Processing of Diffusion-Tensor Magnetic Resonance Images

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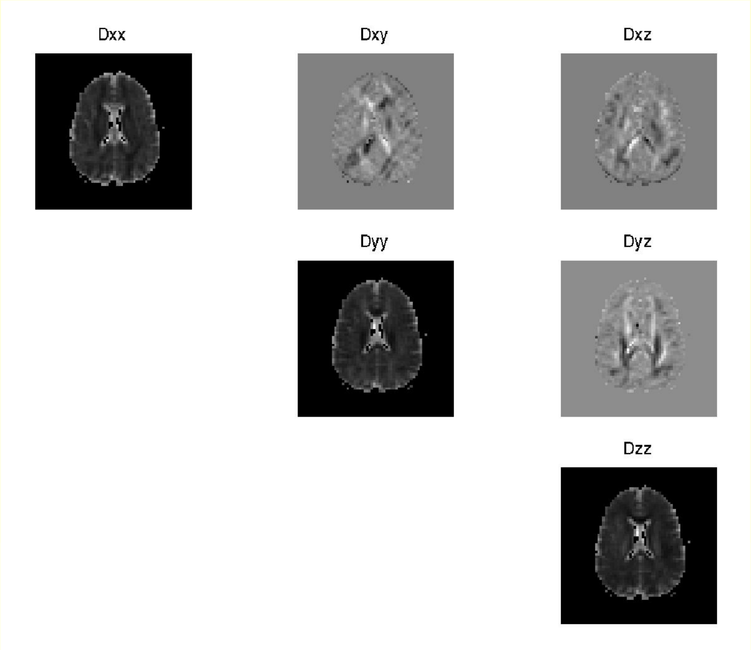
Problem

What is DTMRI?



$$\rho(\mathbf{r}|\tau_d) = \frac{1}{\sqrt{\|\mathbf{D}\|(4\pi\tau_d)}} \exp\left(\frac{-\mathbf{rDr}}{4\tau_d}\right) \quad (1)$$

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Elucidate, non-invasively, the 3-D architectural structure of tissues, organs and organized matter: e.g., find fiber tracks

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Applications

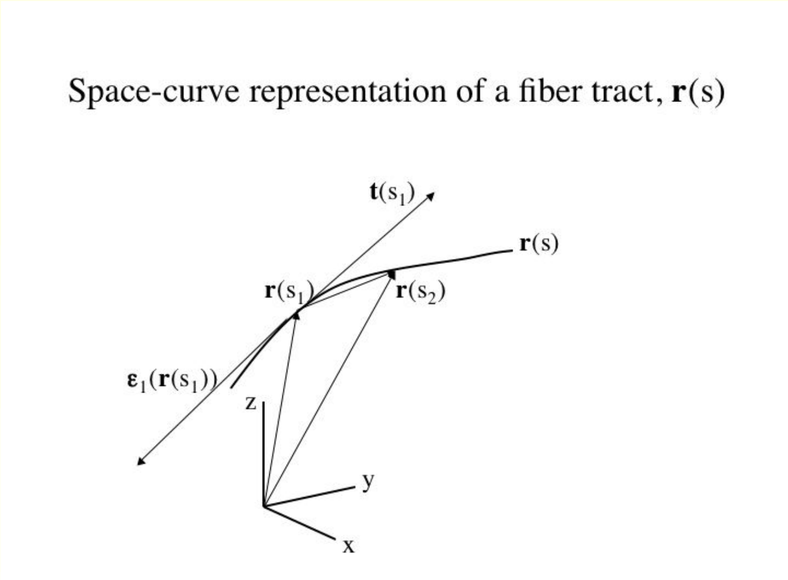
- Mapping connectivity between tissues and organs
- Monitoring structural changes in development, aging and disease

Problem

Tracktography: Find fiber track from DT data

$$\frac{d\mathbf{r}}{ds} = \epsilon_1(\mathbf{r}),\ \mathbf{r}(0) = \mathbf{r}_0$$

where $\epsilon_1(\mathbf{r})$ is the largest eigenvector of diffusion tensor field $\mathbf{D}(\mathbf{r})$.



Difficulties

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- Complex data (non-negative definite matrices, and vectors)
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- Noise
- High dimensionality of data (10^7 values: high computational complexity)
- ...

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- Interpolation (spacially adaptive, non-negative definite preserving +...)
- Numerical DE solver (fast, accurate)

Shift Invariant Space Models

$$V_\Delta(\mathbf{B}) = \left\{ \mathbf{D}(\mathbf{x}) = \sum_{j\in\Lambda} \sum_{\mathbf{k}\in\mathbb{Z}^n} c^j(\mathbf{k})\mathbf{B}^j\left(\frac{\mathbf{x}}{\Delta} - \mathbf{k}\right) : \; c^j \in \ell^2(\mathbb{Z}^n) \right\}, \quad (2)$$

where $\{\mathbf{B}^j : j \in \Lambda\}$ form a generator for the space V_Δ.

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- Discrete tensor data $\{\mathbf{D}(k) : k \in \mathbb{Z}^3\}$ can be approximated with fast filtering algorithms by a member in $\mathbf{D}_\Delta \in V_\Delta$ defined on all $\mathbb{R}^3$, where noise is controlled by size Δ.

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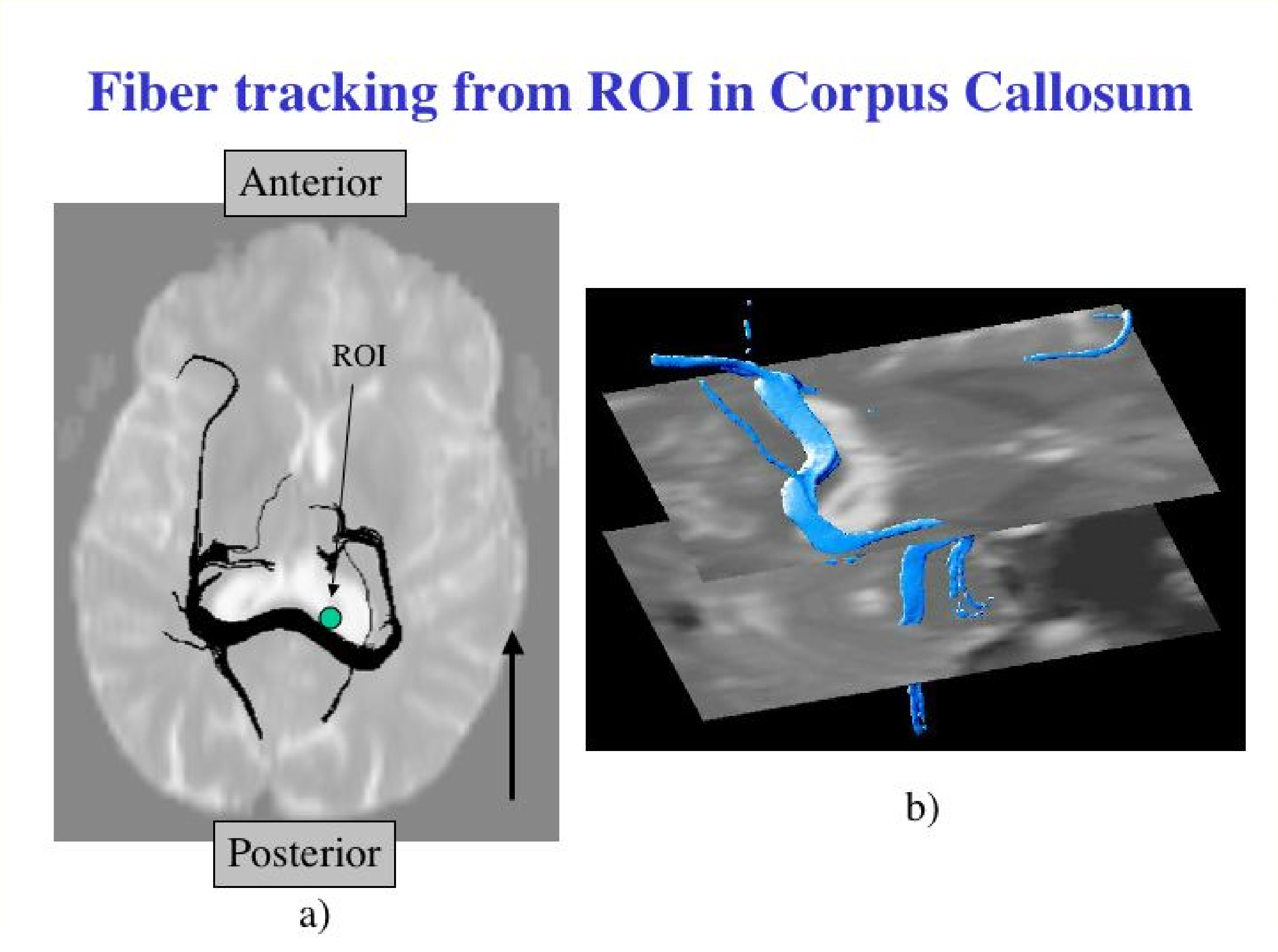
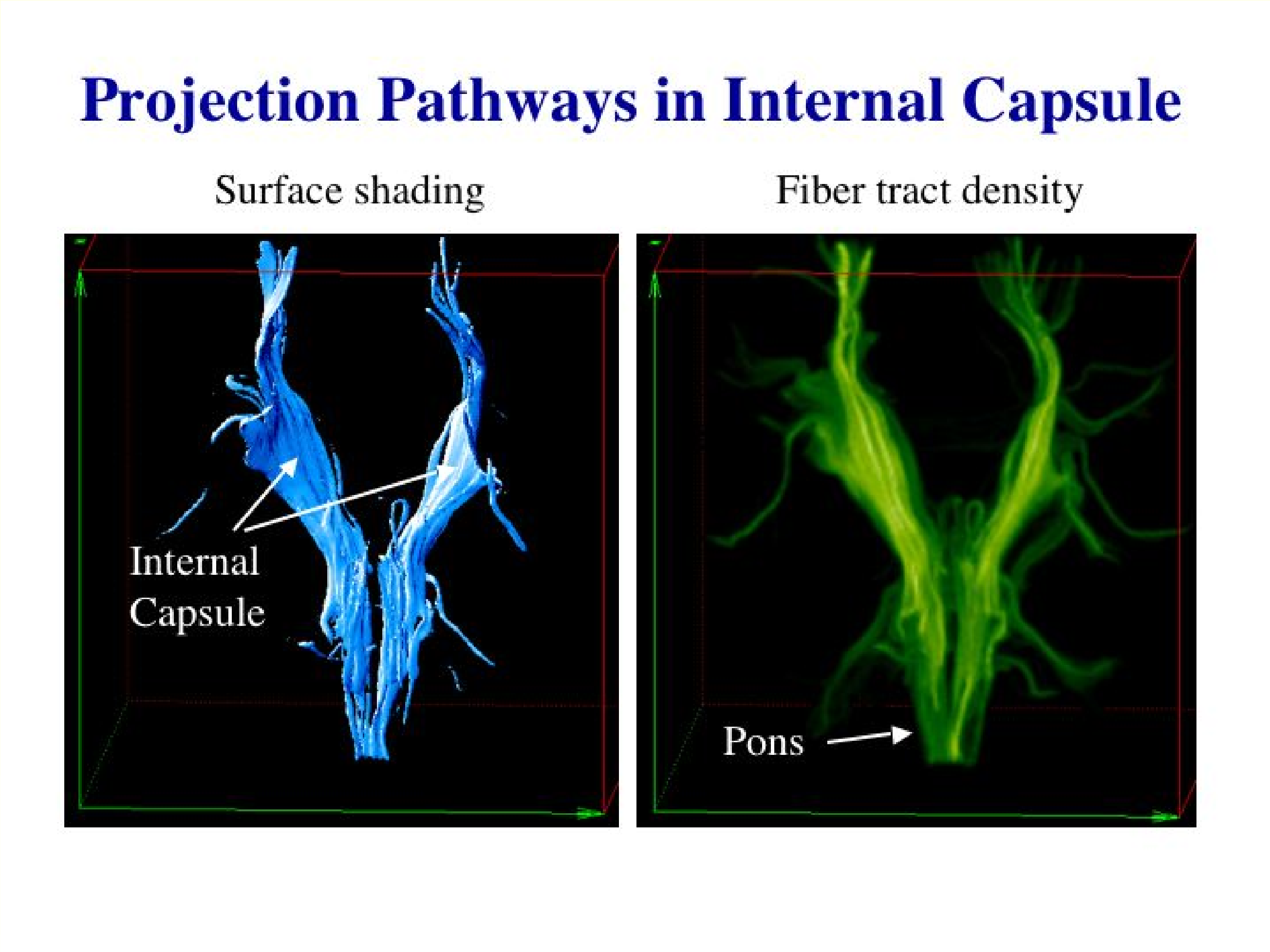
- Use Euler method for integration.

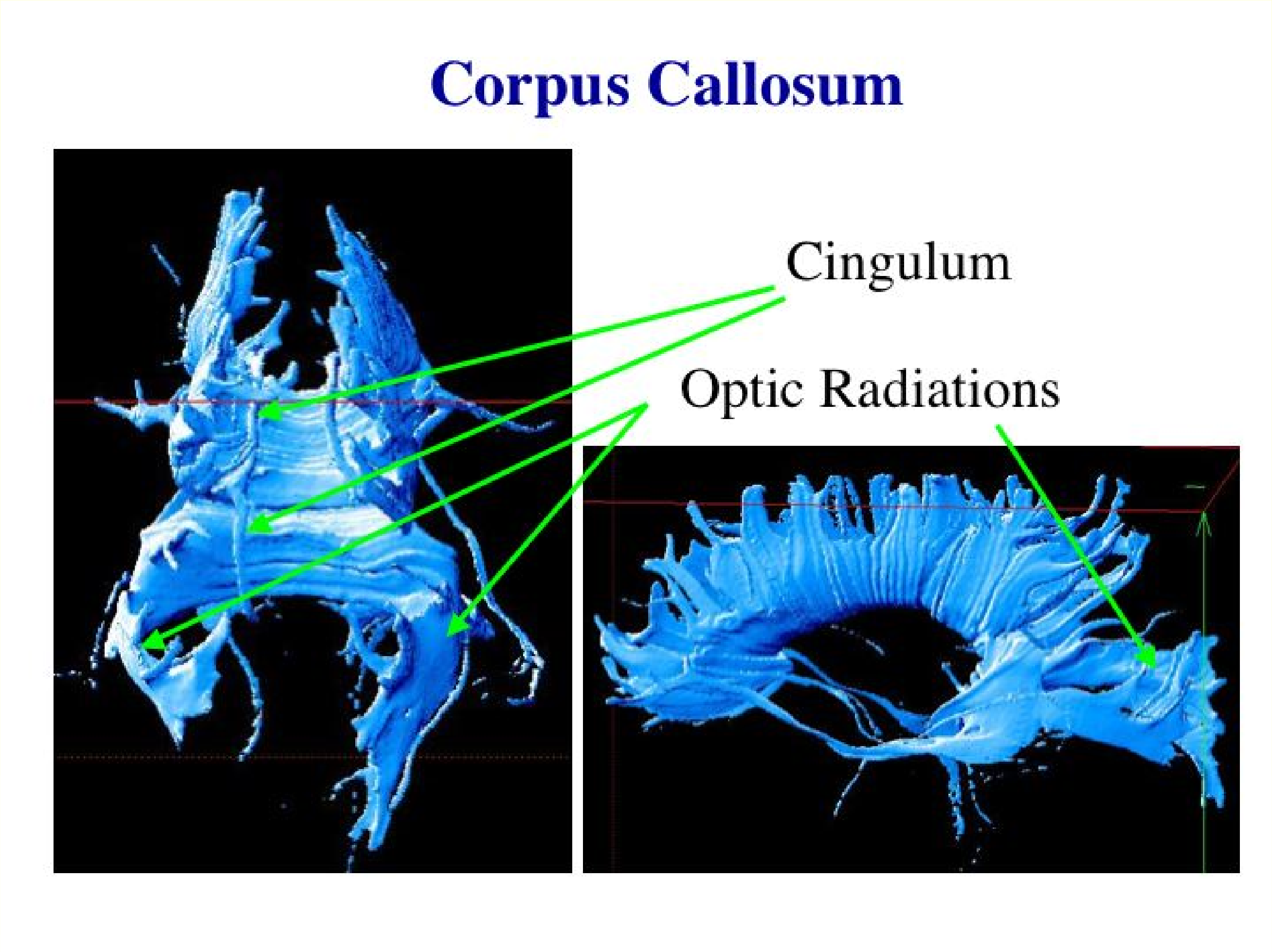
Example 1


Fiber tracking from ROI in Corpus Callosum
Anterior
ROI
Posterior
a)
b)

Example 2


Projection Pathways in Internal Capsule
Surface shading
Fiber tract density
Internal
Capsule
Pons

Example 3



Bayesian Approach

Euler tracking method

$$\mathbf{r}_{i+1} = \mathbf{r}_i + \alpha\epsilon_i$$

where α is integration step size step and ϵ_i is direction vector to be estimated. Let $\mathbf{d} = (\mathbf{D}_{11}\mathbf{D}_{22}\mathbf{D}_{33}\mathbf{D}_{12}D_{13}D_{23})^t$ and let $\overline{\mathbf{d}}$ be the true element tensor vector

$$p(\mathbf{d}|\overline{\mathbf{d}}) = \frac{1}{(2\psi)^3|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{d}-\overline{\mathbf{d}})^t\Sigma^{-1}(\mathbf{d}-\overline{\mathbf{d}})\right]$$

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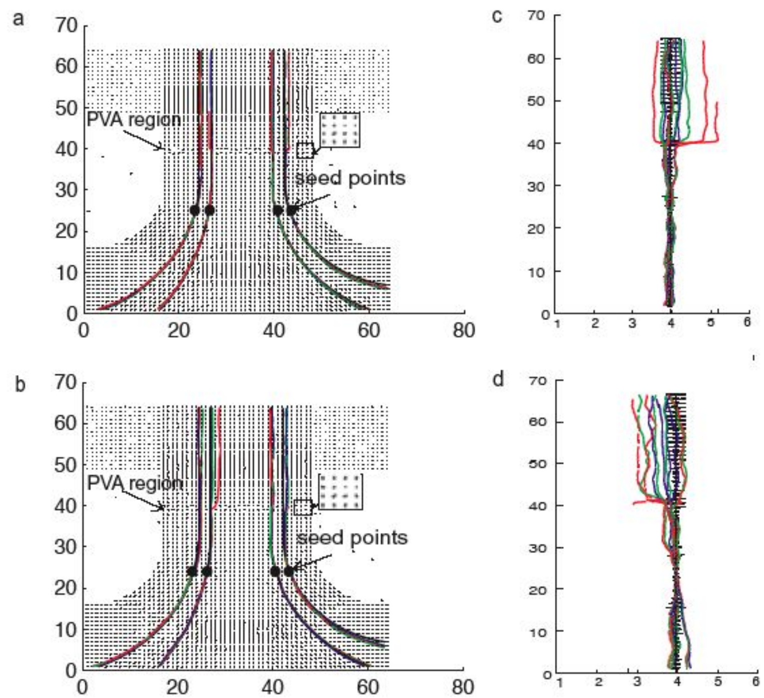
Maximize

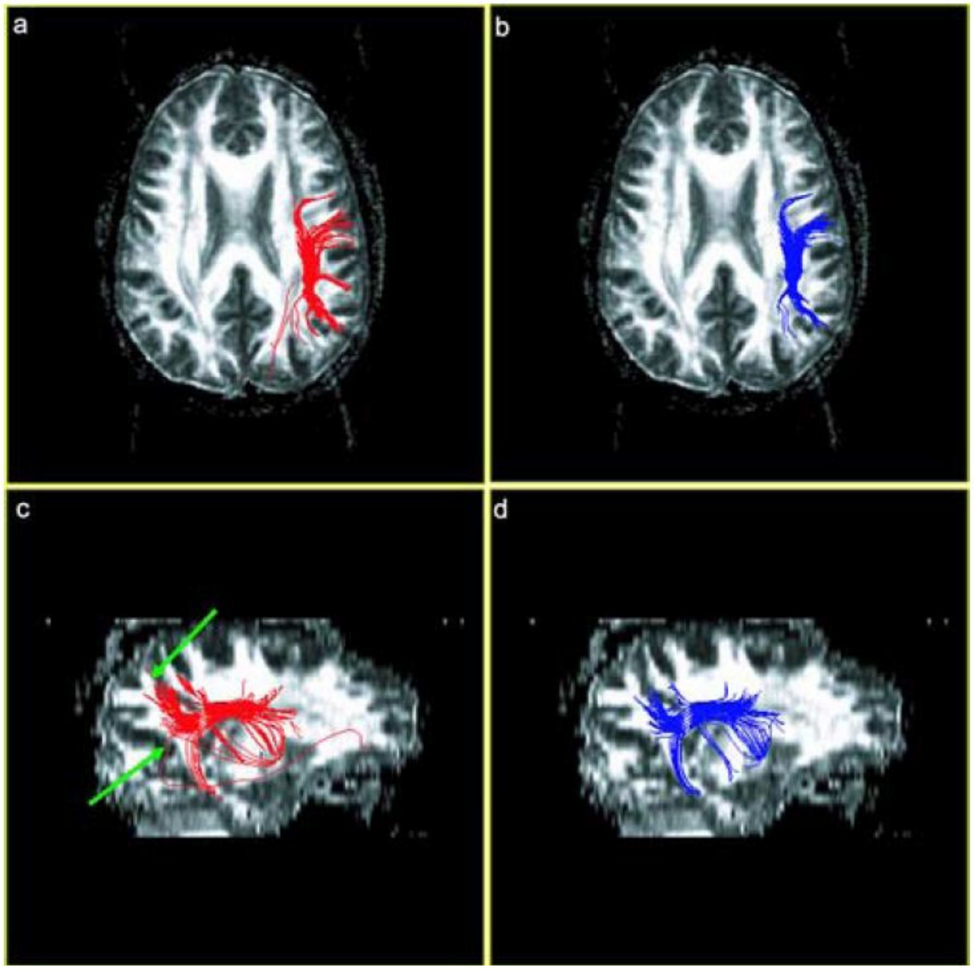
$$p(\overline{\mathbf{d}}|\mathbf{d}) = \frac{p(\mathbf{d}|\overline{\mathbf{d}})p(\overline{\mathbf{d}})}{p(\mathbf{d})}$$

Example 4



(a,b) axial views and (c,d) sagital views. SNR 30 (a,c) and 20 (b,d). Blue= Bayesian, Red= Euler, Green=TEND.

Example 5



Other methods and remaining challenges

Methods

- Deterministic, i.e. 1-point at most 1 path between two points, e.g. Aldroubi and Basser 1998; Mori et al. 1999; Basser et al. 2000; Poupon et al. 2000; Gossl et al. 2002; Parker et al. 2002; Tench et al. 2002; Zhukov et al. 2002, Stampfli et al. 2005, Lazar et al. 2003; ....,

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- Probabilistic, e.g. Hagmann et al. 2003, Koch et al. 2002, Parker et al. 2003,...

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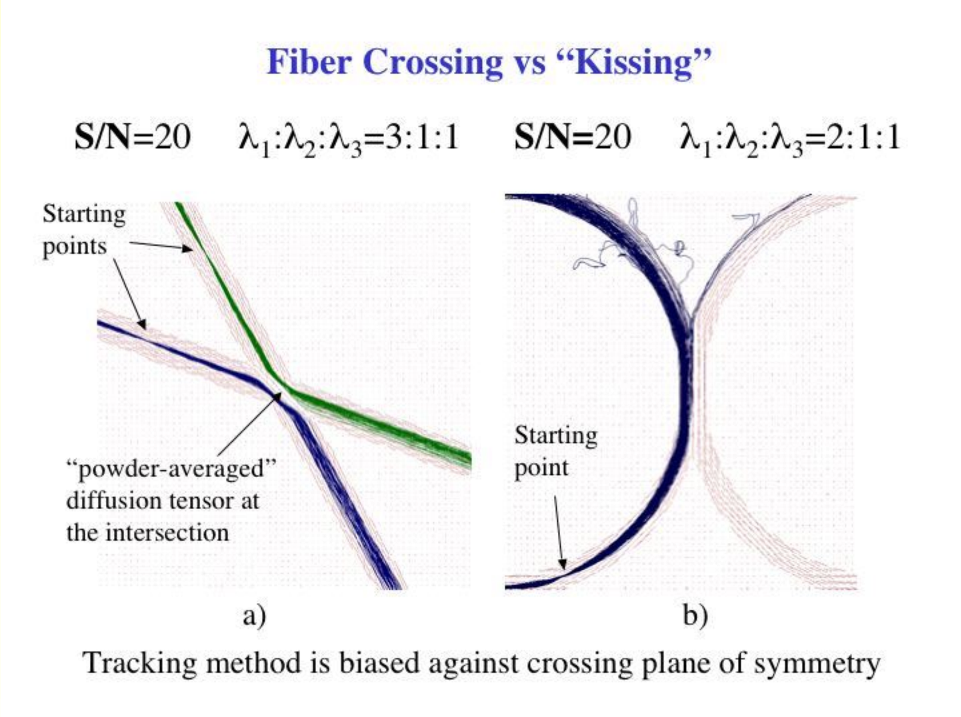
- Methods to deal with PVA, brakes in fibers, crossing or kissing of fibers, and other singular features.



END