

Overlay networks for wireless ad hoc networks

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Motivation

Wireless ad hoc networks have many important applications:

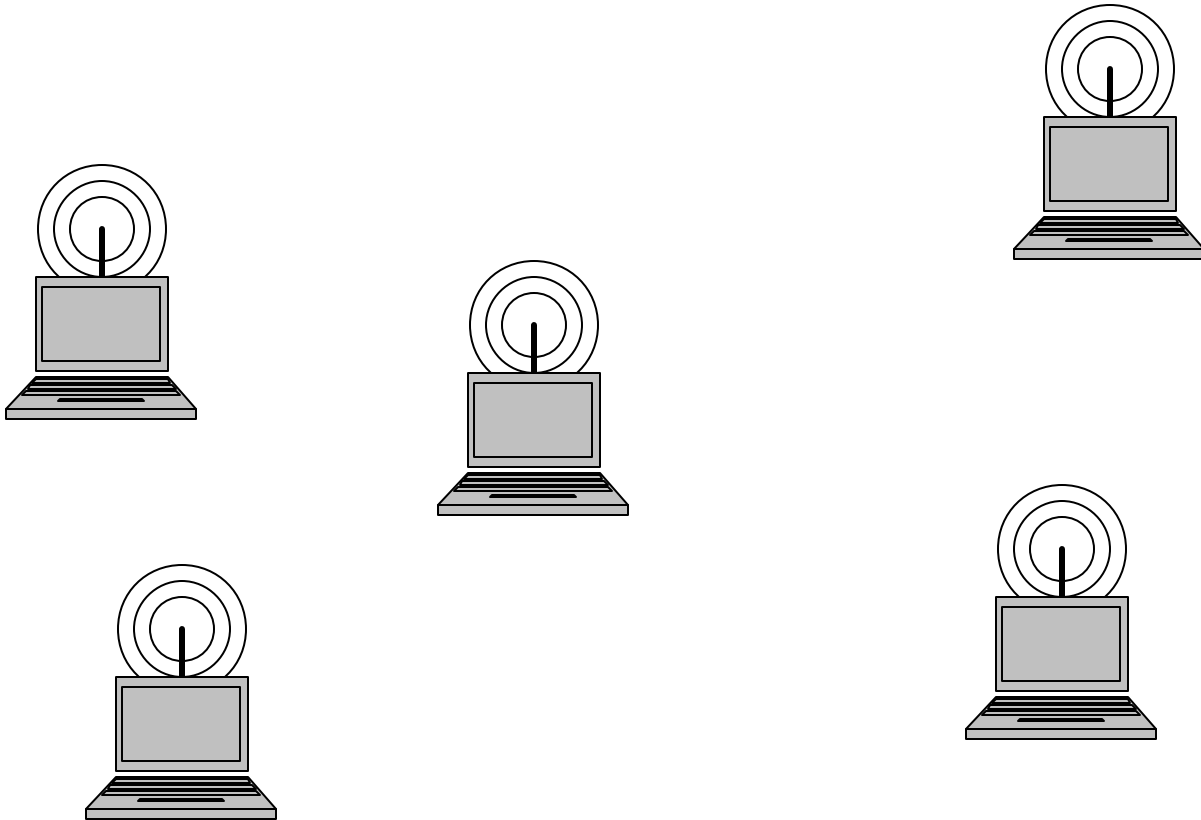
- search and rescue missions
- emergency situations
- military applications
- Signal processing and MAC: OK
- Scheduling: somewhat OK
- Routing: **HARD!!**

Outline

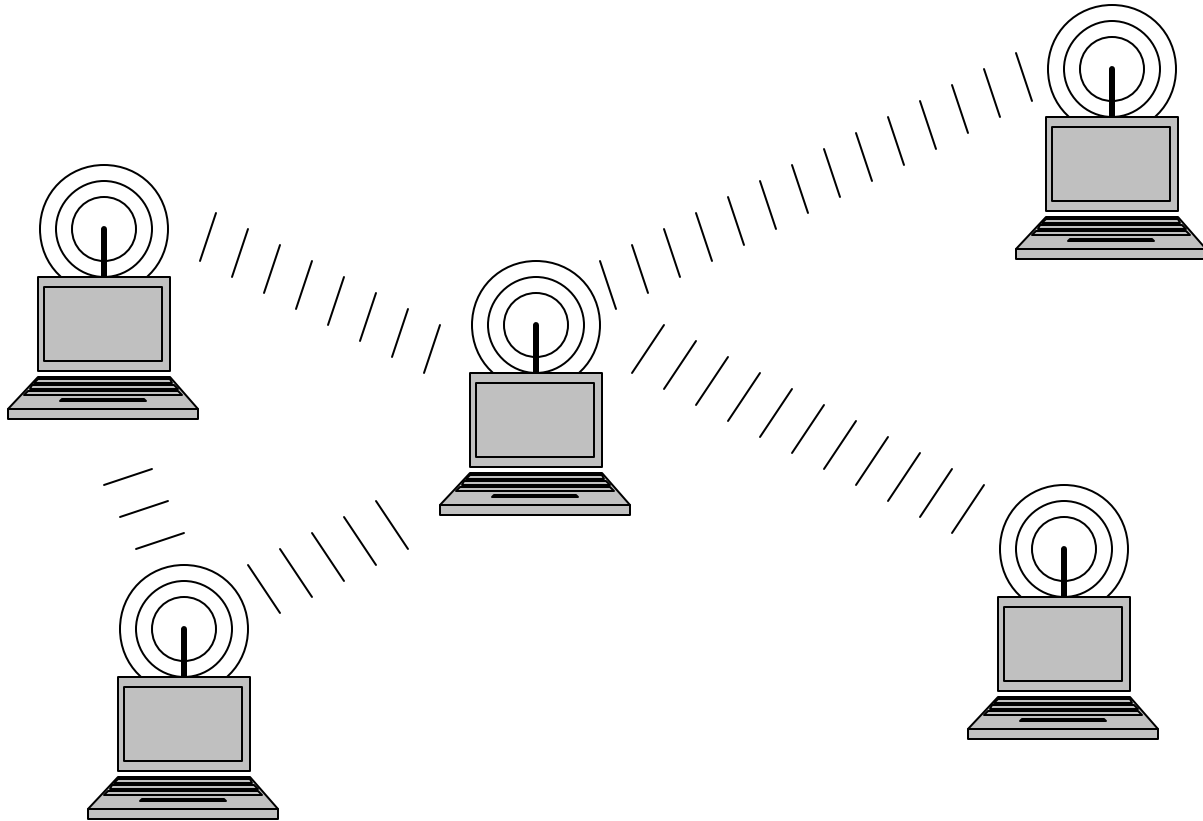
In this talk:

- Basic model and goals
- Basic approach (graph spanners)
- Basic results about spanners
- Examples
- Extensions and problems
- Realistic wireless models and protocols
- References

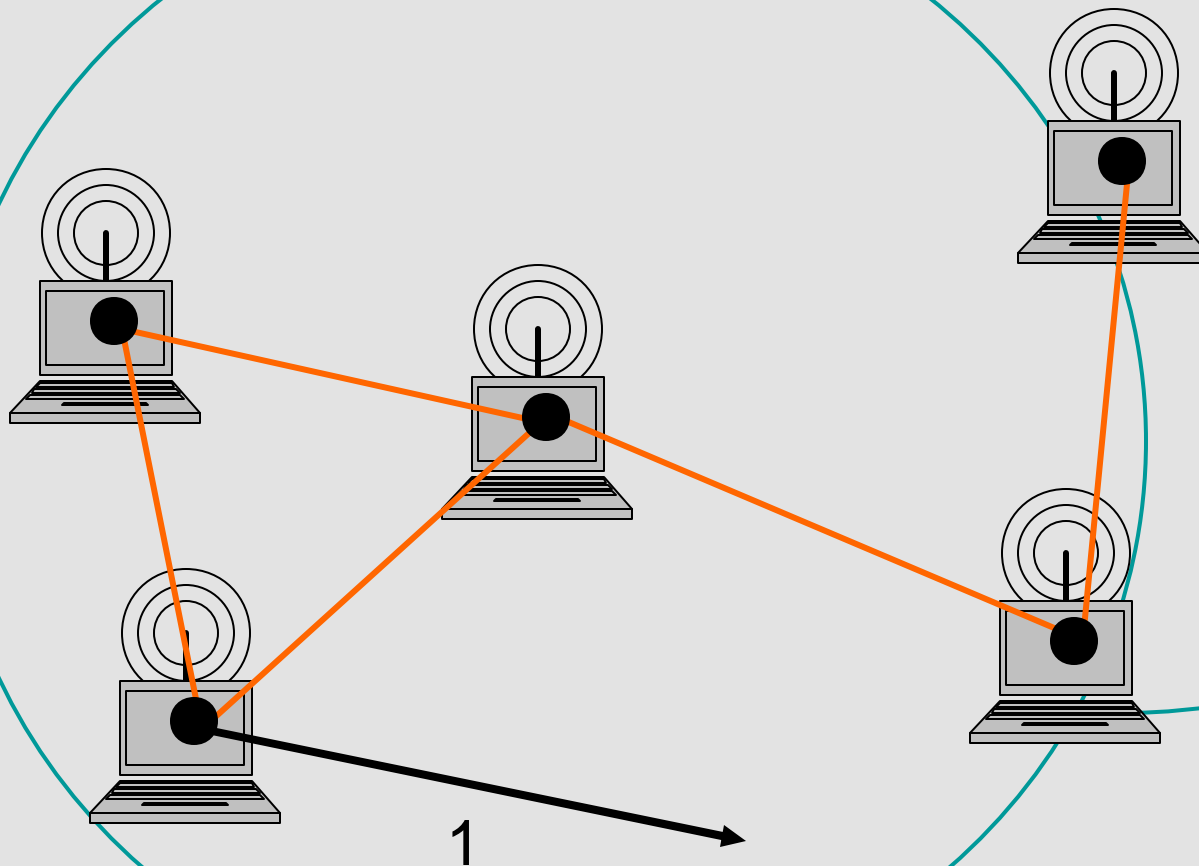
Wireless ad hoc network



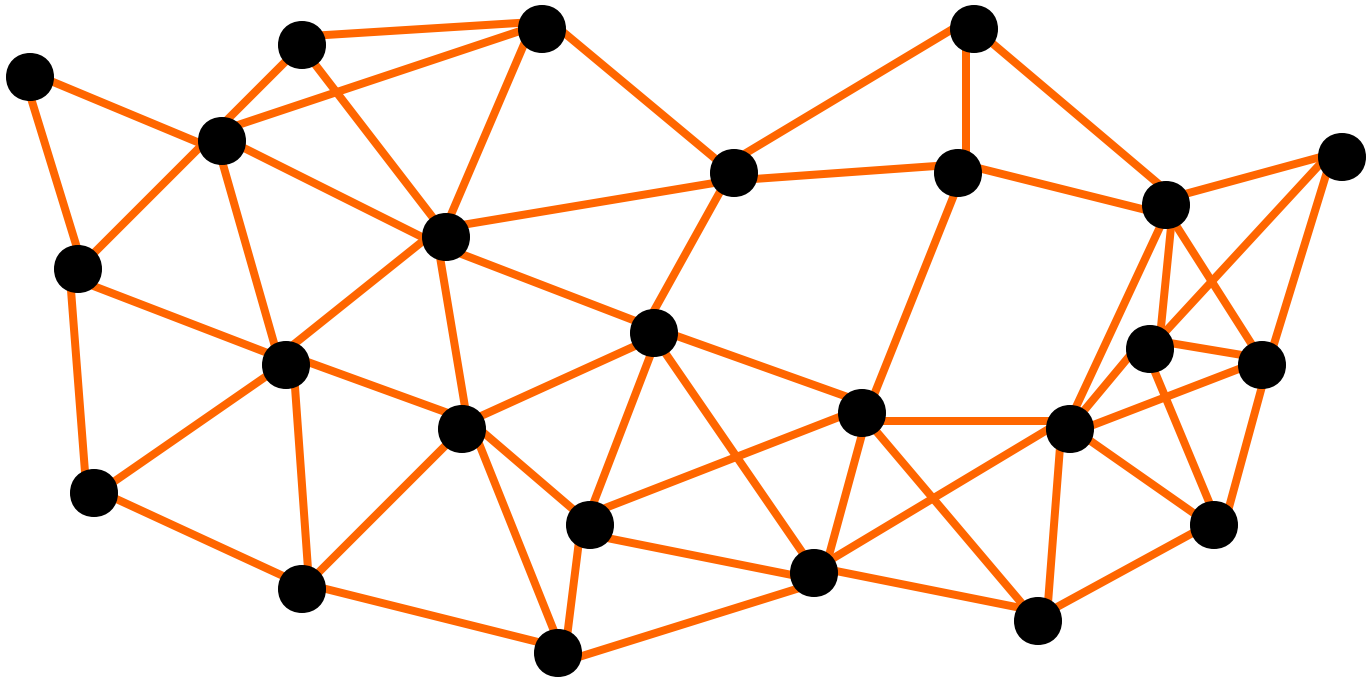
Overlay network



Unit disk graph



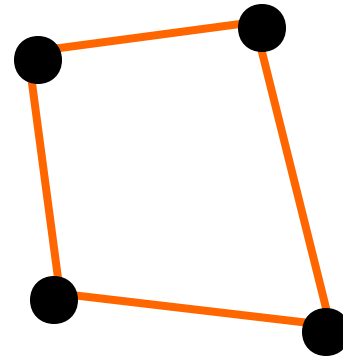
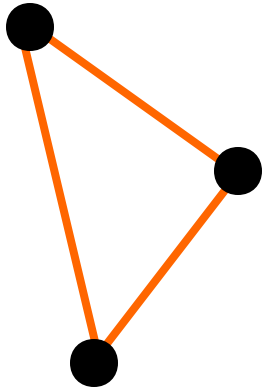
Unit disk graph



Basic Graph Algorithms: Breadth-First Search, Depth-First Search, Shortest Paths, Minimum Spanning Trees, Network Flow, Graphs and Applications

Sparsification is not trivial!

Every node connects to two nearest neighbors.



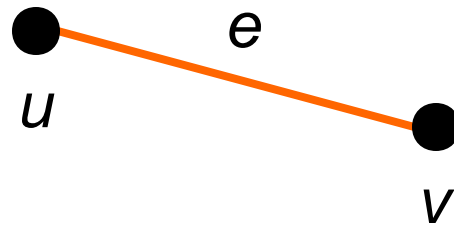
- $< 0.074 \log n$ nearest neighbors: disconnected w.h.p.
 - $> 5.1774 \log n$ nearest neighbors: connected w.h.p.
- if** nodes distributed uniformly at random in convex reg.

Goals of Sparsification

- Guarantee **connectivity**
- Guarantee **energy-efficient** paths resp. paths with **high success probability**
- Maintain **self-routing** network
(no preprocessing for path selection)
- General strategy: **graph spanners**

Assumptions

Node set V distributed in 2-dim Euclidean space



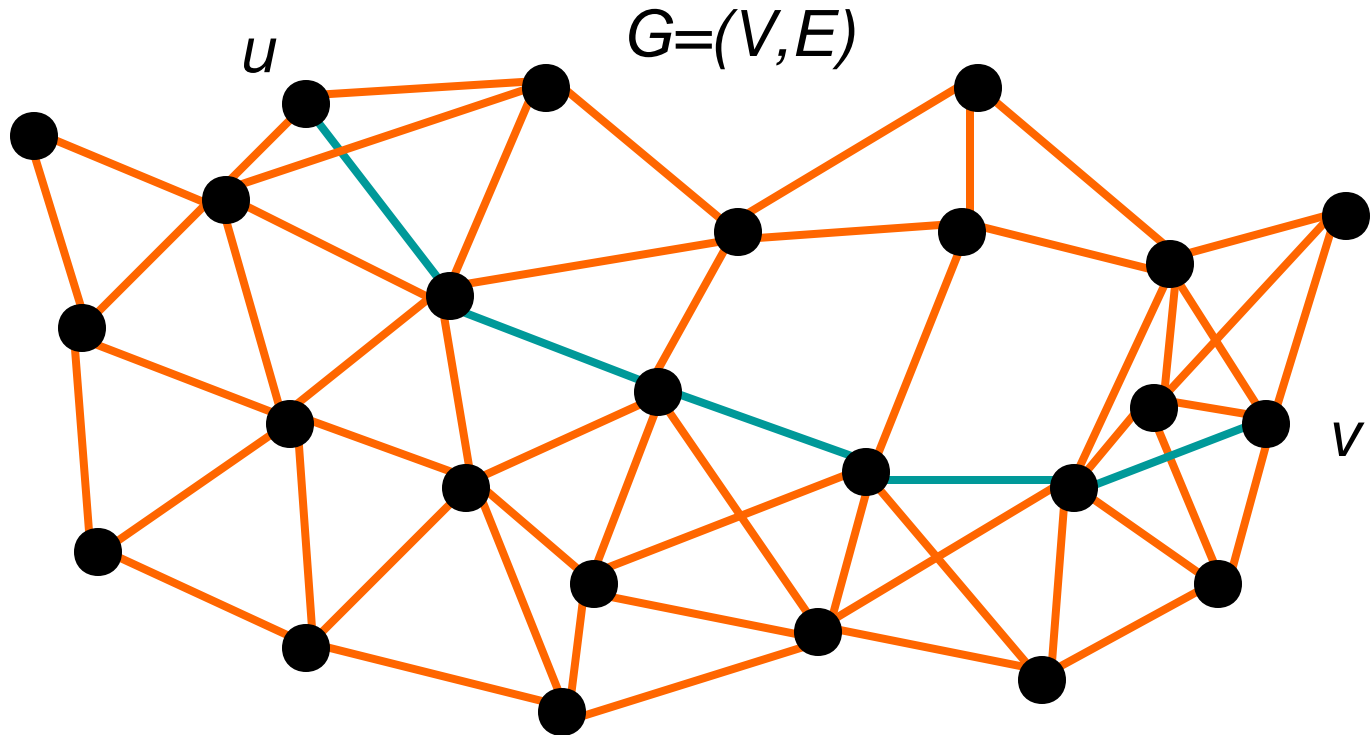
Euclidean distance: $\|u v\|$ or $\|e\|$

Power consumption: $\|u v\|^\delta$ for some $\delta > 2$

δ -cost of path $p=(e_1, \dots, e_k)$:

$$\|p\|^\delta = \sum_{i=1}^k \|e_i\|^\delta$$

Distance



$d_G^\delta(u,v)$: $\min \|p\|^\delta$ over all paths p from u to v in G

Unit disk graph $UDG(V)$: simply $d^\delta(u,v)$

UDG Spanners

Basic idea: S is **spanner** of graph G for some δ :
 subgraph of G with $d_S^\delta(u,v) \leq c \cdot d_G^\delta(u,v)$ for all $u,v \in V$

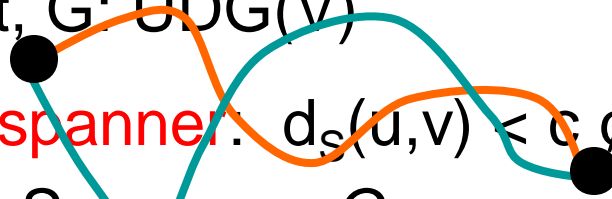
$c > 1$ constant, $G: \text{UDG}(V)$

• **Geometric spanner:** $d_S(u,v) \leq c \cdot d(u,v)$

• **Power spanner:** $d_S^\delta(u,v) \leq c \cdot d^\delta(u,v)$, $\delta > 2$

• **Weak spanner:** path p from u to v within disk of diameter $c \cdot d(u,v)$

• **Topological spanner:** $d_S^0(u,v) \leq c \cdot d^0(u,v)$

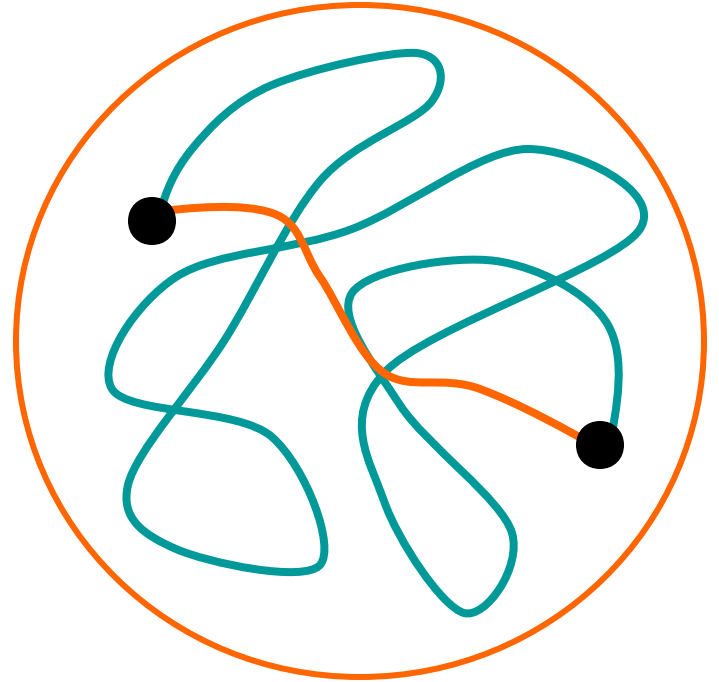


UDG Spanners

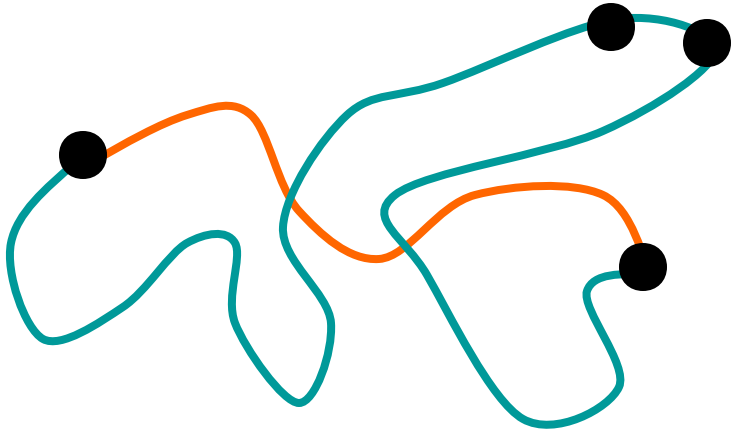
Geometric spanner:



Weak spanner:

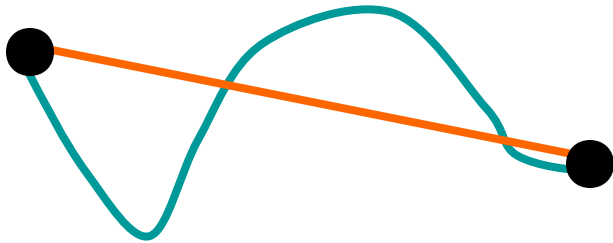


Power spanner:

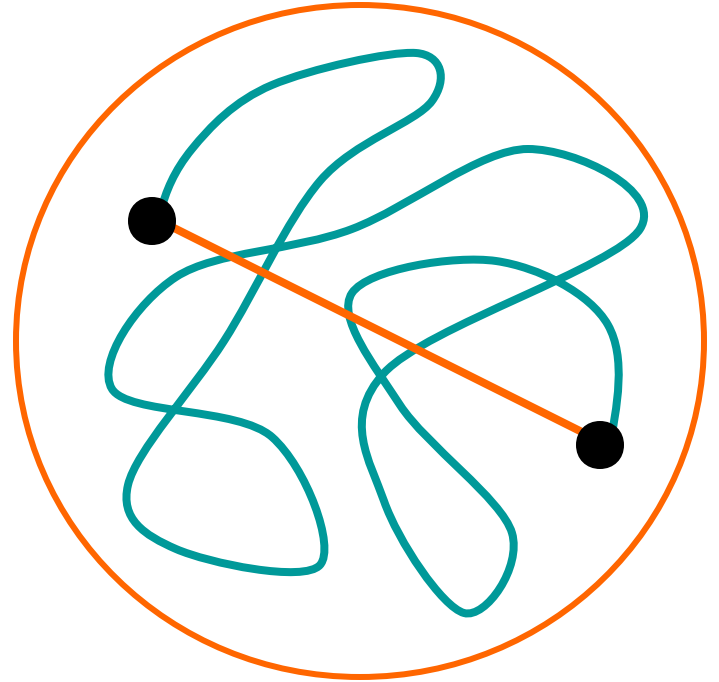


Spanners

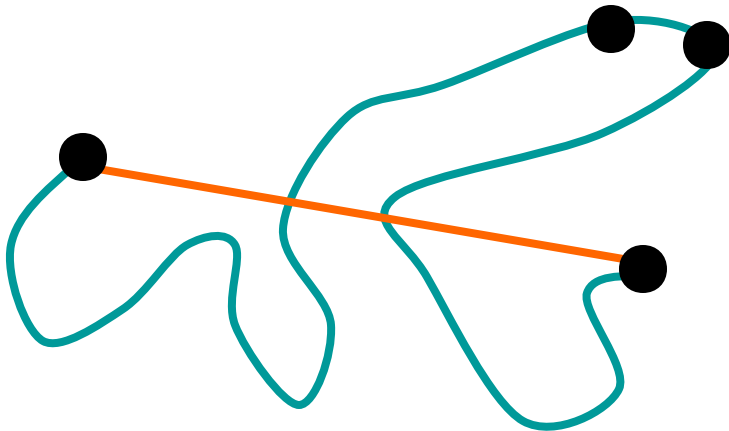
Geometric spanner:



Weak spanner:



Power spanner:



UDG Spanners

Basic $c > 1$ constant

- **Geometric spanner**: $d_S(u, v) < c \cdot \|u - v\|$
- **Power spanner**: $d_S^\delta(u, v) < c \cdot \|u - v\|^\delta, \delta \geq 2$
- **Weak spanner**: path p from u to v within disk of diameter $c \cdot \|u - v\|$
- **Constrained spanner**: there is a path p satisfying constraint above and $\|e\| \leq \|u - v\|$ for all $e \in p$

**$E(\text{constrained spanner}) \cap \text{UDG}(V)$:
spanner of $\text{UDG}(V)$**

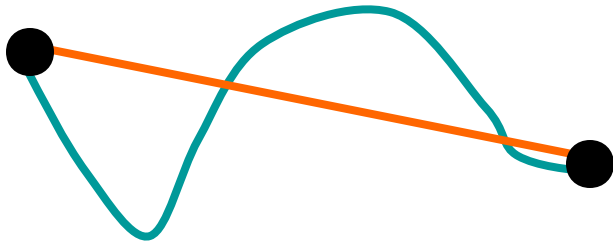
Spanner Properties

geometric \supset weak \supset power spanner

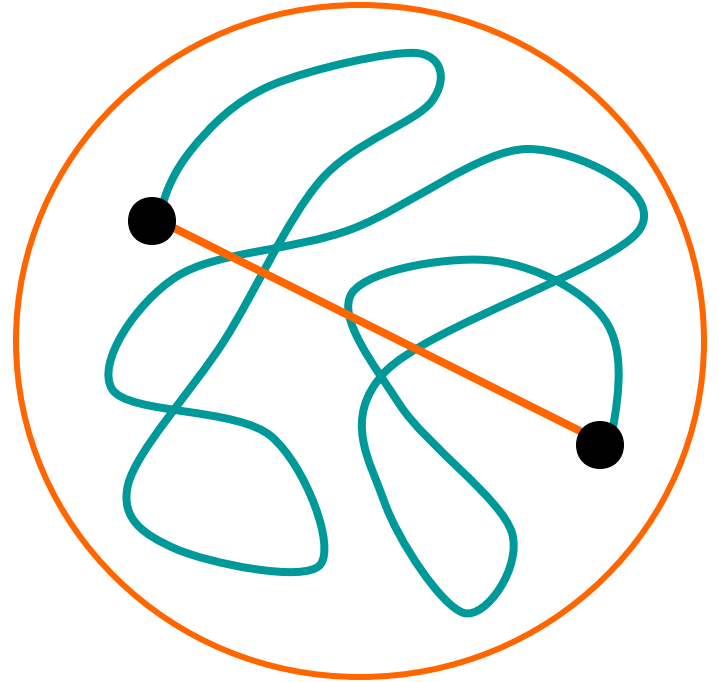
- Geometric spanner \supset power spanner
- Geometric spanner \supset weak spanner
- Weak spanner $\not\supset$ geometric spanner
- Power spanner $\not\supset$ weak spanner
- Weak spanner \supset power spanner

Spanners

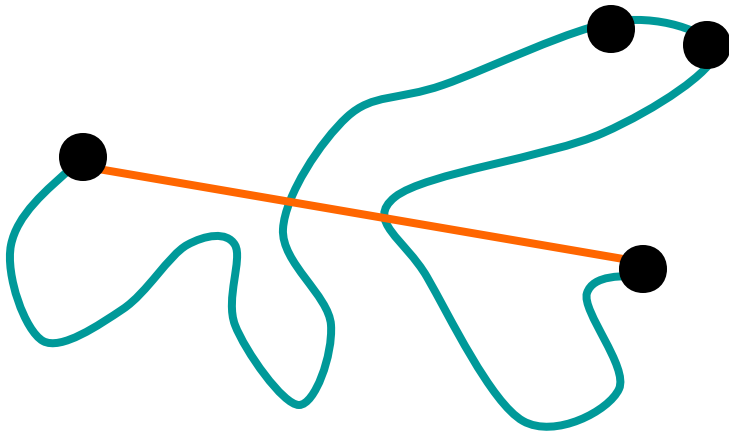
Geometric spanner:



Weak spanner:



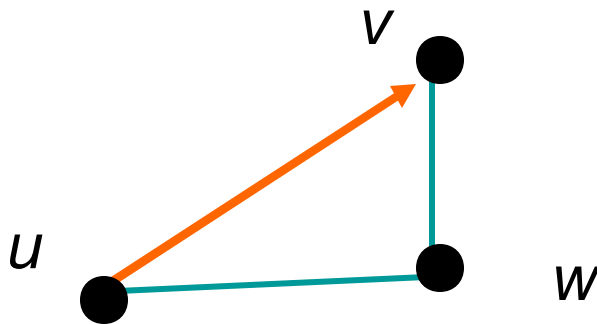
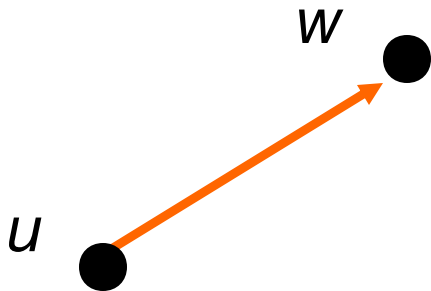
Power spanner:



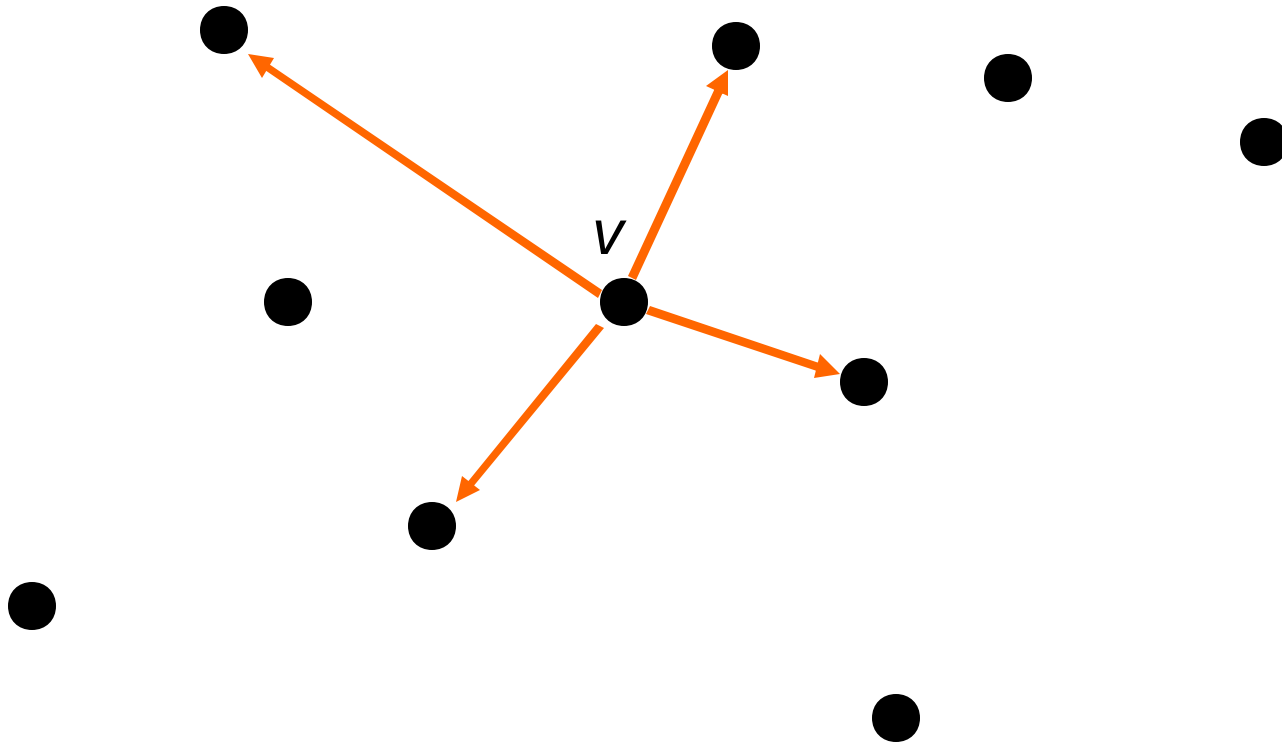
Proximity graphs

$G=(V,E)$ is **proximity graph** of V if $\forall u,w \in V$:

- either $(u,w) \in E$
- or $(u,v) \in E$ for some v : $\|v-w\| < \|u-w\|$



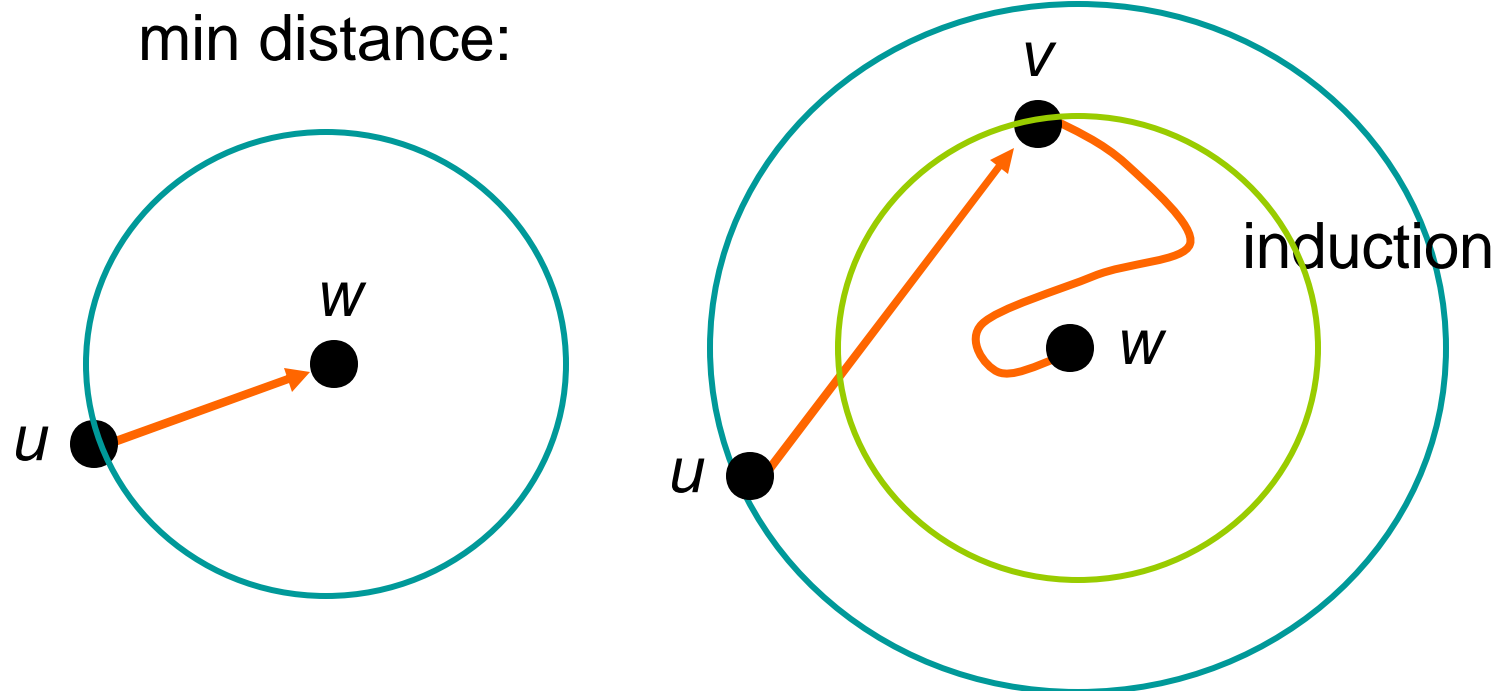
Example



Proximity graphs

Every proximity graph of V is a **weak 2-spanner**.

Proof: by induction on distance

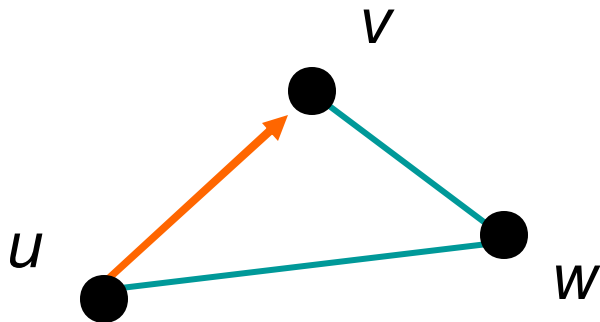


Relative neighborhood graphs

$G=(V,E)$ is a **RNG** of V if for all $u,w \in V$:

- either $(u,w) \in E$
- or $(u,v) \in E$ for some v :

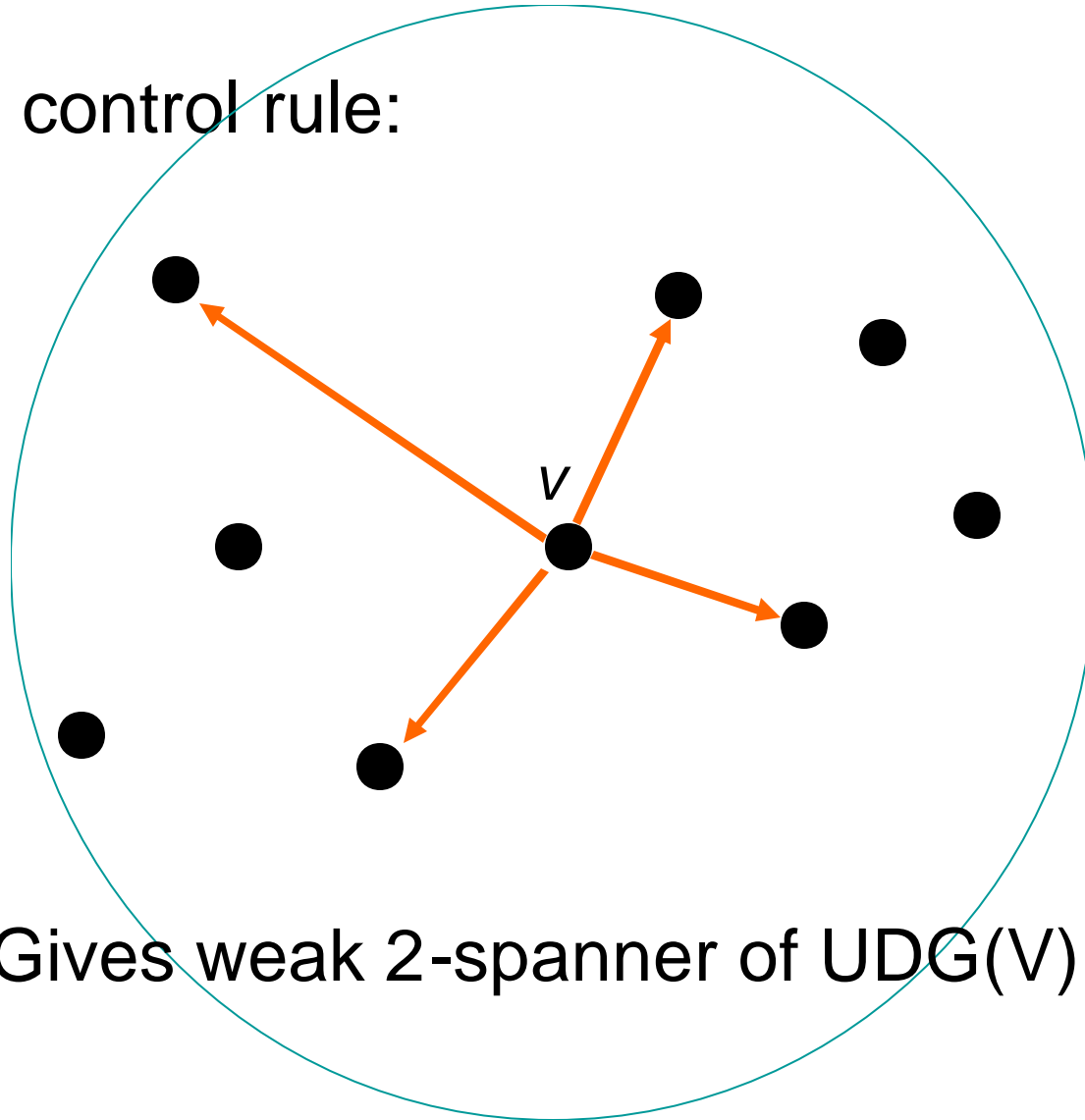
$$\|u v\| \leq \|u w\| \text{ and } \|v w\| < \|u w\|$$



RNG: **constrained**
proximity graph

Relative neighborhood graphs

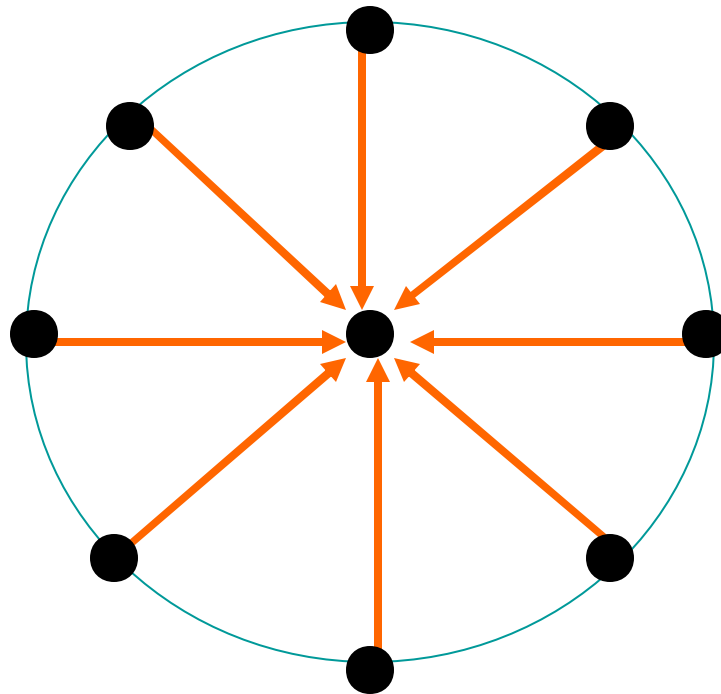
Local control rule:



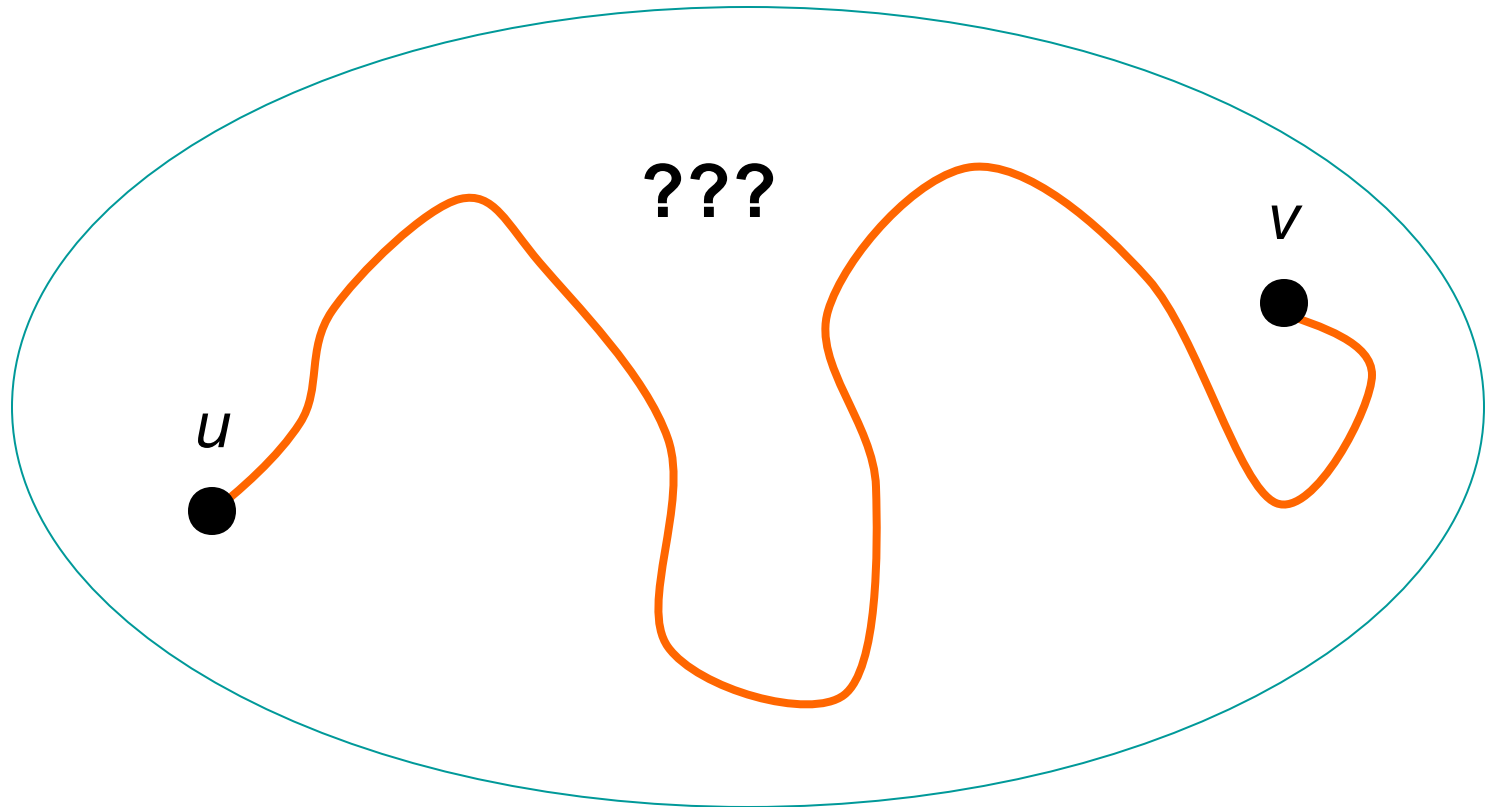
Gives weak 2-spanner of $UDG(V)$.

Relative neighborhood graphs

Problem: Minimal RNGs have outdegree at most 6 but indegree can be large.



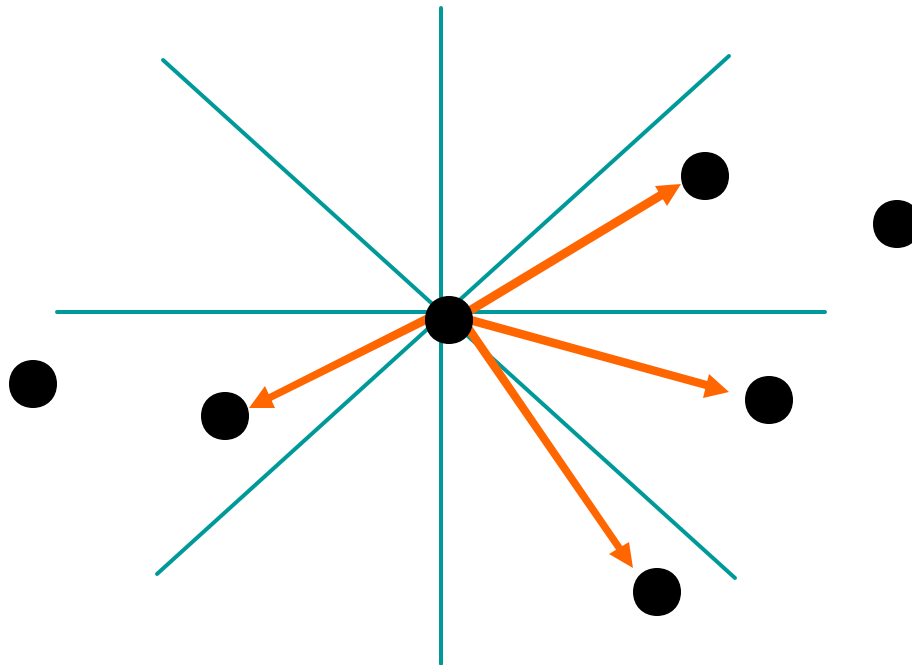
Routing in RNGs



What is the appropriate algorithm?
Bellman's algorithm works on paths

Sector-based spanners

Yao graph or θ -graph (special RNG):

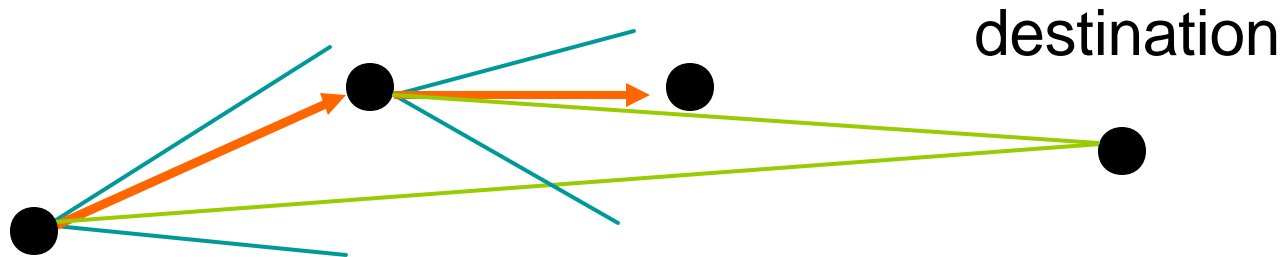


Rule: Connect
to nearest
node in each
sector

Sector based spanners

θ -graphs with >6 sectors are **geometric spanners**

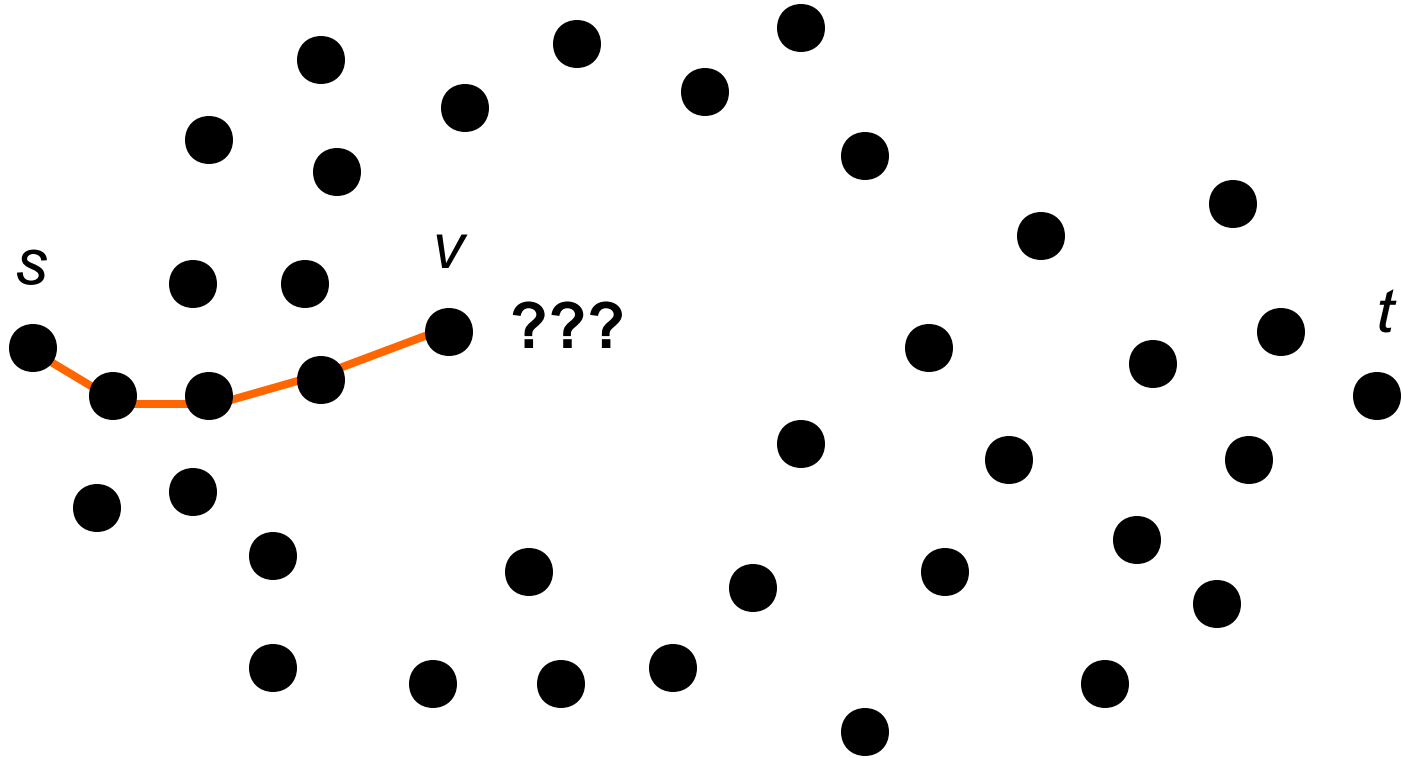
Routing:



Go to node in sector of destination.
Requires dense node distribution, GPS!

Sector based graphs

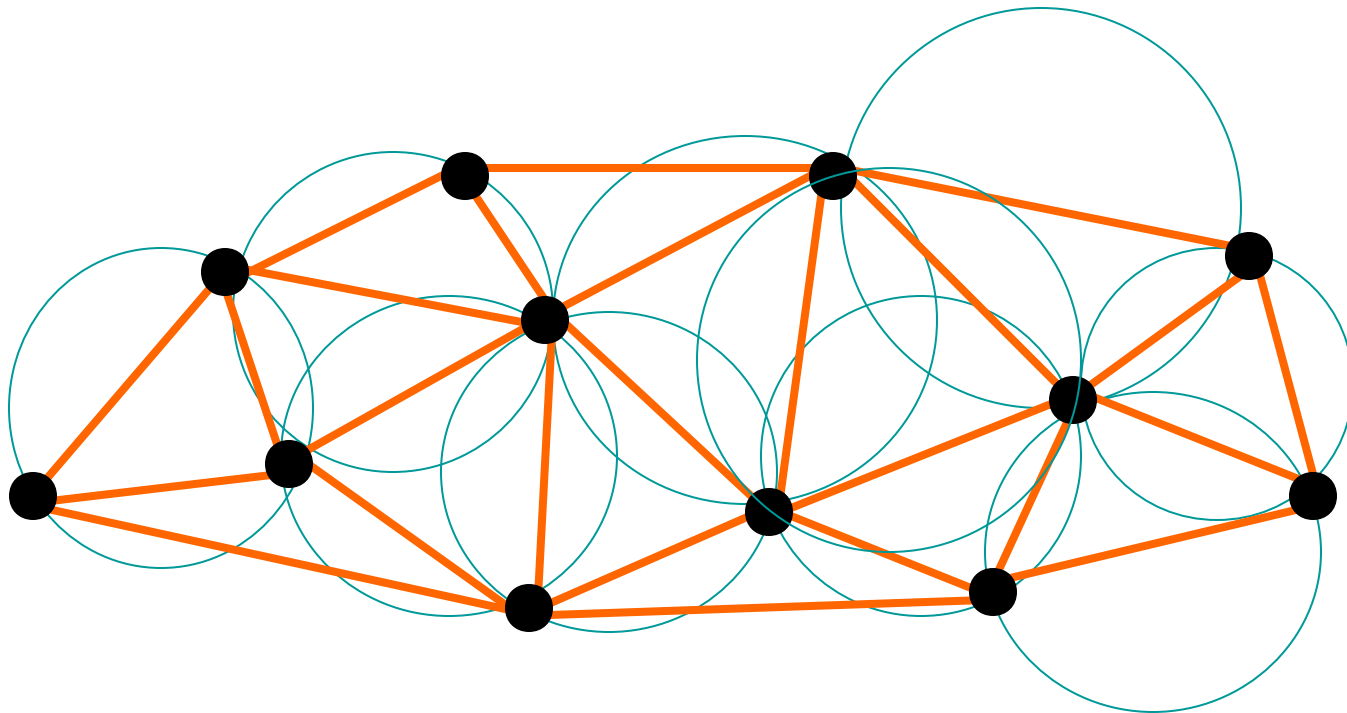
Distribution not dense:



Delaunay-based spanners

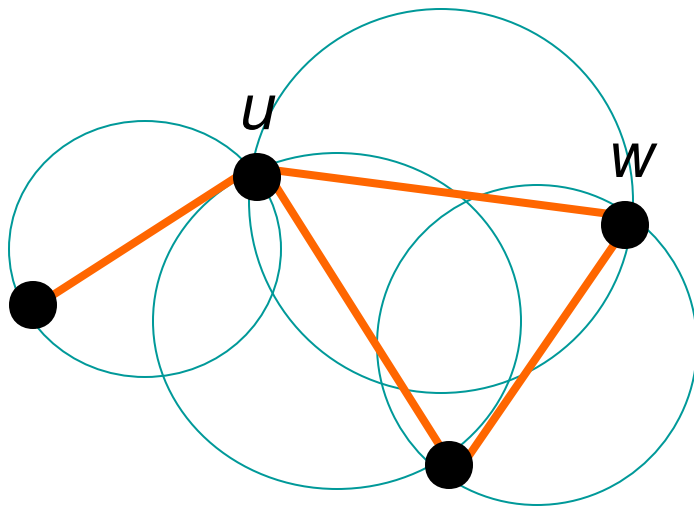
Idea: use variants of Delaunay graph

Delaunay graph $\text{Del}(V)$ of V : contains all $\{u,v\}$: $\exists w \in V$ where $O(u,v,w)$ does not contain any node of V



Delaunay-based spanners

Gabriel graph $GG(V)$ of V : contains all $\{u,w\}$ with no $v \in V$ s.t. $\|u-v\|^2 + \|v-w\|^2 < \|u-w\|^2$



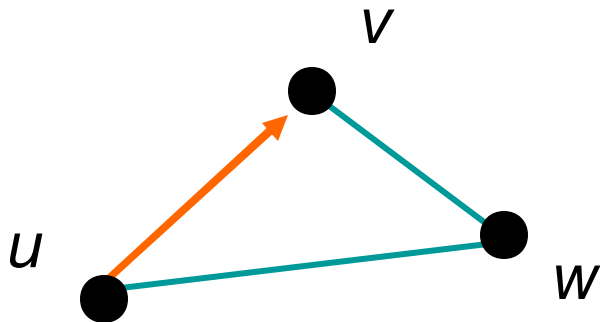
$GG(V) \frac{1}{2} Del(V)$

Relative neighborhood graph

$G=(V,E)$ is a **RNG** of V if for all $u,w \in V$:

- either $(u,w) \in E$
- or $(u,v) \in E$ for some v :

$$\|u v\| \leq \|u w\| \text{ and } \|v w\| < \|u w\|$$

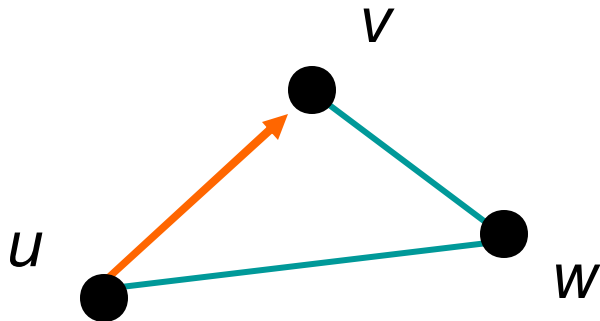


RNG: **constrained**
proximity graph

RNG for path loss

The GG satisfies for all $u, w \in V$:

- either $(u, w) \in E$
- or $(u, v) \in E$ for some v :
 $c(u, v) \leq c(u, w)$ and $c(v, w) < c(u, w)$



GG: **constrained**
proximity graph for
path loss with $\delta=2$

Delaunay-based spanners

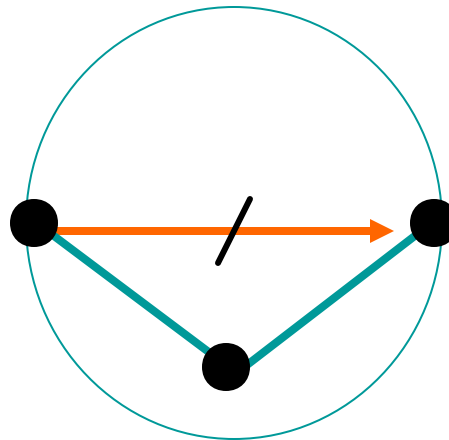
$GG(V)$ is an **optimal power spanner**.

Proof: Let p be energy-optimal path for $\{u,v\}$.
Consider any edge $\{x,y\}$ in p .

$\{x,y\} \in GG(V)$:



$\{x,y\} \notin GG(V)$:

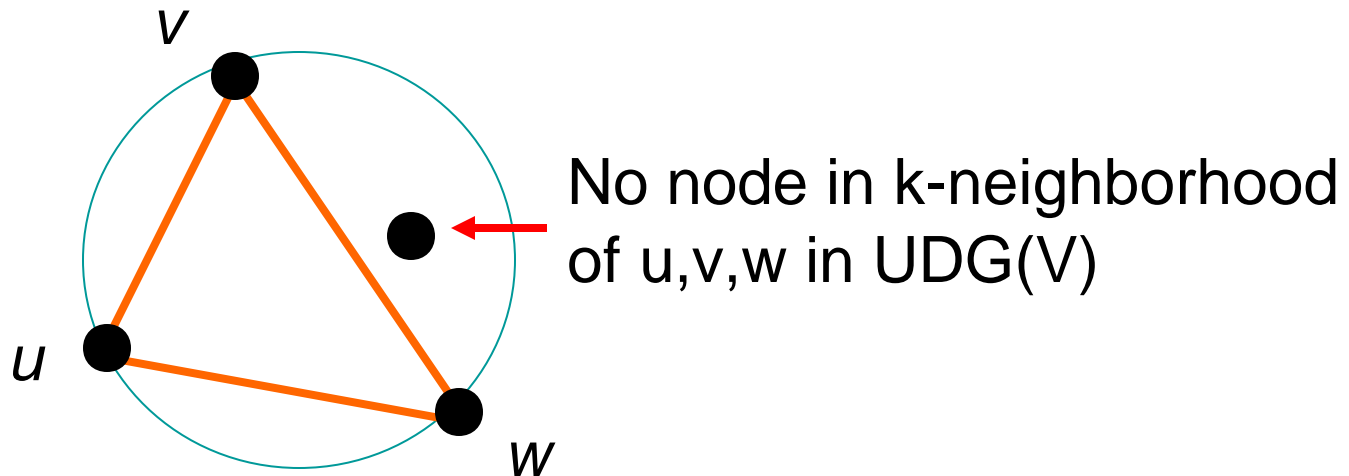


Contradiction!

Delaunay-based spanners

Problem with Gabriel graphs: can have high degree, can be poor geometric spanner

Alternative: **k-localized Delaunay graphs** $\text{LDel}^{(k)}(V)$



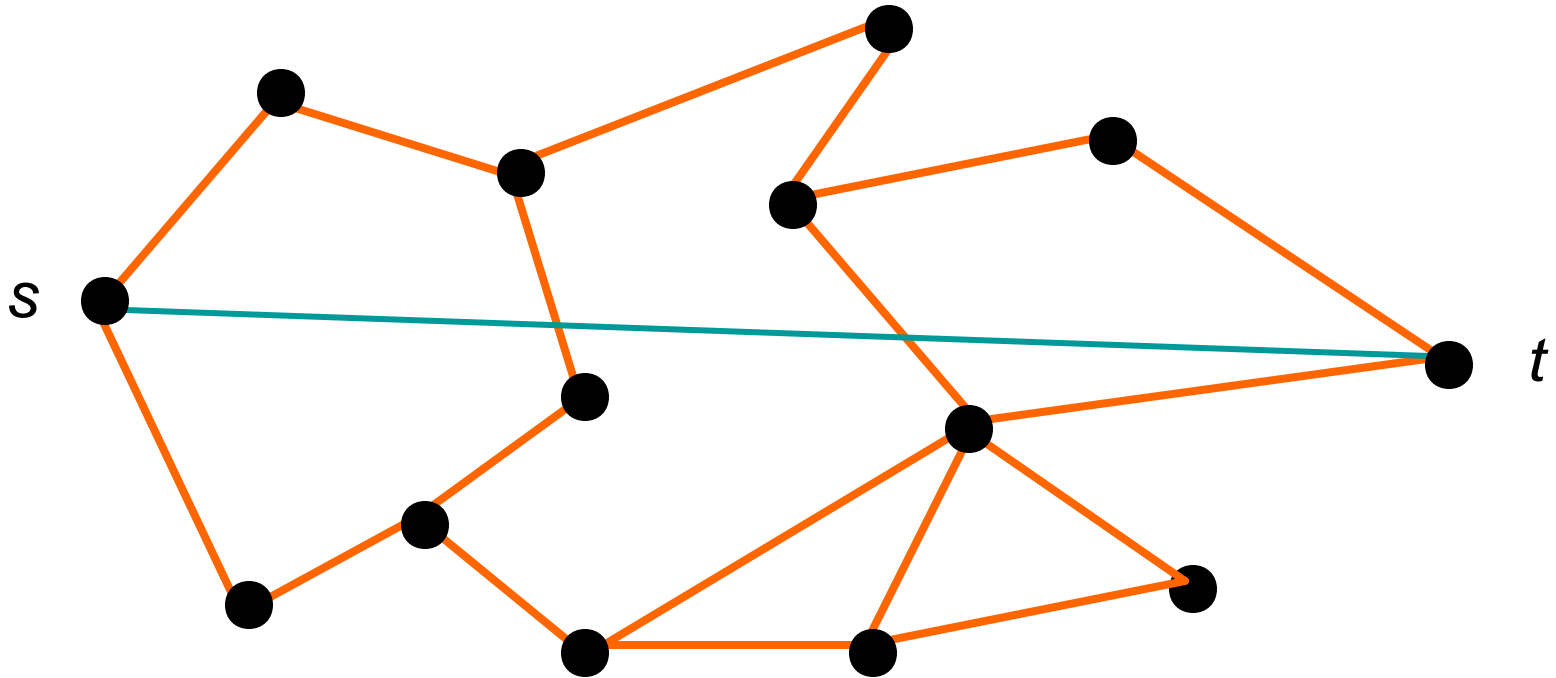
Delaunay-based spanners

- $\text{LDel}^{(1)}(V)$: not planar
- $\text{LDel}^{(2)}(V)$: **planar** and **geometric spanner**

Planarity important for routing!

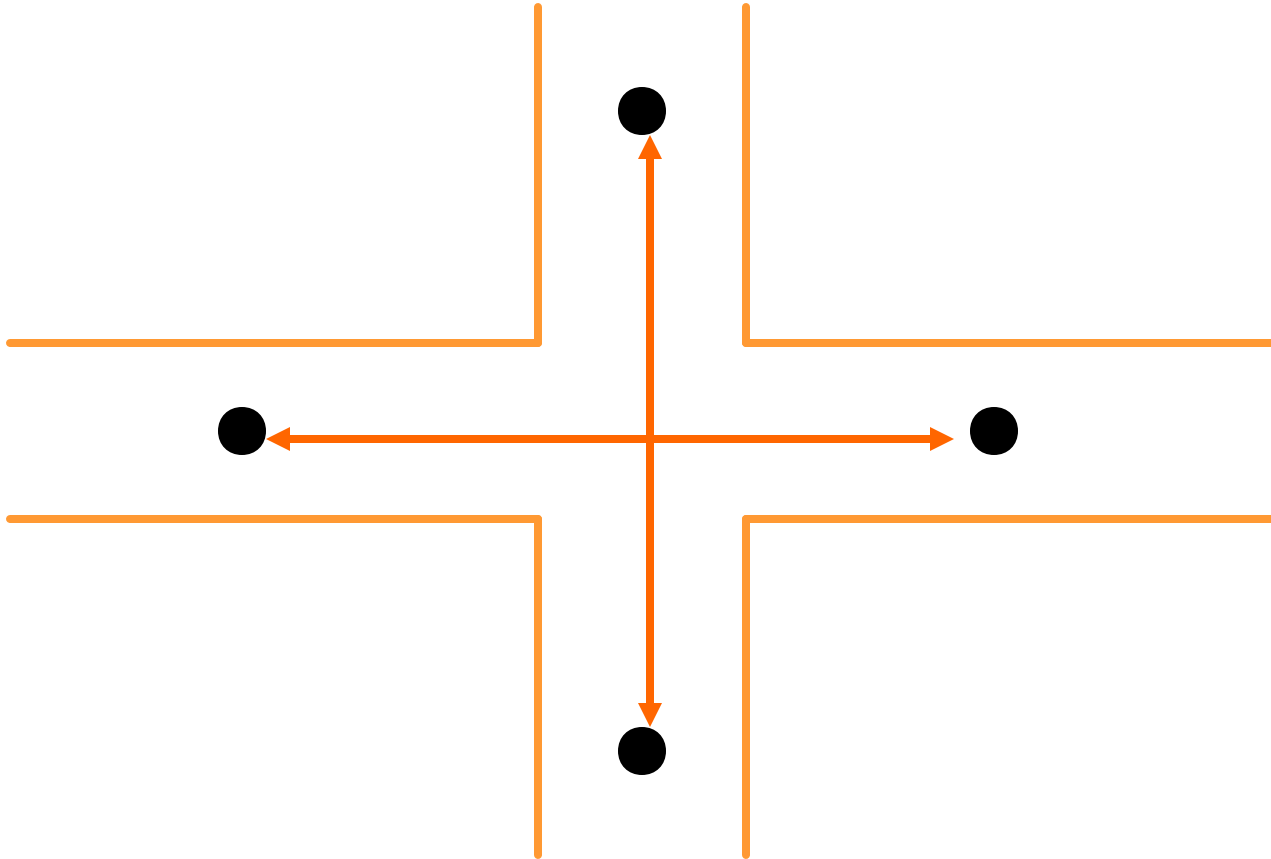
Routing in planar graphs

Face routing:



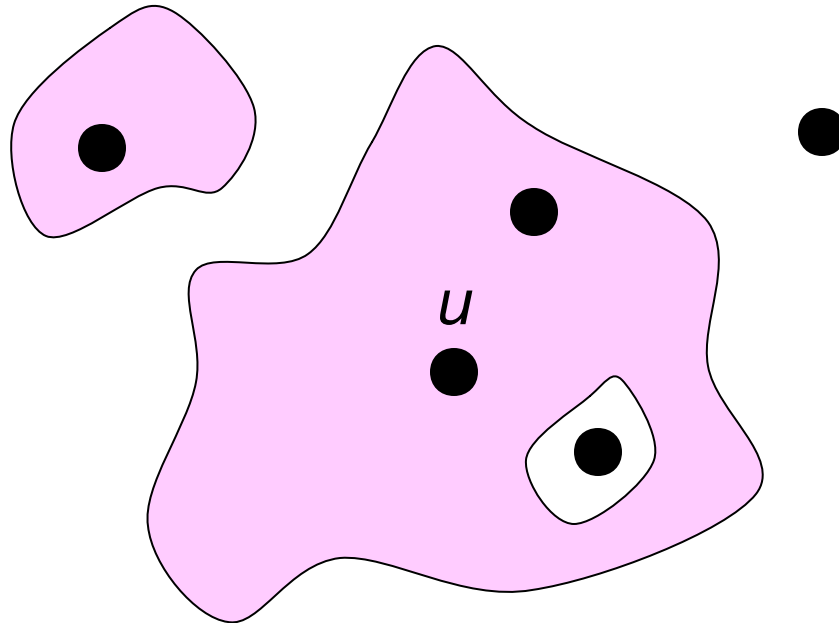
**That's nice, but
do these strategies work
in practice???**

Problem: Unit Disk Model



In reality, **hard** to maintain planar overlay network.

Reality



Cost function c and constant $\delta > 0$ s.t. for any two points v and w

- $c(v, w) \in [(1-\delta) \|v - w\|, (1+\delta) \|v - w\|]$
- $c(v, w) = c(w, v)$

Realistic wireless model

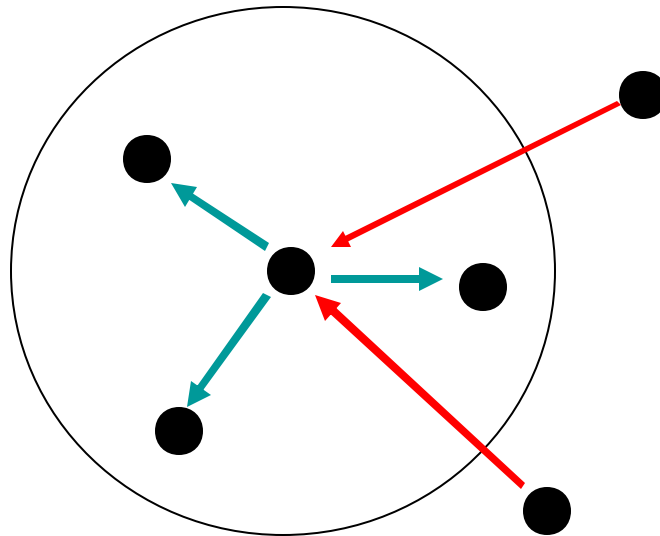
When using cost function c :

- Proximity graphs: OK
- Relative neighborhood graphs: OK
- θ -graphs: $\theta < \arccos(\delta^2 + 1/2)/(1 - \delta^2)$, $\delta < 1/2$
- Gabriel graphs, localized Delaunay graphs:
not planar but other spanner results OK
- Face routing extendable to more realistic model?
- Is GPS necessary?

Problem: GPS

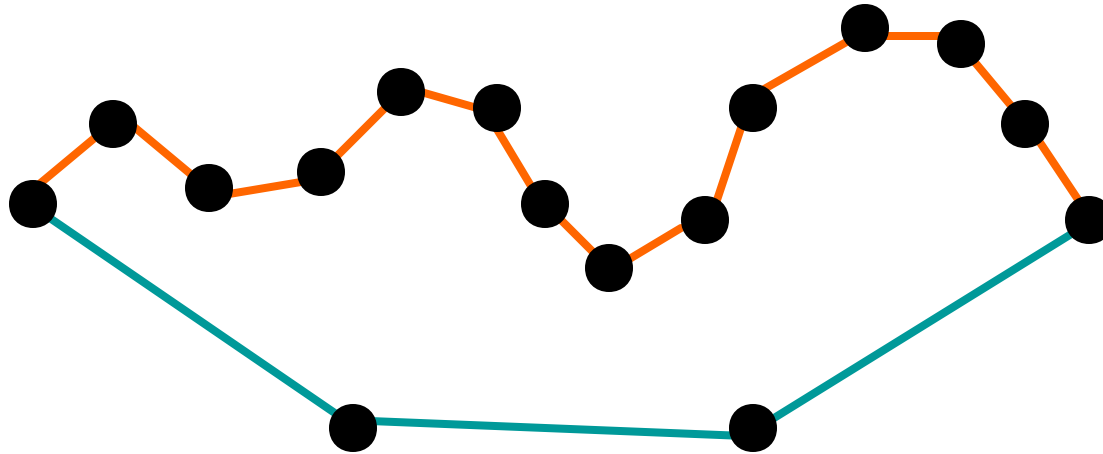
GPS not necessary for topology control, but
can we avoid GPS for routing?

Possible solution: use force approach

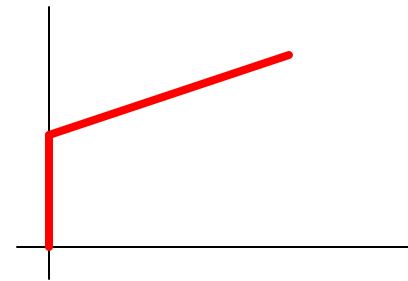


Problem: Cost Model

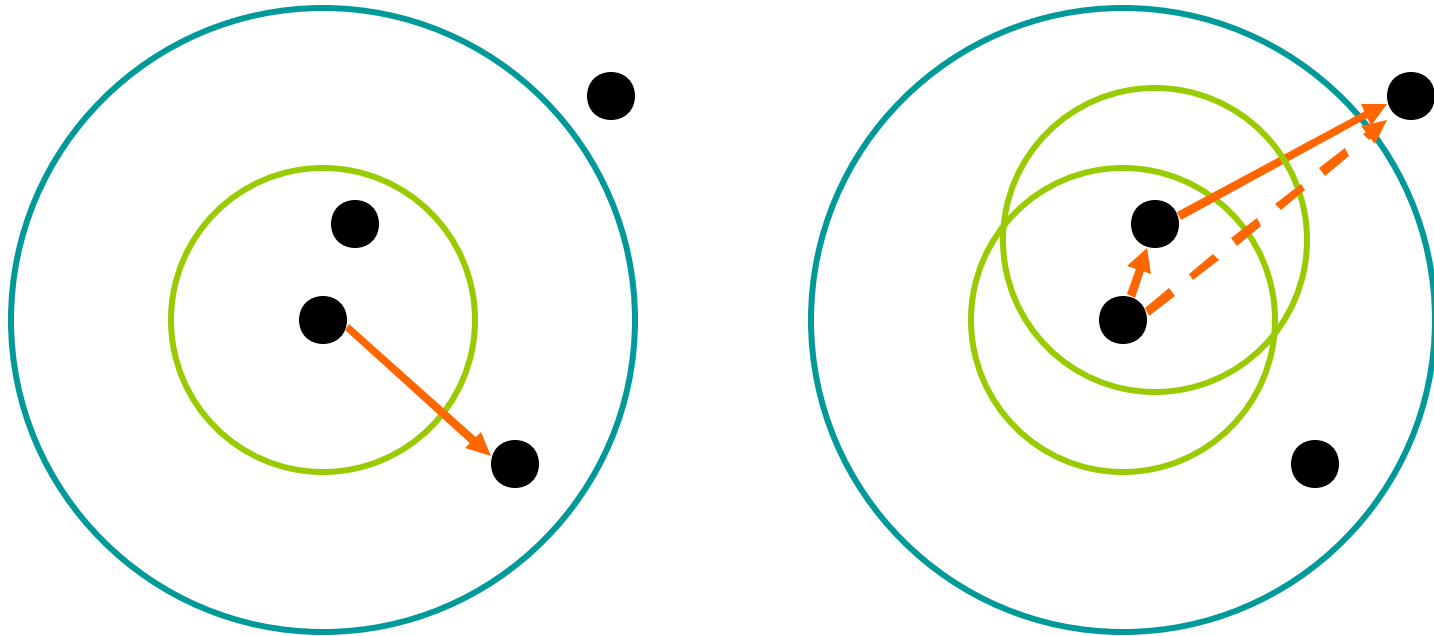
Power spanner prefers — over —



Energy consumption:



Solution

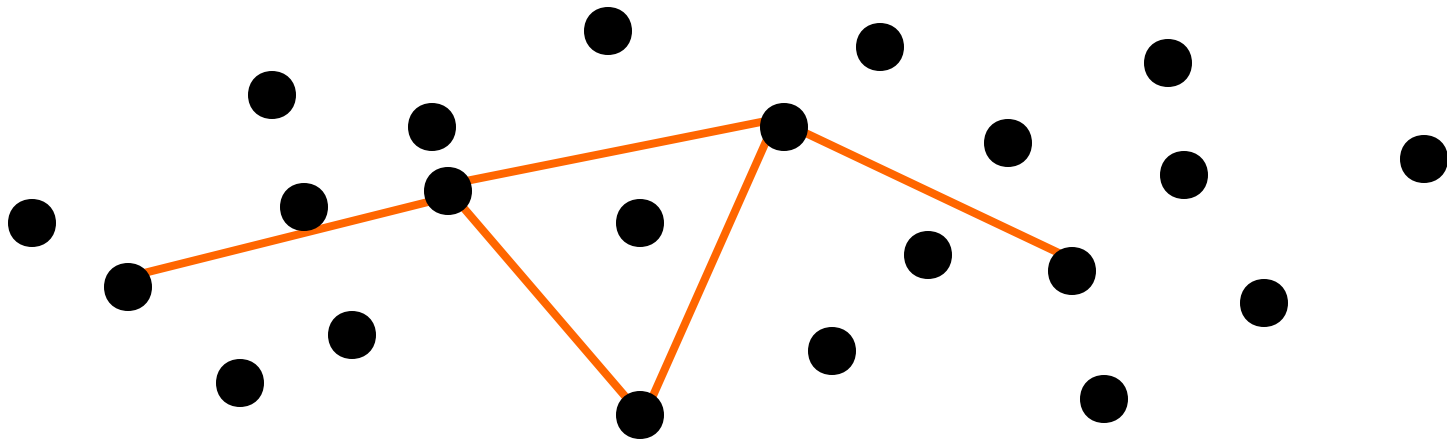


Only allow long edges (except last).

Problem: Contention

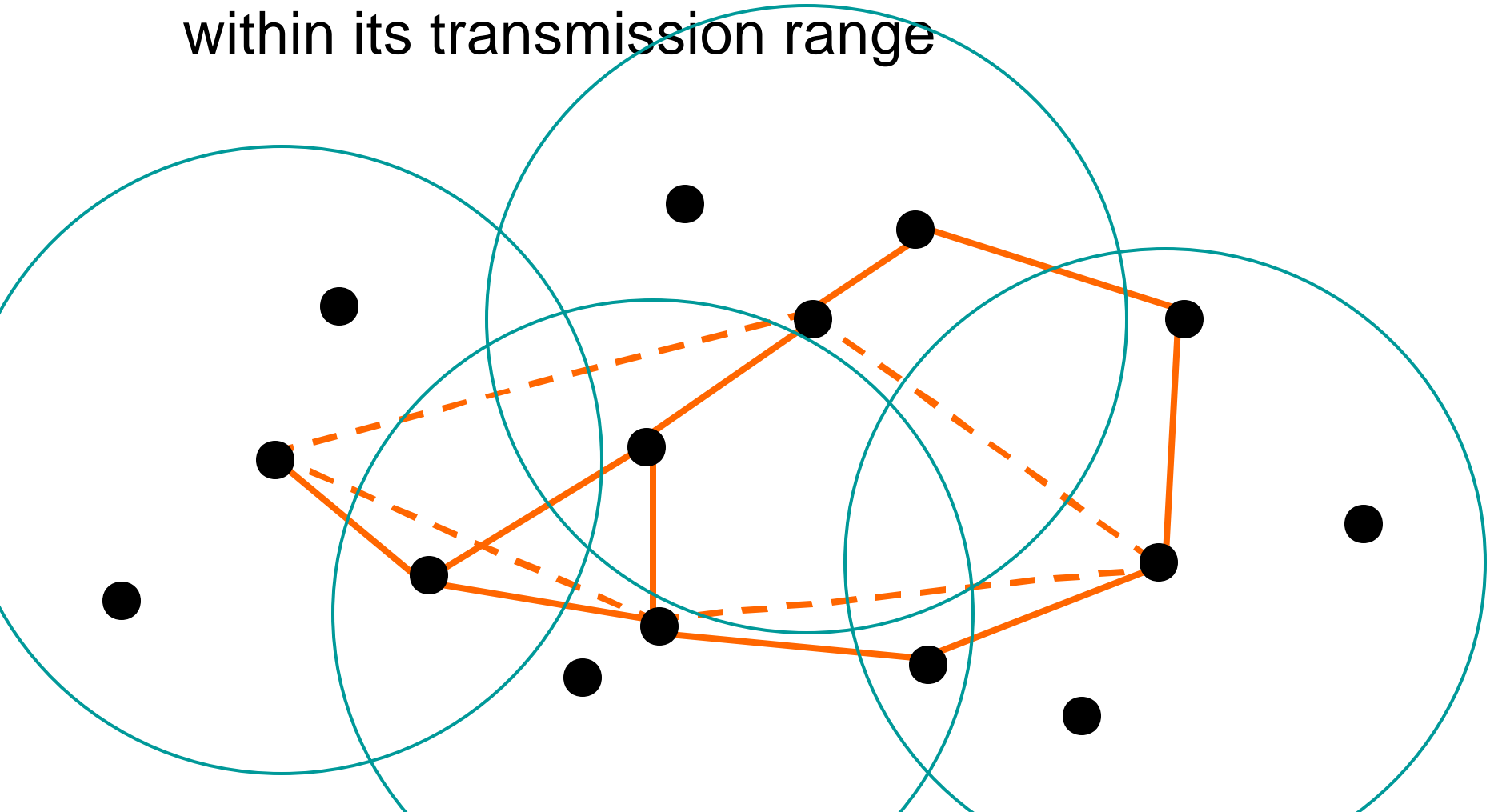
Still too much contention because every node participates in routing.

Better to have **highway system** (i.e., only few nodes act as relay nodes).



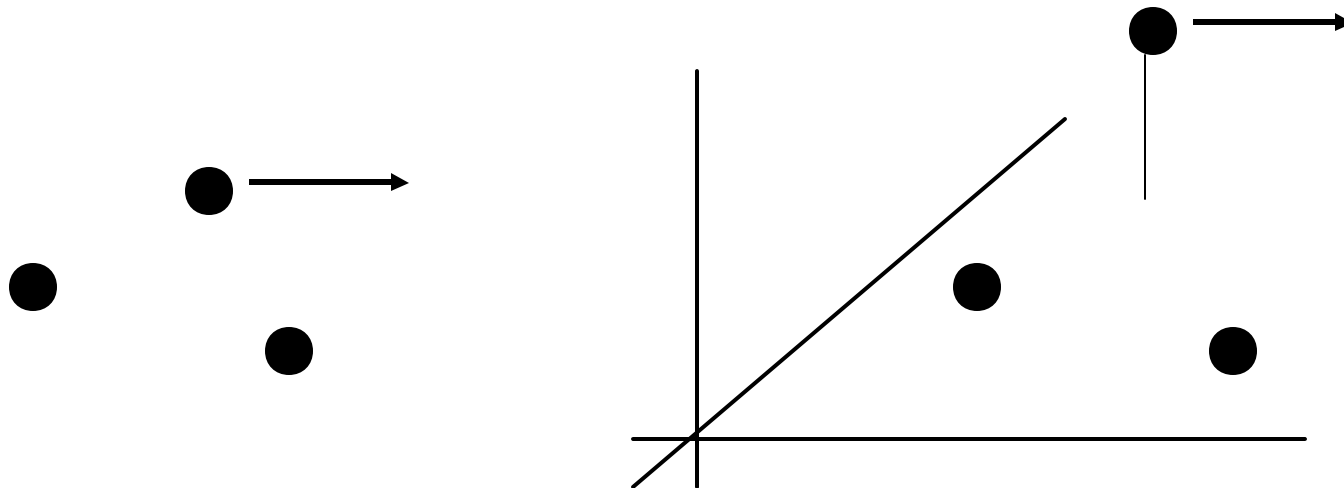
Solution

Construct **dominating set**: every ● has ● within its transmission range

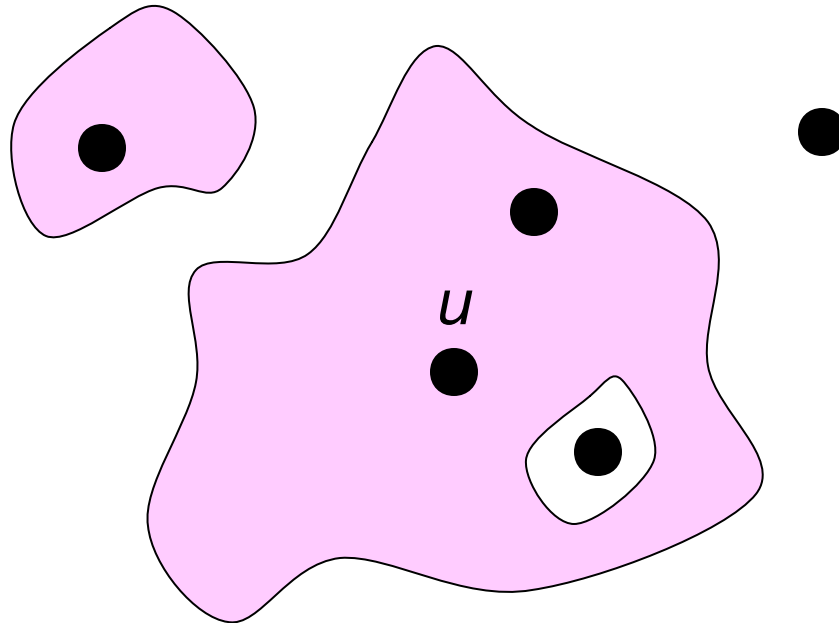


Problem: Mobility

Solution: view mobility as another dimension



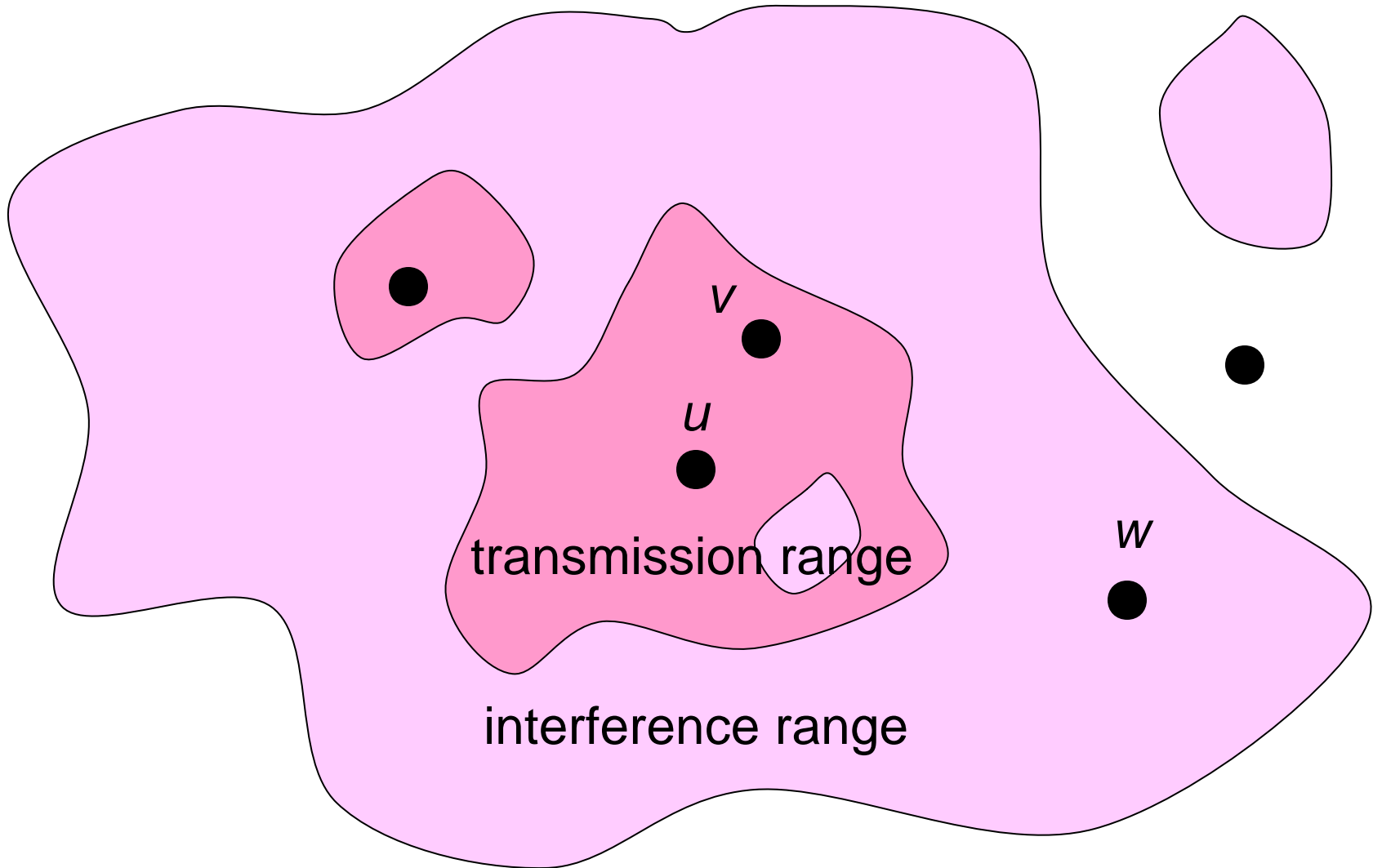
Problem: Protocol Design



Cost function c and constant $\delta > 0$ s.t. for any two points v and w

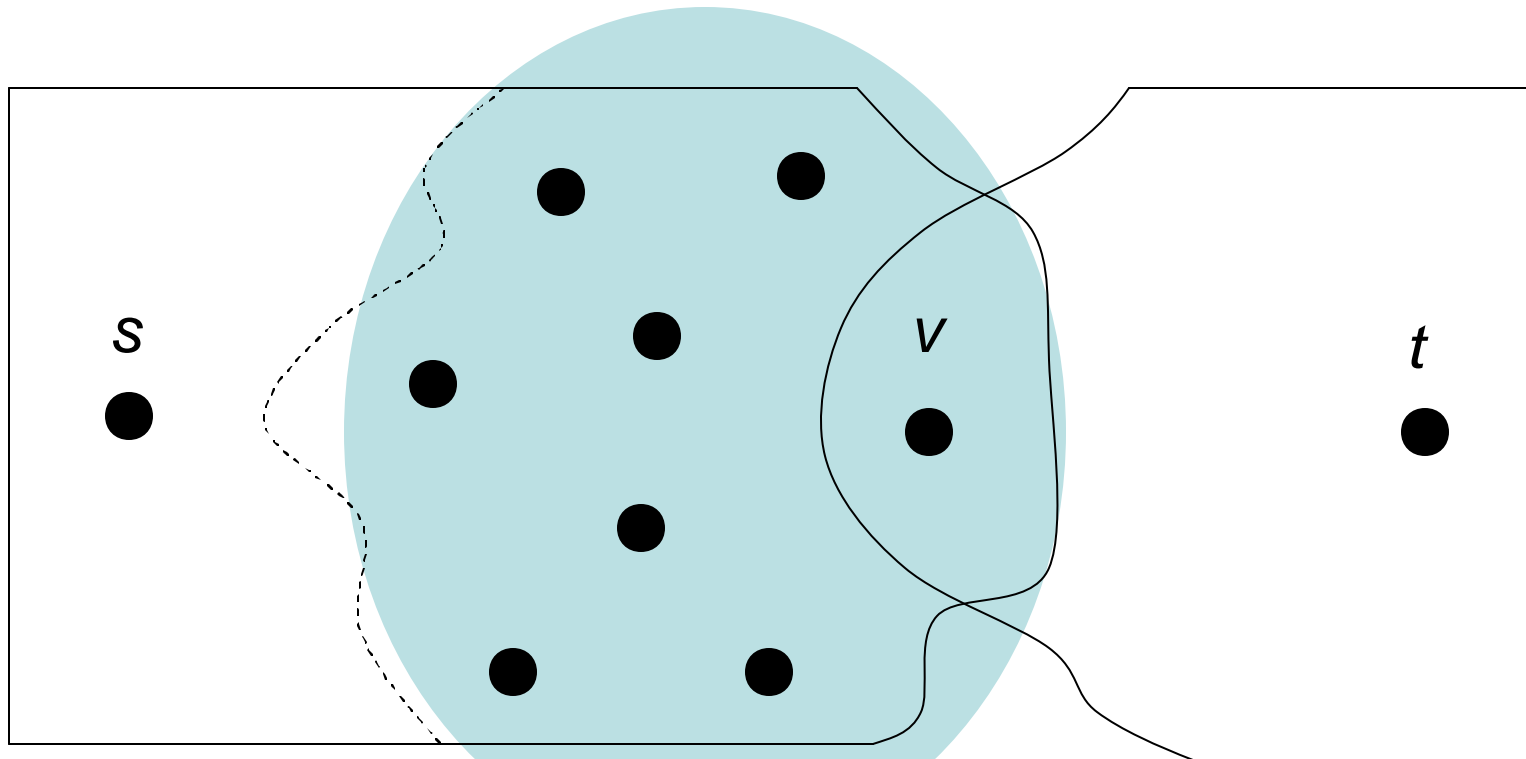
- $c(v, w) \in [(1-\delta) \|v - w\|, (1+\delta) \|v - w\|]$
- $c(v, w) = c(w, v)$

Realistic wireless model



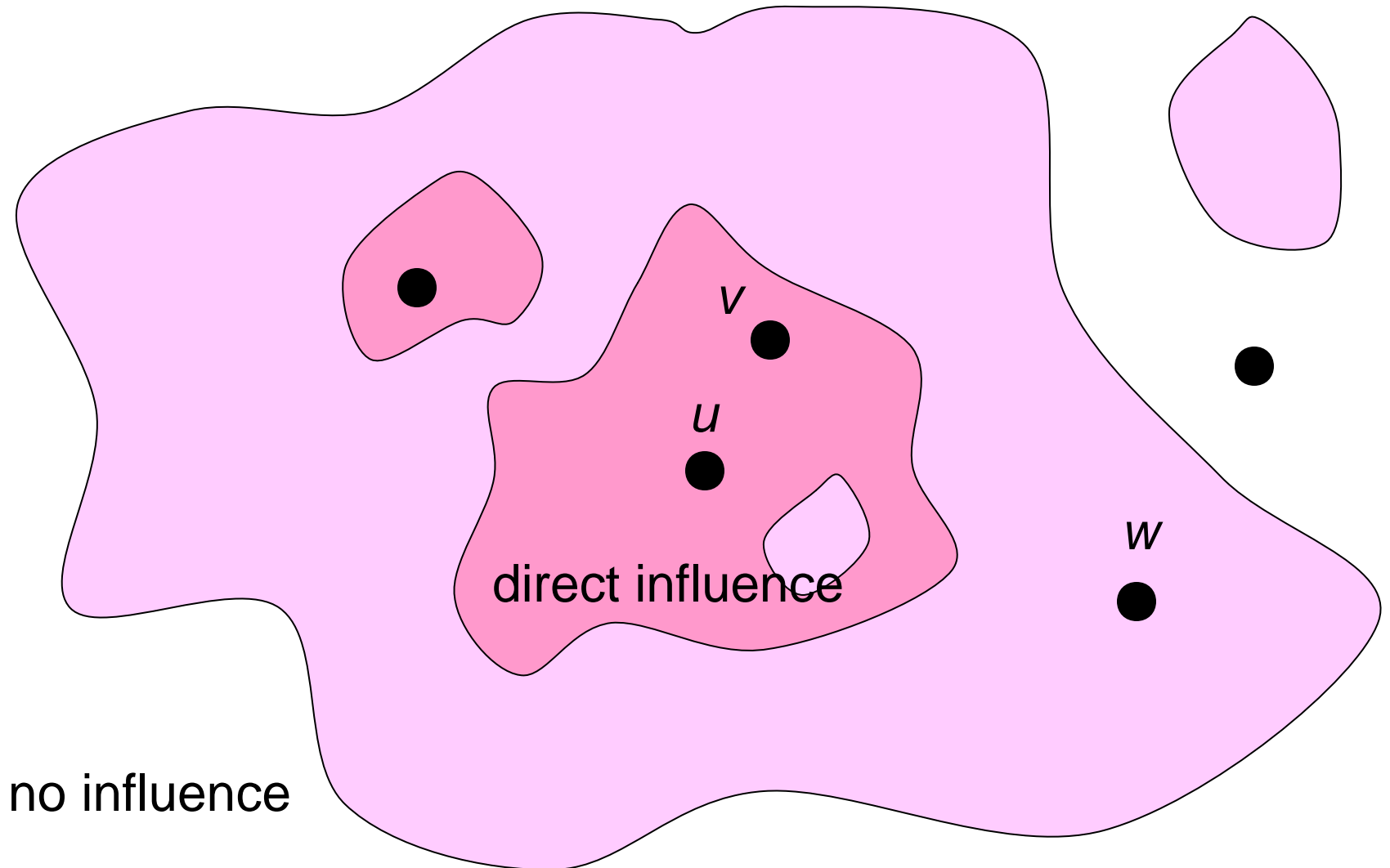
Realistic wireless model

Design of algorithms much more complicated!



Estimate of density difficult without physical carrier sensing

Physical carrier sensing



Future problems

Overlay networks are **hot** topic!

Problems:

- Local self-stabilization
- Robustness against adversarial behavior
- Distributed optimization
- Unified network model (???)

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Stochastic distribution of nodes:

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Realistic wireless models:

- K. Kothapalli, M. Onus, A. Richa and C. Scheideler. Constant density spanners for wireless ad-hoc networks. In Proc. of 17th ACM Symp. on Parallel Algorithms and Architectures (SPAA), 2005.

Lists are not meant to be comprehensive but should be sufficient to get started in this field.