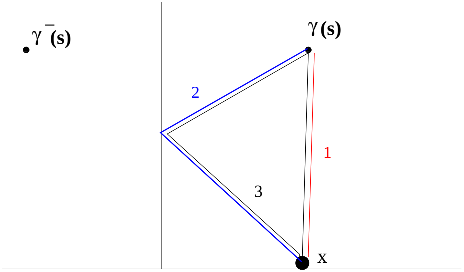
Artifact Analysis

- Artifact avoidance:
  1. Side-scan mode
  2. High-curvature for flight-track weakens artifact by smearing it, enhanced by long (dwell) integration times



Scattering from a nearby wall

- We now switch attention to a target near a wall (or ground)
- Waves can scatter in three ways (plus a fourth way which reverses path 3):



Scattering from a target with a nearby wall

- We represent the wall ($x_1 = 0$) using the method of images, placing a virtual source at $(-\gamma_1(s), \gamma_2(s), \gamma_3(s))$
- The data d now becomes

$$d = \sum_{i=1}^{3} \mathcal{F}_i V$$

and we enforce a Dirichlet condition at the wall

- Where $\mathcal{F}_i$ is an operator of the same form as $\mathcal{F}$ with a modified amplitude and phase

$$\begin{aligned} \phi_1 &= -i\omega(t - 2|x - \gamma(s)|) \\ \phi_2 &= -i\omega(t - |x - \gamma(s)| + |x - \gamma^-(s)|) \\ \phi_3 &= -i\omega(t - 2|x - \gamma^-(s)|) \end{aligned}$$

Scattering from a target with a nearby wall

- We assume scatterers are located in $x_1 > 0$
- Idea: Examine singularities in data (s, t, σ_i, τ) due to the three different paths
- We'll show how to arrange $\sigma_1 \neq \sigma_3$
- Observe $\sigma_2 = (\sigma_1 + \sigma_2)/2$, which means none of the σ_i's can ever have common values!
- Latter point is key to avoiding further artifacts in backprojection (see later)

Scattering from a target with a nearby wall

- That we might arrange $\sigma_1 \neq \sigma_3$ is readily seen graphically for the case when RADAR flies perpendicularly away from wall ...
- σ_1, σ_3 are proportional to the direction cosines of the range vectors from the source and virtual source respectively
- Thus σ_1, σ_3 are monotonically increasing and decreasing respectively, and only agree for RADAR located on wall (dis-allowed)

Scattering from a target with a nearby wall

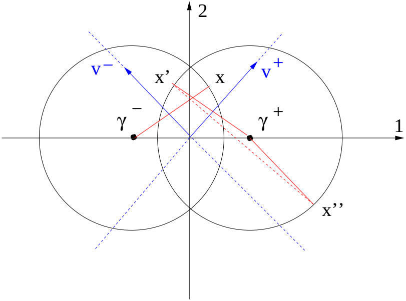
- Significance of distinct σ_i's ...

$$\mathcal{F}^* d = \sum_{i=1}^{3} \sum_{j=1}^{3} \mathcal{F}_i^* \mathcal{F}_j \, V$$

- Singularities are reconstructed properly from the diagonal terms ($i = j$) as we've seen from first part of talk ... provided we employ side-scan mode, or other methods
- Once we establish the how to arrange distinct σ_i's, we will have arranged $\Lambda_i^* \circ \Lambda_j = \emptyset$
- So, no artifacts arise from off-diagonal terms ($i \neq j$)

Conditions for disjoint σ_i's

- As noted, we just need to ensure $\sigma_1 \neq \sigma_3$



- Figure shows that x-scatterer produces a σ_3 equal to σ_1 due to x''-scatterer in experiment 1.
- Thus data could be backprojected along experiment 1 to produce x''-scatterer, which could be an artifact
- BUT x'' artifact is avoided due to Side-scan mode!

Conditions for disjoint σ_i's

- Algebraically, we see this from ($\sigma_1 = \sigma_3$):

$$\frac{(\gamma_1 + x_1)v_1 + (\gamma_2 - x_2)v_2 + (\gamma_3 - x_3)v_3}{|\gamma^- - x|} =$$

$$\frac{(\gamma_1 - x_1'')v_1 + (\gamma_2 - x_2'')v_2 + (\gamma_3 - x_3'')v_3}{|\gamma^+ - x|}$$

- In composing $\Lambda_1^* \circ \Lambda_3$ we are composing points $((x'', \xi''), (s, t, \sigma_1, \tau))$ and $((s, t, \sigma_3, \tau), (x, \xi))$
- Where $t = |\gamma^- - x| = |\gamma^+ - x''|$, which allows us to cancel denominators
- As x and x'' are at same depth beneath respective sources, we cancel last term in denominator, yielding

Conditions for disjoint σ_i's

- Condition for $\sigma_1 = \sigma_3$

$$V^{(2)} \cdot \begin{bmatrix} x_1 + x_1'' \\ x_2'' - x_2 \end{bmatrix} = 0$$

- where

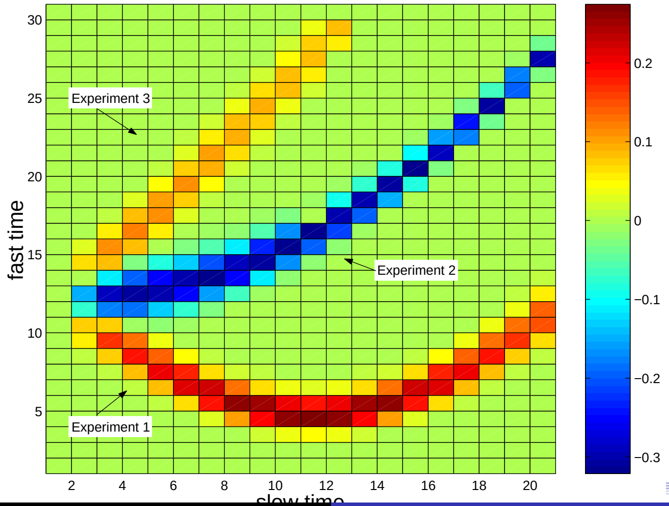
$$V^{(2)} = [v_1 \; v_2]^T$$

- If we fly perpendicular to the wall ($v_2 = 0$), then $x = x''$ and $x_1 = 0$ (x is on wall), which is vacuous, since scatterers assumed to have $x_1 > 0$
- So no artifact-scatterers away from wall

Conditions for disjoint σ_i's

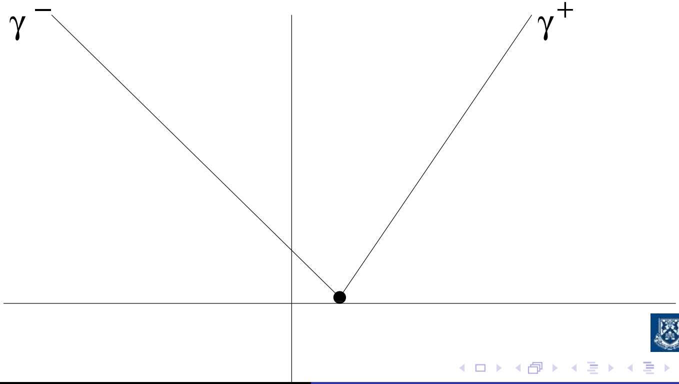
- If we fly parallel to wall ($v_1 = 0$), then $x_2 = X_2''$, so artifact-scatterers lie on intersection of a line perpendicular to the wall and a circle centred at γ^+
- In general
$$v_1 x_1 + v_2 x_2 = f(x'')$$
which gives a series of parallel lines (perpendicular to $\dot{\gamma}$) to be intersected with a circle centred at γ^+
- This confirms graphical assertion that side-scan mode will avoid mixed backprojection artifacts as well as the original left-right ambiguity artifacts simultaneously

Data Set


fast time
slow time
30
25
20
15
10
5
2
4
6
8
10
12
14
16
18
20
0.2
0.1
0
−0.1
−0.2
−0.3
Experiment 3
Experiment 2
Experiment 1

Angular Resolution Increased

- Virtual source has benefit of making aspects of scatterers visible that would not otherwise be visible
- In effect, for scatterers located close to the wall, we double the 'opening angle'



Intersecting Lagrangians

- Allowing scatterers to be located on wall, the following is the situation ...
- Without the wall, the operator $\mathcal{F}^*\mathcal{F}$ is associated to the class of singular FIO's ($I^{p,l}(\Lambda_1, \Lambda_2)$) with cleanly intersecting Lagrangians
- Insertion of the wall results in composition of operators associated to triply intersecting Lagrangians (see Neumann version of previous data set).
- No calculus for singular FIO's associated to tripple-intersecting Lagrangians
- So, if you want to consider scatterers on the wall, you will have to carry out explicit analysis to see if inversion can be arranged

Summary & Acknowledgement

- Simple message is that side-scan SAR (Sonar) avoids left-right ambiguity artifacts as well as those that might have been expected due to multiple scattering with the wall
- Acknowledgement: Thanks to Science Foundation Ireland (SFI) and NSF for supporting this research.
- Invitation: Go see 'Praire Home Companion' rehearsal show in St. Paul Friday evening (*Lake Wobegon: Where the women are strong, the men are good-looking, and all the children are above average*)