Large System Analysis of Wireless Communications Systems:

Which way to infinity?

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Asymptotic Analysis: Classical Examples

- Coding theorems (Shannon)
  - Block length $\longrightarrow \infty$
- Discrete adaptive estimation
  - Continuous-time limit
- Stochastic systems
  - Fluid limits, heavy traffic limits

Wireless System Parameters

- Degrees of freedom
  - Bandwidth
  - Dimension of signal subspace
  - Antennas
  - Signatures in space/time/frequency
- Users
- Training interval (adaptive estimation)
- Power
- Mathematical tool: random matrix theory

Outline

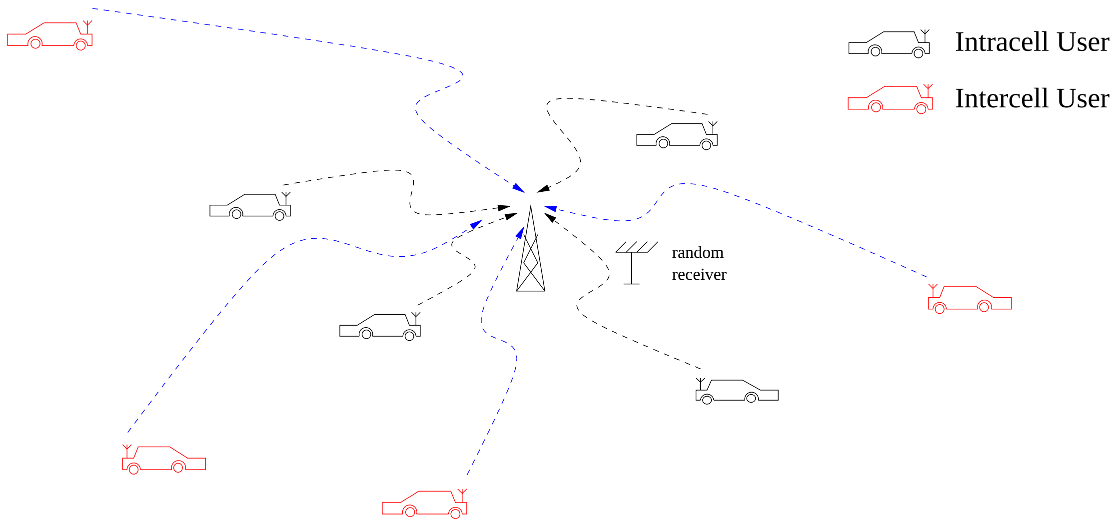
- Performance of CDMA and multi-antenna systems
  - Joint work with Matthew Peacock, Iain Collings

    University of Sydney

- Transient behavior of adaptive Least Squares estimators
  - Joint work with Weimin Xiao, Matthew Peacock, Iain Collings, Yakun Sun

- Signature optimization with limited feedback
  - Joint work with Wiroonsak Santipach

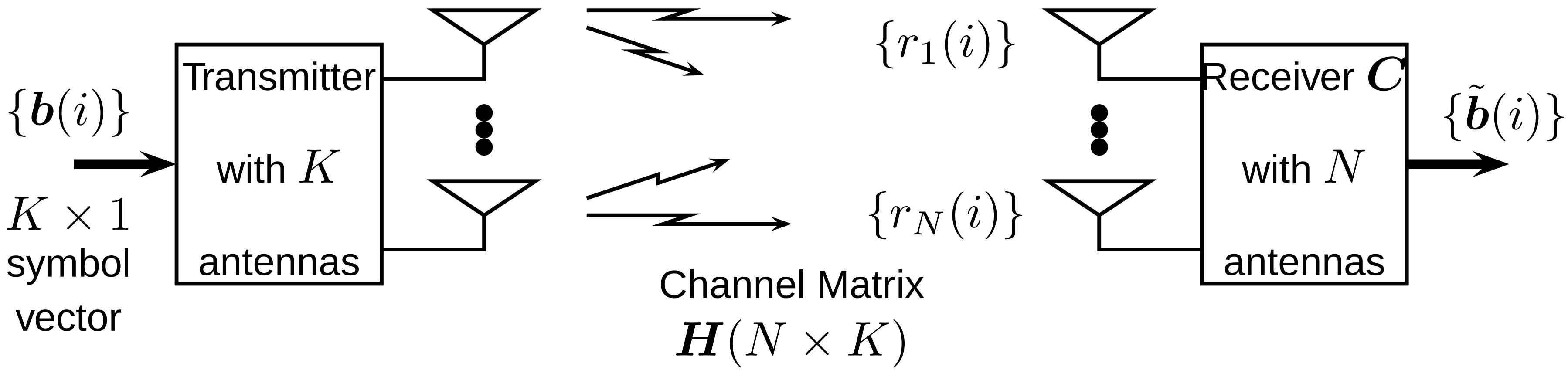
Code-Division Multiple Access (CDMA)



Received $N \times 1$ vector: $\mathbf{r} = \sum_{k=1}^{K} \sqrt{P_k} b_k \mathbf{s}_k + \mathbf{n} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n}$

- $\mathbf{s}_k$ is the $N \times 1$ random *i.i.d.* signature for user k, $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_K]$
- b_k is the symbol for user k, $\mathbf{b} = [b_1, \cdots, b_K]^T$
- P_k is the power for user k, $\mathbf{P} = \text{diag}[P_1, \cdots, P_K]$
- $\mathbf{n}$ is additive white Gaussian noise, $E[\mathbf{n}\mathbf{n}^\dagger] = \sigma_n^2 \mathbf{I}$

Multiple Antenna Model



$$\boldsymbol{r}(i) = \boldsymbol{H}\boldsymbol{b}(i) + \boldsymbol{n}(i), \; E[\boldsymbol{n}(i)\boldsymbol{n}^{\dagger}(i)] = \sigma^2 \boldsymbol{I}$$

- $\boldsymbol{H}$ is random with *i.i.d.* elements (flat fading assumption)
- Multi-Input/Multi-Output (MIMO) channel

Random Matrix Channel Models

- Ideal, synchronous CDMA: $\mathbf{r} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n}$

  $\mathbf{S} \doteq N \times K$ *i.i.d.* (or isometric) random signature matrix

  $\mathbf{P} \doteq K \times K$ diagonal power matrix

  $\mathbf{b} \doteq K \times 1$ symbol vector

  $\mathbf{n} \doteq N \times 1$ noise vector (cov = $\sigma^2\mathbf{I}$)

- Synchronous forward-link CDMA with multipath: $\mathbf{r} = \mathbf{H}\mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n}$

  $\mathbf{H} \doteq N \times N$ diagonal channel matrix

- Multi-user/multi-signature CDMA: $\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{n}$

  $\mathbf{S}_k \doteq N \times J_k$ random signature matrix (*i.i.d.* elements) for user k

Preface

- For an extensive overview of large system analysis, see

  A. M. Tulino and S. Verdú, "Large Matrix Theory and its Application to Wireless Systems", *Foundations and Trends in Communications and Information Theory*, Vol. 1, No. 1, 2004.

- This talk: direct derivations of selected large system results.

Linear Minimum Mean Squared Error (MMSE) Receiver

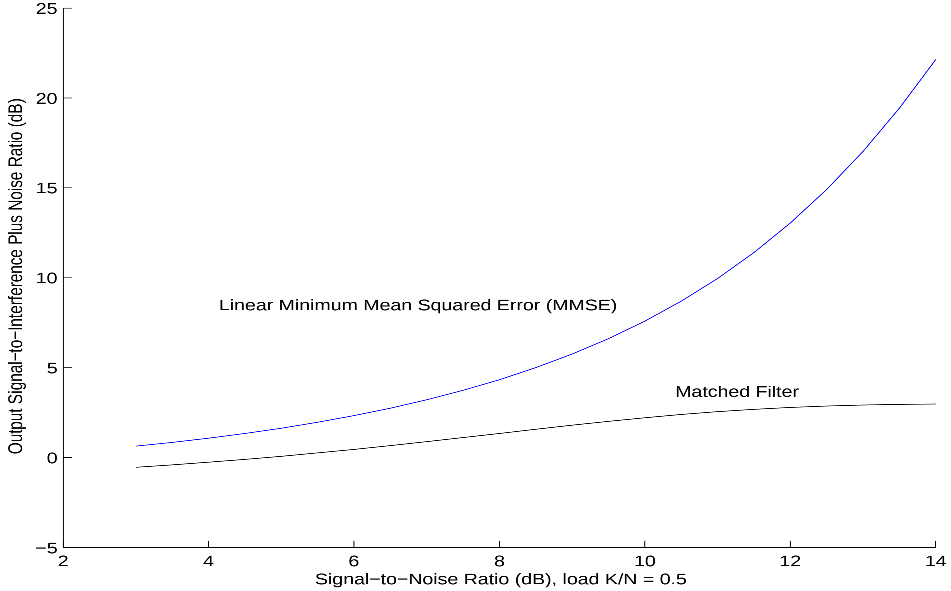
$$\mathbf{c}_k = \arg\min \mathsf{E}\left[|b_k - \mathbf{c}_k^\dagger \mathbf{r}|^2\right] = \mathbf{R}^{-1}\mathbf{s}_k, \quad \text{where} \quad \mathbf{R} = \mathbf{S}\mathbf{P}\mathbf{S}^\dagger + \sigma^2\mathbf{I}$$

Output Signal-to-Interference Plus Noise Ratio (SINR), conditioned on $\mathbf{S}_k$:

$$\mathsf{SINR}_k = P_k \mathsf{E}_{\mathbf{s}_k}\{\mathbf{s}_k^\dagger \mathbf{R}_k^{-1}\mathbf{s}_k\} = P_k \frac{1}{N}\,\mathrm{trace}\{\mathbf{R}_k^{-1}\}$$

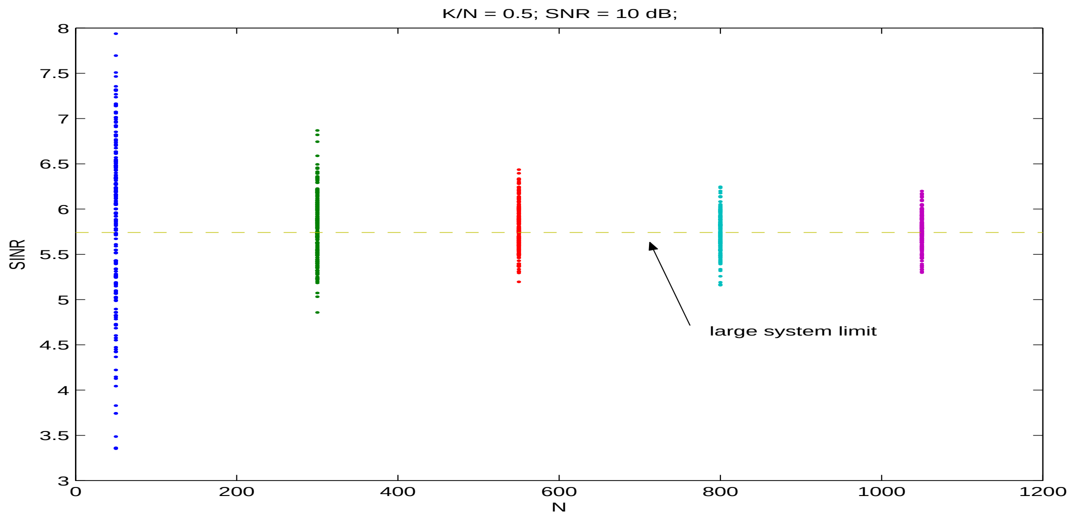
$$\mathbf{R}_k = \mathbf{S}_k\mathbf{P}\mathbf{S}_k^\dagger + \sigma^2\mathbf{I}, \qquad \mathbf{S}_k = [\mathbf{s}_1 \cdots \mathbf{s}_{k-1}\mathbf{s}_{k+1} \cdots \mathbf{s}_K]$$

Problem: Evaluate $\mathsf{E}_{\mathbf{S}_k}[\mathsf{SINR}_k]$.



Large System SINR (Tse-Hanly)

Users $K \to \infty$; Degrees of freedom $N \to \infty$; $\beta = K/N$ fixed



$$\lim_{(K,N)\to\infty} \mathsf{SINR}_k = \mathsf{SINR}_k^\infty = P_k \gamma^\infty, \quad \text{and} \quad \gamma^\infty = \frac{1}{\sigma^2 + \beta \int \frac{P\,dF(P)}{1+\gamma^\infty P}}$$

where $\{P_1, \cdots, P_K\} \xrightarrow{\mathcal{D}} F(\cdot)$

The Matrix Inversion Lemma (Theme)

- The received covariance matrix

$$\mathbf{R} = \mathsf{E}\left[\mathbf{r}\mathbf{r}^\dagger\right] = \mathbf{S}\mathbf{P}\mathbf{S}^\dagger + \sigma^2\mathbf{I}$$

- The received interference-plus-noise covariance matrix

$$\mathbf{R}_k = \mathbf{S}_k\mathbf{P}\mathbf{S}_k^\dagger + \sigma^2\mathbf{I}$$

where $\mathbf{S}_k = \left[\mathbf{s}_1 \cdots \mathbf{s}_{k-1}\mathbf{s}_{k+1} \cdots \mathbf{s}_K\right]$

- MIL:

$$\mathbf{R}^{-1} = \mathbf{R}_k^{-1} - \frac{\mathbf{R}_k^{-1}\mathbf{s}_k\mathbf{s}_k^\dagger\mathbf{R}_k^{-1}}{1 + \mathbf{s}_k^\dagger\mathbf{R}_k^{-1}\mathbf{s}_k}$$

Derivation of Tse-Hanly Formula

$$
\begin{aligned}
1 &= \lim_{N\to\infty} \frac{1}{N} \operatorname{trace}(\mathbf{R}^{-1}\mathbf{R}) \\
&= \lim_{N\to\infty} \frac{1}{N} \operatorname{trace}\left[\mathbf{R}^{-1}(\sigma^2\mathbf{I} + \mathbf{SPS}^\dagger)\right] \\
&= \sigma^2 \lim_{N\to\infty} \frac{1}{N} \operatorname{trace}\mathbf{R}^{-1} + \lim_{N\to\infty} \frac{1}{N} \sum_{k=2}^{K} \frac{P_k(\mathbf{s}_k^\dagger \mathbf{R}_k^{-1}\mathbf{s}_k)}{1 + P_k(\mathbf{s}_k^\dagger \mathbf{R}_k^{-1}\mathbf{s}_k)}
\end{aligned}
$$

Now,

$$
\gamma^\infty = \lim_{N\to\infty} (\mathbf{s}_k^\dagger \mathbf{R}_k^{-1}\mathbf{s}_k) = \lim \frac{1}{N} \operatorname{trace}\{\mathbf{R}_k^{-1}\} = \lim \frac{1}{N} \operatorname{trace}\{\mathbf{R}^{-1}\}
$$

Hence

$$
1 = \sigma^2\gamma^\infty + \beta \int \frac{P\gamma^\infty}{1 + P\gamma^\infty} dF(P)
$$

Forward-Link CDMA

(Debbah et al; Li-Tulino-Verdú; Peacock et al)

$$\mathbf{r} = \mathbf{HSP}^{1/2}\mathbf{b} + \mathbf{n}$$

- Large system SINR for linear MMSE receiver is

$$\text{SINR}_k^\infty = P_k \rho^\infty \quad \text{where} \quad \rho^\infty = \lim_{(N,K)\to\infty} \frac{1}{N} \text{trace}(\mathbf{H}^\dagger \mathbf{R}^{-1} \mathbf{H})$$

$$\text{and} \quad \mathbf{R} = \mathbf{HSPS}^\dagger \mathbf{H}^\dagger + \sigma^2 \mathbf{I}$$

- Same type of calculation gives:

$$1 = \sigma^2 \gamma^\infty + \beta \rho^\infty \int \frac{P}{1 + P\rho^\infty} dF(P)$$

where

$$\gamma^\infty = \lim_{(K,N)\to\infty} \frac{1}{N} \text{trace}(\mathbf{R}^{-1})$$

- How to compute γ^∞?

Covariance Matrix Expansions

- Before: used the matrix inversion lemma (MIL) to remove signature $\mathbf{s}_k$ from $\mathbf{R}^{-1}$.
- Now apply MIL to alternate expansion of $\mathbf{R}$.
  - Observe that $\mathbf{r}$ and $\mathbf{V}\mathbf{r}$ have the same SINR, where $\mathbf{V}$ is random unitary.
  - Replace $\mathbf{H}\mathbf{H}^\dagger$ by $\mathbf{V}\mathbf{D}\mathbf{V}^\dagger$, where $\mathbf{D}$ is diagonal, so that

$$\mathsf{SINR} = \frac{1}{N}\sum_{n=1}^{N} d_n \mathbf{v}_n^\dagger \mathbf{R}^{-1}\mathbf{v}_n$$

  - Remove $\mathbf{v}_k$ from $\mathbf{R}$ (gives three terms), apply MIL three times to corresponding expansion of $\mathbf{R}^{-1}$.
  - Evaluating limit of terms in identity $\mathrm{trace}(\mathbf{R}\mathbf{R}^{-1}) = 1$ gives

$$\mathsf{SINR}^\infty = P_k \mathsf{E}\left[\frac{H}{\sigma^2 + H\tau^\infty}\right] \qquad \tau^\infty = \beta \mathsf{E}\left[\frac{P}{1 + \frac{P}{P_k}\mathsf{SINR}^\infty}\right]$$

where P (power) and H (channel gain) are scalar random variables.

Multi-User/Multi-Signature CDMA

$$\mathbf{r} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \mathbf{n}$$

User k has J_k signatures ($\mathbf{S}_k$ is $N \times J_k$), and each signature has

$$\mathsf{SINR}_k = \frac{1}{N} \operatorname{trace}\left(\mathbf{H}_k^{\dagger} \mathbf{R}^{-1} \mathbf{H}_k \right)$$

where

$$\mathbf{R} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{S}_k \mathbf{S}_k^{\dagger} \mathbf{H}_k^{\dagger} + \sigma^2 \mathbf{I}_N$$

Large system limit (fixed, finite K):

Signatures $J_k \to \infty$; Degrees of freedom $N \to \infty$;

$\beta_k = J_k / N$ constant for $k = 1, \cdots, K$

Free Probability Approach

$$\text{SINR}_k^{\infty} = \lim_{(J_1,\cdots,J_K,N)\to\infty} \text{trace}\, \frac{1}{N}\left(\mathbf{R}^{-1}\mathbf{H}_k\mathbf{H}_k^{\dagger}\right)$$
$$= \lim \frac{1}{N}\sum_{i=1}^{N}\lambda_i(\mathbf{R}^{-1}\mathbf{H}_k\mathbf{H}_k^{\dagger})$$

where $\{\lambda_i(\mathbf{A}_N)\}$ is the set of eigenvalues of $\mathbf{A}_N$, which converges to a deterministic asymptotic eigenvalue distribution (aed) as $(J_1, \cdots, J_K, N) \to \infty$.

For matrices $\mathbf{A}_N$ and $\mathbf{B}_N$, which are asymptotically free[a]

- The aed of $\mathbf{A}_N + \mathbf{B}_N$ can be computed via the R-transform:
  $R(\mathbf{A}_N + \mathbf{B}_N) = R(\mathbf{A}_N) + R(\mathbf{B}_N)$
- The aed of $\mathbf{A}_N\mathbf{B}_N$ can be computed via the S-transform:
  $S(\mathbf{A}_N\mathbf{B}_N) = S(\mathbf{A}_N)S(\mathbf{B}_N)$

[a] Example: unitarily invariant random matrices are asymptotically free (i.e., $\mathbf{A}$ and $\mathbf{U}\mathbf{A}$ have the same distribution, where $\mathbf{U}$ is unitary).

Free Approximation

- Recall that

$$\mathbf{R} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{S}_k \mathbf{S}_k^{\dagger} \mathbf{H}_k^{\dagger} + \sigma^2 \mathbf{I}_N$$

- The matrices $\mathbf{M}_k = \mathbf{H}_k^{\dagger} \mathbf{S}_k \mathbf{S}_k^{\dagger} \mathbf{H}_k^{\dagger}$ are not asymptotically free!
- Replace each $\mathbf{M}_k$ by unitarily invariant matrix $\mathbf{U}\mathbf{M}_k$ (same eigenvalue distribution). Can then compute asymptotic eigenvalue distribution of $\mathbf{R}^{-1}\mathbf{H}_k\mathbf{H}_k^{\dagger}$ via R-and S-transforms.
  - Accurate approximation for loads $\beta_k < 1$.

Exact SINR via Covariance Matrix Expansions

- As before, expand and simplify the identity $\frac{1}{N}\,\mathrm{trace}(\mathbf{R}^{-1}\mathbf{R}) = 1$, applying the matrix inversion lemma to:
  - Remove a signature from each user.
  - Remove a column of $\mathbf{V}$ for each user.
- Result:

$$\mathsf{SINR}_k^\infty = \mathsf{E}\left[\frac{H_k}{\sigma^2 + \sum_j \frac{\beta_j H_j}{1+\mathsf{SINR}_j^\infty}}\right] \qquad \gamma^\infty = \frac{1}{\sigma^2}\left(1 - \sum_{k=1}^{K} \frac{\beta_k \mathsf{SINR}_k^\infty}{1+\mathsf{SINR}_k^\infty}\right)$$

where H_k is a scalar r.v. distributed according to the channel gains for user k.

- Matrix expansion technique also applies to free matrices, and can be used to derive aed's of sums and products (i.e., R- and S-transforms).
- Exact SINR can also be derived by applying results due to Girko.

Spectral Efficiency

$$C(\sigma^2) = \frac{1}{N} \log |\mathbf{R}| \longrightarrow - \int_{-\infty}^{-\sigma^2} \left(G(x) + \frac{1}{x} \right) dx$$

as $(J_1, \cdots, J_K, N) \to \infty$ where

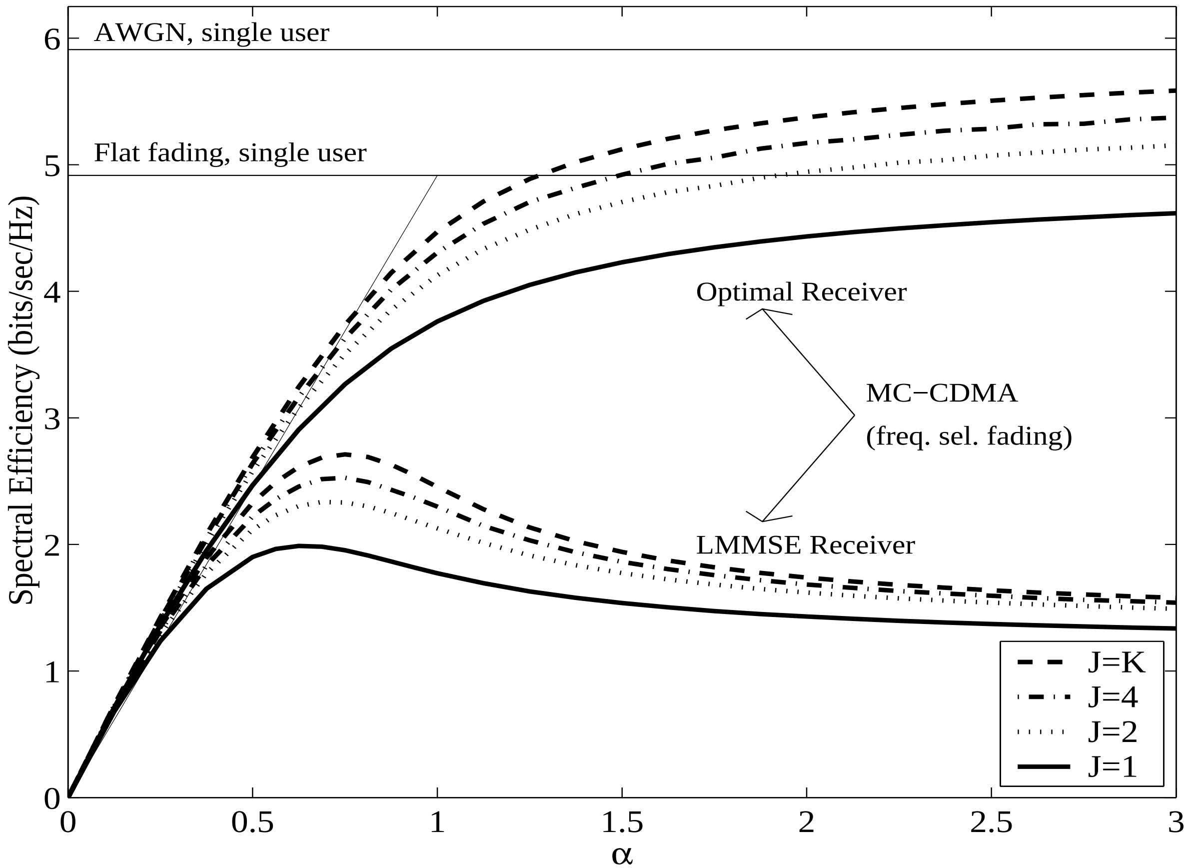
$$G(x) = \lim_{N \to \infty} \frac{1}{N} \operatorname{trace}(\mathbf{R}^{-1})$$

with σ^2 in $\mathbf{R}$ replaced by $-x$.

(Stieltjes transform of the asymptotic eigenvalue distribution of $\mathbf{R}$.

Note that $G(-\sigma^2)$ is the SINR for the linear MMSE detector.)

Asymptotic Sum Spectral Efficiency

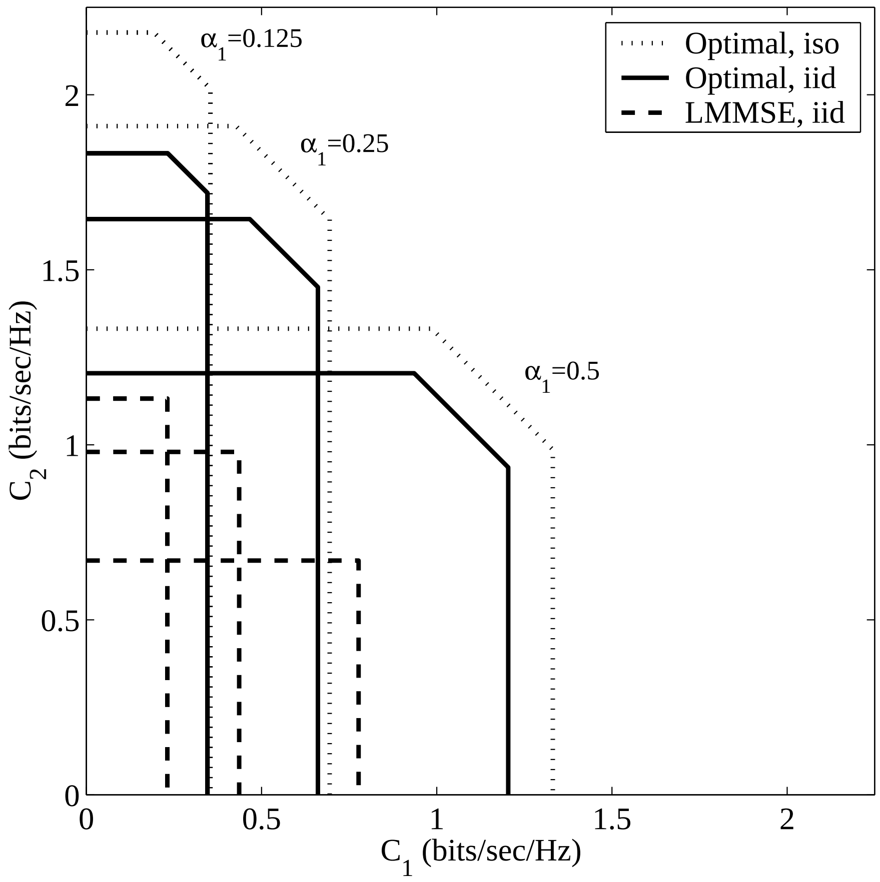


Spectral efficiency increases with K

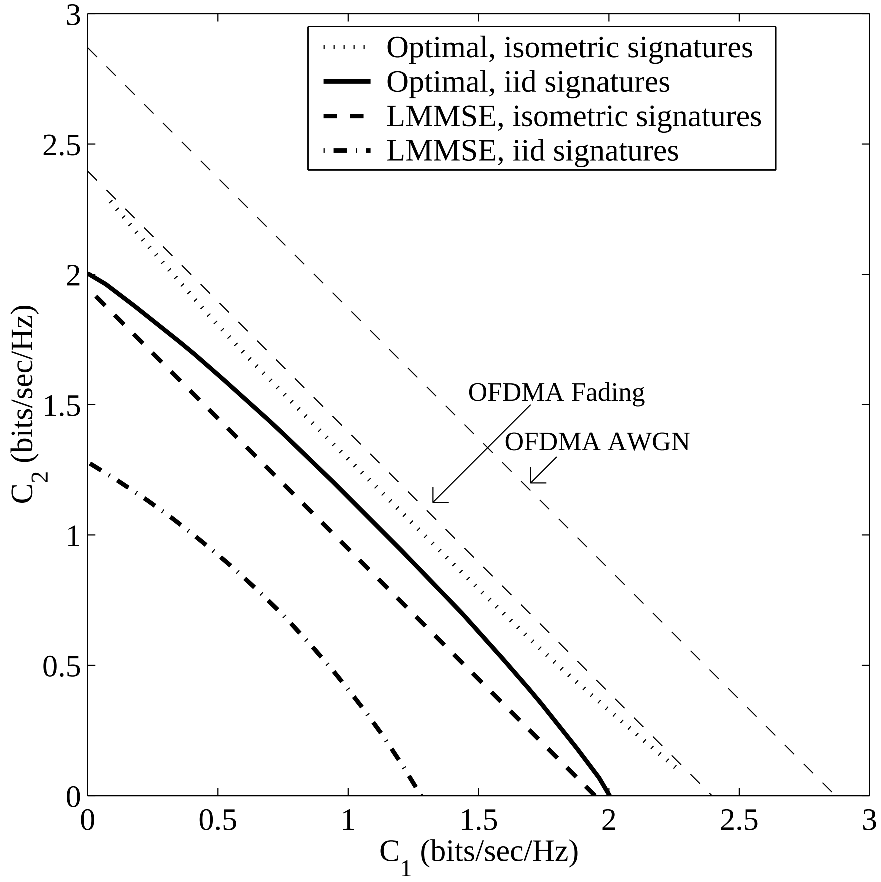
Isometric vs. i.i.d. Signatures

- i.i.d. Signatures: Elements of signature matrices $\mathbf{S}_k$, $k = 1, \cdots, K$, are *i.i.d.* random variables.
- Isometric Signatures: $\mathbf{S}_k$, $k = 1, \cdots, K$, are independent, *unitary* matrices ($\mathbf{S}_k^\dagger \mathbf{S}_k = \mathbf{I}$)

Asymptotic Spectral Efficiency Region: Two Equal-Power Users



(a) Three regions with $\beta_1 + \beta_2 = 1$.



(b) Union of all regions with $\beta_1 + \beta_2 = 1$.

Least Squares Estimation

- System model:

$$\mathbf{r}(i) = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b}(i) + \mathbf{n}(i)$$

  time index $i = 1, 2, \cdots$

- Linear MMSE receiver: $\mathbf{c}_k = \mathbf{R}^{-1}\mathbf{s}_k$
  - Problem: may not know $\mathbf{R} = \mathbf{S}\mathbf{P}\mathbf{S}^\dagger + \sigma_n^2\mathbf{I}$

- Least Squares estimates (with training sequence):

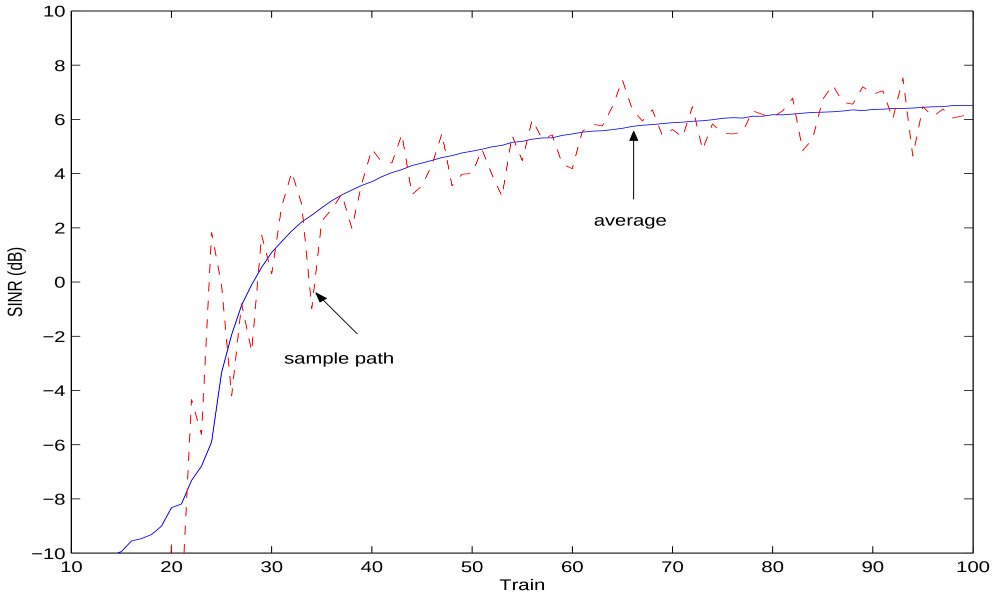
$$\hat{\mathbf{c}}_k(i) = \arg\min_{\mathbf{c}_k} \sum_{j=1}^{i} |b_k(j) - \mathbf{c}_k^\dagger \mathbf{r}(j)|^2 = \hat{\mathbf{R}}^{-1}(i)\hat{\mathbf{s}}_k(i)$$

  where $\hat{\mathbf{R}}(i) = \sum_{j=1}^{i} \mathbf{r}(j)\mathbf{r}^\dagger(j)$ and $\hat{\mathbf{s}}_k(i) = \sum_{j=1}^{i} b_k^*(j)\mathbf{r}(j)$

- Output SINR:

$$\hat{\mathsf{SINR}}_k(i) = \frac{|\hat{\mathbf{c}}_k^\dagger(i)\mathbf{s}_k|^2}{\hat{\mathbf{c}}_k^\dagger \mathbf{R}_k(i)\hat{\mathbf{c}}_k(i)}$$

SINR Transient Behavior



- For random $\mathbf{S}$, $\mathbf{b}$, and $\mathbf{n}$, compute averaged $\widehat{\text{SINR}}_k(i) = \dfrac{|\hat{\mathbf{c}}_k^\dagger(i)\mathbf{s}_1|^2}{\hat{\mathbf{c}}_k^\dagger(i)\mathbf{R}_k(i)\hat{\mathbf{c}}_k(i)}$
- Classical problem; very difficult
- Prior work:
  - Reed, Mallet, Brennan (1974): For Gaussian $\mathbf{S}$, $\widehat{\text{SINR}}_k(i)/\text{SINR}_k \approx 1/2$ for $i = 2N$.
  - Asymptotic convergence (large i) and approximate analyses

Large System Transient Behavior

Let $(K, N, i) \to \infty$ with fixed $\beta = K/N, \quad \bar{T} = i/N$

- If $\mathbf{S}$ has *i.i.d.* elements, then the SINR for each k converges to an asymptotic limit $\hat{\text{SINR}}^{\infty}(\beta, \bar{T})$. Furthermore, $\hat{\text{SINR}}^{\infty}(\beta, \bar{T}) \to \text{SINR}^{\infty}(\beta)$ as $\bar{T} \to \infty$.
- Can show

$$\hat{\text{SINR}}^{\infty}(\beta, \bar{T}) = \frac{\text{SINR}^{\infty}(\beta)}{1 + \frac{1}{\bar{T}-1}\left(1 + \frac{1}{\text{SINR}^{\infty}(\beta)}\right)}$$

- $\hat{\text{SINR}}^{\infty}(\beta, \bar{T} = 1) = 0$, $\hat{\text{SINR}}^{\infty}(\beta, \bar{T} = 2) \approx \text{SINR}^{\infty}(\beta)/2$
- Independent of particular distribution of $\mathbf{S}$.
- Accurate for moderate values of N (e.g., ≥ 16).

Derivation Outline

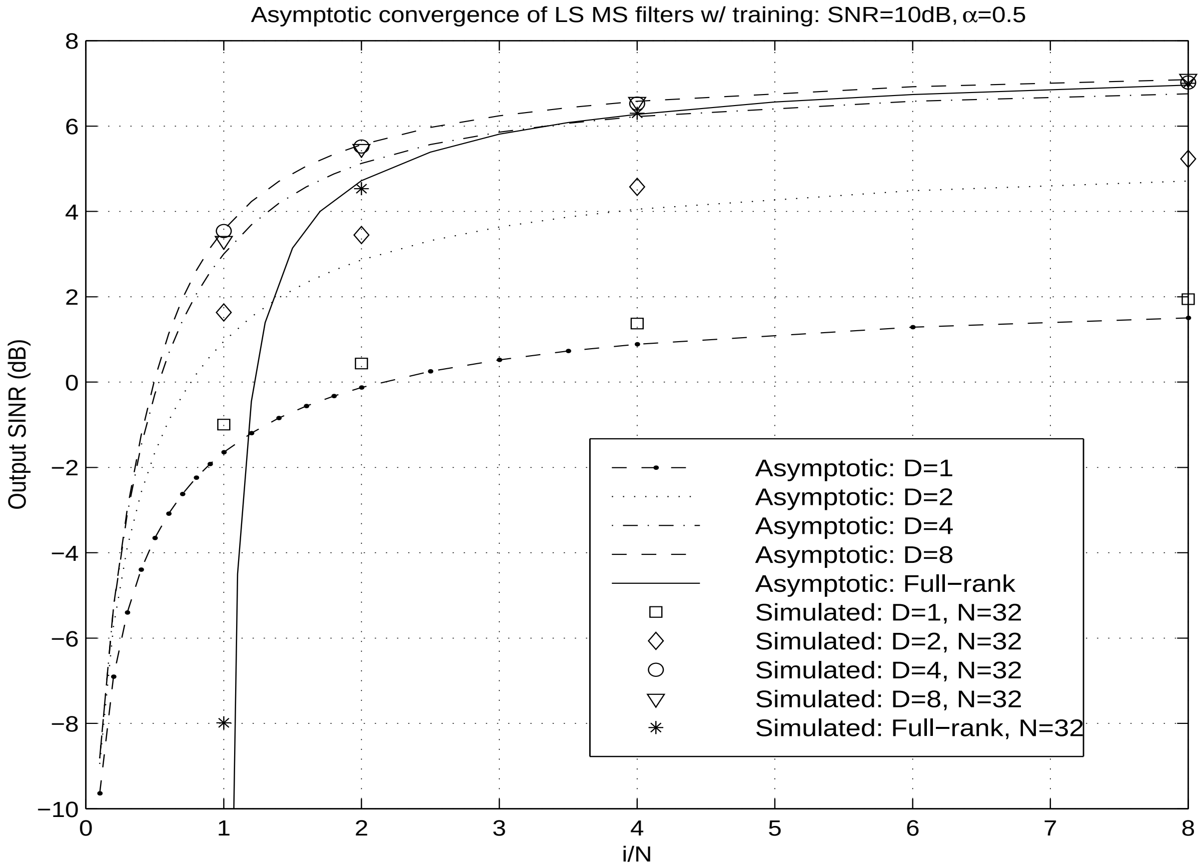
- Problem reduces to computing $\lim \frac{1}{N} \mathrm{trace}[\hat{\mathbf{R}}(i)]$, where the sample covariance matrix (ignoring noise),

$$
\begin{aligned}
\hat{\mathbf{R}}(i) &= \frac{1}{i}\sum_{j=1}^{i} \mathbf{r}(j)\mathbf{r}^{\dagger}(j) = \frac{1}{i}\mathbf{S}\mathbf{B}(i)\mathbf{P}\mathbf{B}^{\dagger}(i)\mathbf{S}^{\dagger} \\
&= \mathbf{S}\mathbf{V}_B(i)\Lambda_B(i)\mathbf{V}_B^{\dagger}(i)\mathbf{S}^{\dagger} \\
&= \mathbf{V}_S(i)\Lambda_B(i)\mathbf{V}_S^{\dagger}
\end{aligned}
$$

  where $\mathbf{B}(i) = [\mathbf{b}(1)\cdots\mathbf{b}(i)]$.

- For Gaussian $\mathbf{S}$, $\mathbf{V}_S(i)$ is *i.i.d*, and the diagonal elements of $\Lambda_B(i)$ converge to the eigenvalue distribution of $\mathbf{B}\mathbf{P}\mathbf{B}^{\dagger}$.
- Can apply large matrix results from the literature (Silverstein), or evaluate directly using covariance matrix expansions.
- Can generalize to include additive noise.

Large System Transient Behavior



$\beta = 1/2$, SNR=10 dB

Extensions

- Data windowing:

$$\hat{\mathbf{c}}_k(i) = \arg\min_{\mathbf{c}_k} \sum_{j=1}^{i} w_j |b_k(i) - \mathbf{c}_k^\dagger \mathbf{r}(i)|^2$$

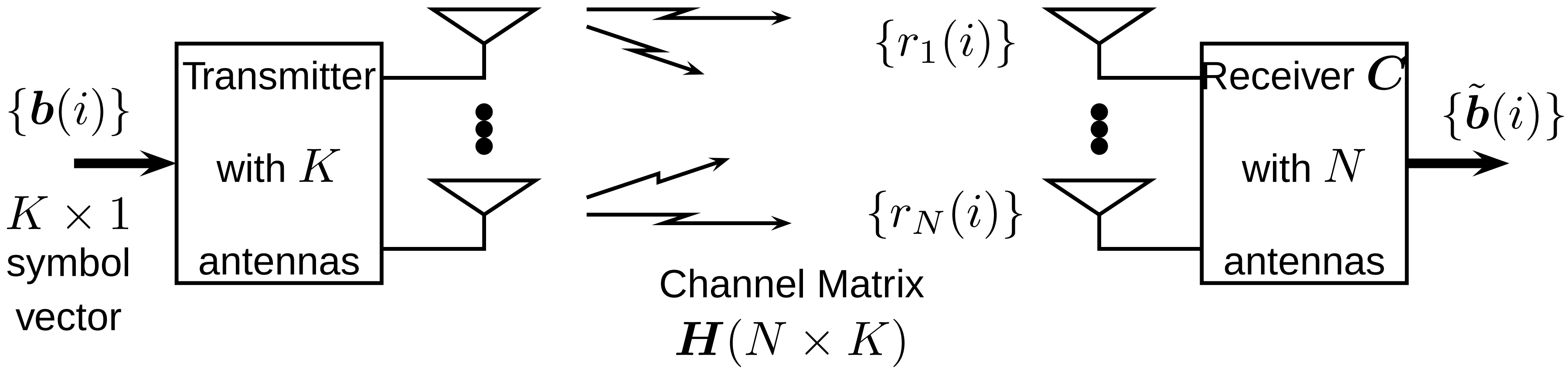
where the data windowing sequence $\{w_1, \cdots, w_i\} \xrightarrow{\mathcal{D}} F_w(\cdot)$ as $i \to \infty$

- Diagonal loading: $\hat{\mathbf{R}}(i) = \sum_{j=1}^{i} \mathbf{r}(j)\mathbf{r}^\dagger(j) + \delta\mathbf{I}$ where $\delta > 0$ is the diagonal loading factor, and prevents ill-conditioning.

- $\mathbf{r} = \mathbf{C}\mathbf{b} + \mathbf{n}$ for any $\mathbf{C}$ with a deterministic, well-defined asymptotic eigenvalue distribution (includes equalization).
  - Large system transient behavior depends only on the asymptotic SINR (as $\bar{T} \to \infty$).

- Reduced-rank least squares filtering
  - Constrain filter to lie in lower-dimensional subspace.
  - Can reduce complexity, improve convergence.

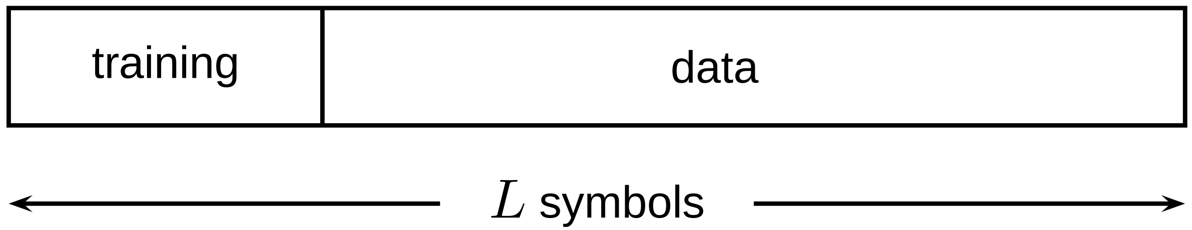
Training for MIMO Channels



$$\boldsymbol{r}(i) = \boldsymbol{H}\boldsymbol{b}(i) + \boldsymbol{n}(i), \;\; E[\boldsymbol{n}(i)\boldsymbol{n}^\dagger(i)] = \sigma^2 \boldsymbol{I}$$

Estimate of kth symbol: $\tilde{b}_k(i) = \boldsymbol{c}_k^\dagger \boldsymbol{r}(i)$

Packet:



How long should the training interval be?

Performance Metric: Capacity

- Total capacity (bits/transmit antenna):

$$C_{\text{MIMO}} \geq C = \frac{1}{K}\sum_{k=1}^{K}\left(1 - \frac{\bar{T}}{\bar{L}}\right)\log[1 + \hat{\text{SINR}}_k(\bar{T})]$$

where $\bar{T} = T$ training symbols/N, and $\bar{L} = L/N$ (normalized packet length)

- Problem: select $\bar{T}$ to maximize C

- Related work:
  - Hassibi and Hochwald (2001),
  - Vikalo, Hassibi, Hochwald, and Kailath (2004)
    - Optimal (maximum-likelihood) receiver; Training for channel estimation
  - Misra, Swami, and Tong (2003)
    - Cutoff rate, Gauss/Markov channel model

Optimal Training

- As $(K, N, T, L) \to \infty$ with fixed $\beta = K/N$, $\bar{T} = T/N$, and $\bar{L} = L/N$,

$$C \to \bar{C}(\beta, \bar{T}, \bar{L}) = \left(1 - \frac{\bar{T}}{\bar{L}}\right) \log \left(1 + \frac{\mathsf{SINR}^{\infty}(\beta)}{1 + \frac{1}{\bar{T}-1}[1 + \frac{1}{\mathsf{SINR}^{\infty}(\beta)}]}\right)$$

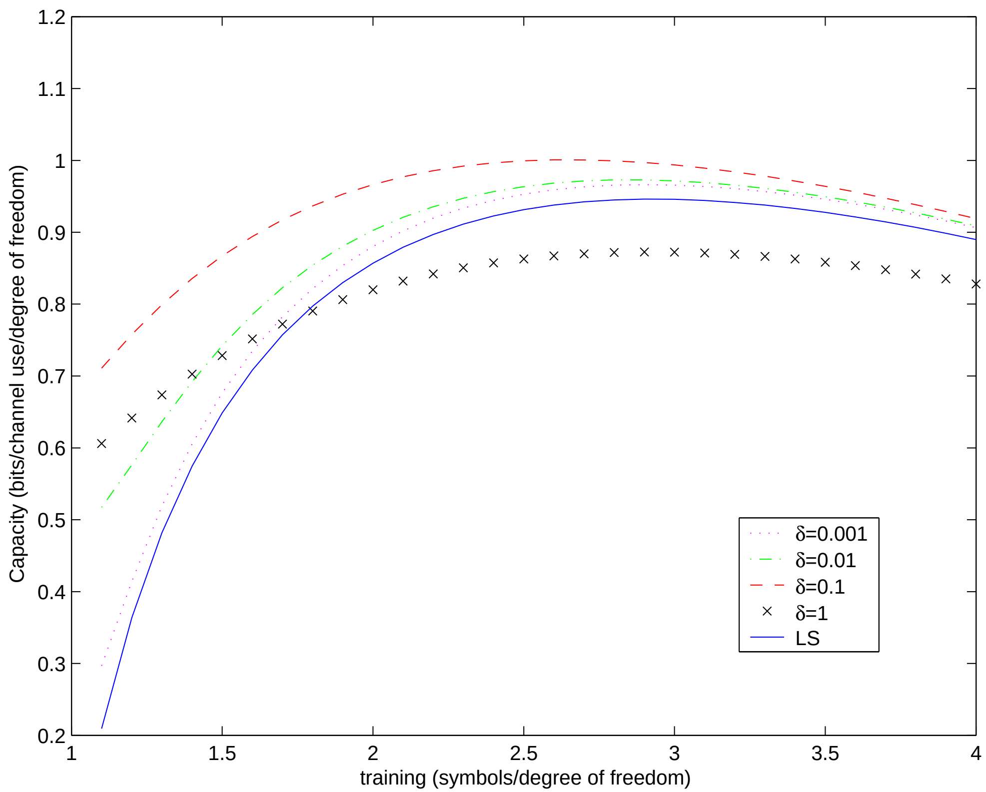
- Theorem: The optimal $\bar{T}$, which maximizes $\bar{C}$, satisfies

$$\lim_{\bar{L}\to\infty} \frac{\bar{T}}{\sqrt{\bar{L}}} = \sqrt{\frac{1}{\log[1 + \mathsf{SINR}^{\infty}(\beta)]}}$$

- As the SNR $\to \infty$ (i.e., $\sigma^2 \to 0$), $T/N \to 1$

- As the SNR $\to 0$, $T/N \to (L/N + 1)/2$

MIMO Capacity Versus Training Length



SNR=10 dB; $K/N = 1$; $L = 10N$, δ is the diagonal loading

Conclusions

- Can accurately characterize the performance of many finite-size random matrix channel models in terms of large system limits.
- Relevant large matrix results can be derived via elementary matrix manipulations (covariance matrix expansions), without direct application of results from free probability.
  - Allows derivation of new large system performance results.
- Results provide substantial insight.