Communication Strategies and Coding for Relaying

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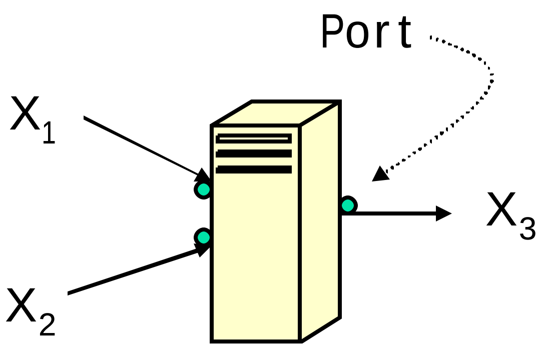
Outline

1) Network nodes
2) Wireline relaying
   - How data compression helps, and (later) why
3) Wireless relaying
   - Block-Markov coding
   - Half-duplex devices and timing modulation
   - Distributed codes

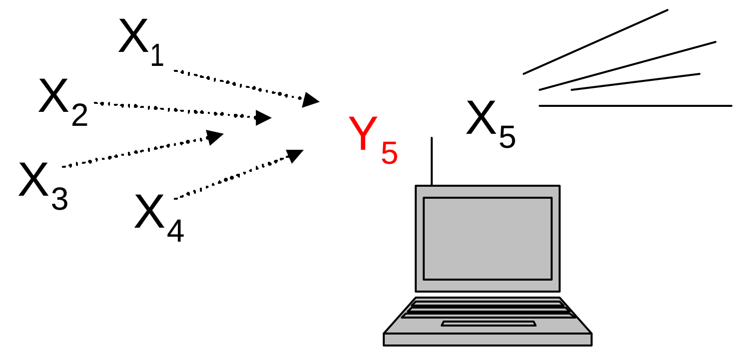
1) Network Nodes

- Wireline



Node constraints:
Suppose k ports can be active at once, e.g., k=1

- Wireless

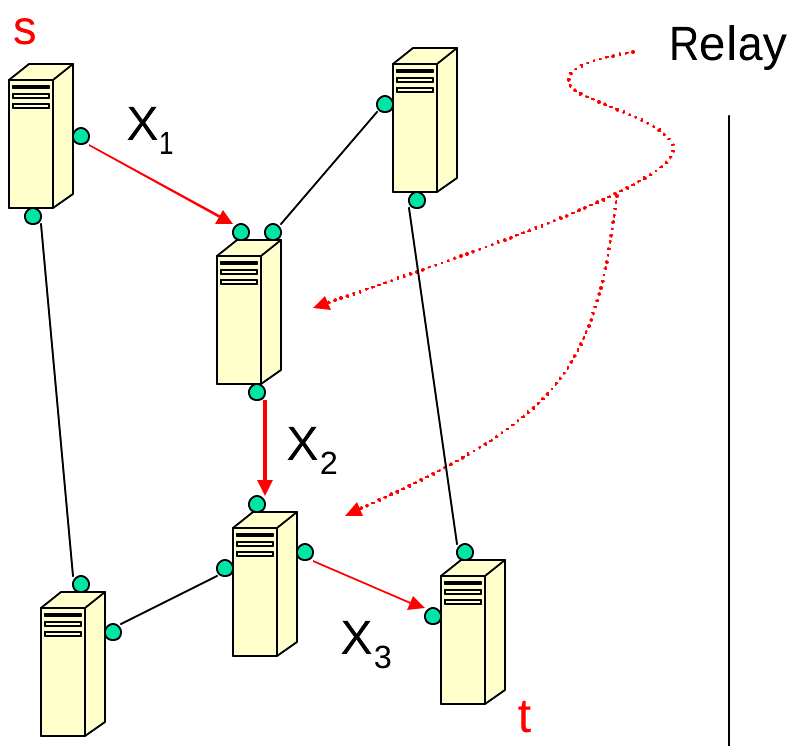


Half-duplex constraint:

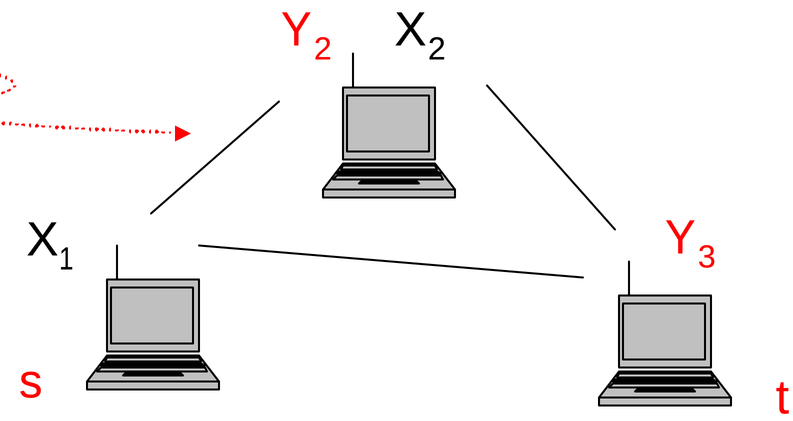
$$Y_t = \begin{cases} Z_t + \sum_{s \neq t} \frac{h_{st}}{d_{st}^{a/2}} X_s & \text{if } X_t = 0 \\ 0 & \text{else} \end{cases}$$

Networks

- Wireline



- Wireless

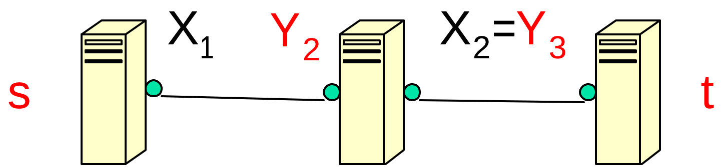


$$Y_2 = \begin{cases} Z_2 + \dfrac{h_{12}}{d_{12}^{a/2}} X_1 & \text{if } X_2 = 0 \\ 0 & \text{else} \end{cases}$$

$$Y_3 = Z_3 + \frac{h_{13}}{d_{13}^{a/2}} X_1 + \frac{h_{23}}{d_{23}^{a/2}} X_2$$

2) Wireline Relaying

- 3 node example:



- Suppose 1 port/node can be active simultaneously. A link (channel) model:

$$\text{if } X_2=0 \text{ then } Y_2=X_1$$
$$\text{if } X_2?0 \text{ then } Y_2=0$$

- Suppose the random variables are bits*.

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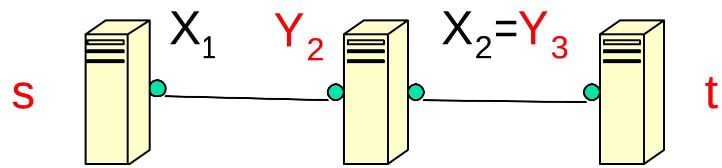
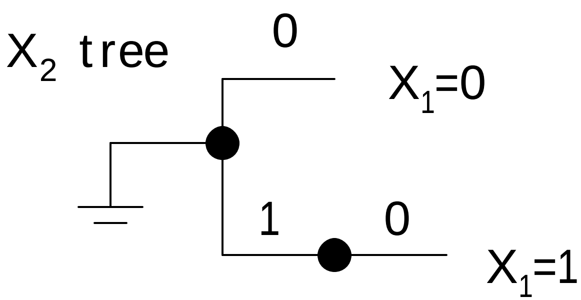
*Usually the X_t are packets, and not bits, but the following gives the general idea



- Guess: capacity is ½ bit/use (or packet/use) ?
- A "decompression" code at node 2:

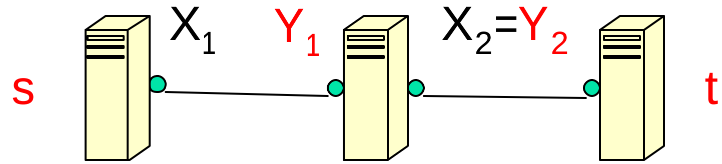


Node 2 transmits appropriate branch labels upon receiving X_1. For example:

X_1 = 0, 1, X0, 0, 1, X1, X1, X0

X_2 = 0, 0, 10, 0, 0, 10, 10, 10, 0

- 1st network edge: every X_2 word has one zero
- 2nd network edge: R = $1/E[L_2]$ = 2/3 bits/use !



- Better compression codes (e.g., Huffman codes, arithmetic source codes) achieve
  R = 0.773 bits/use with $\Pr[X_2=0] = 0.773$*.

- How can we understand this gain?
  Is 0.773 the capacity of this network?

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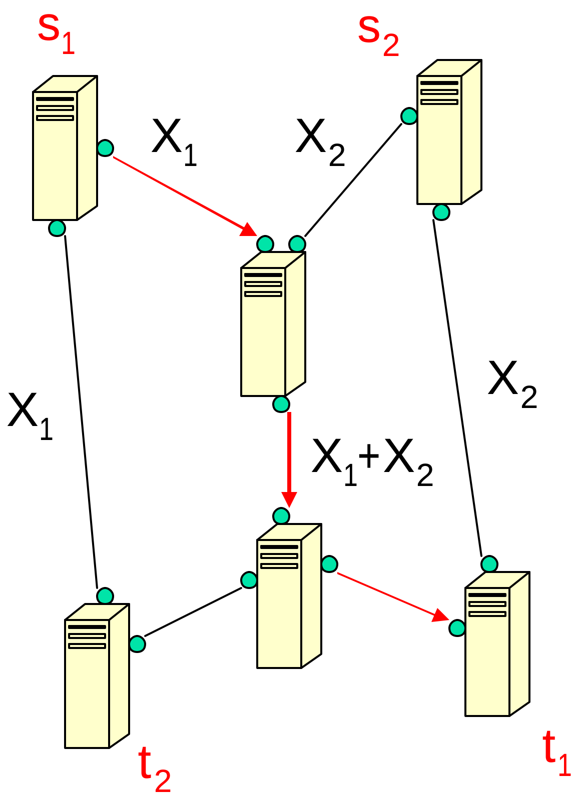
*This is when $\Pr[X_2=0] = h(\Pr[X_2=0])$, where
$h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is Shannon's binary entropy function

- Network implications:

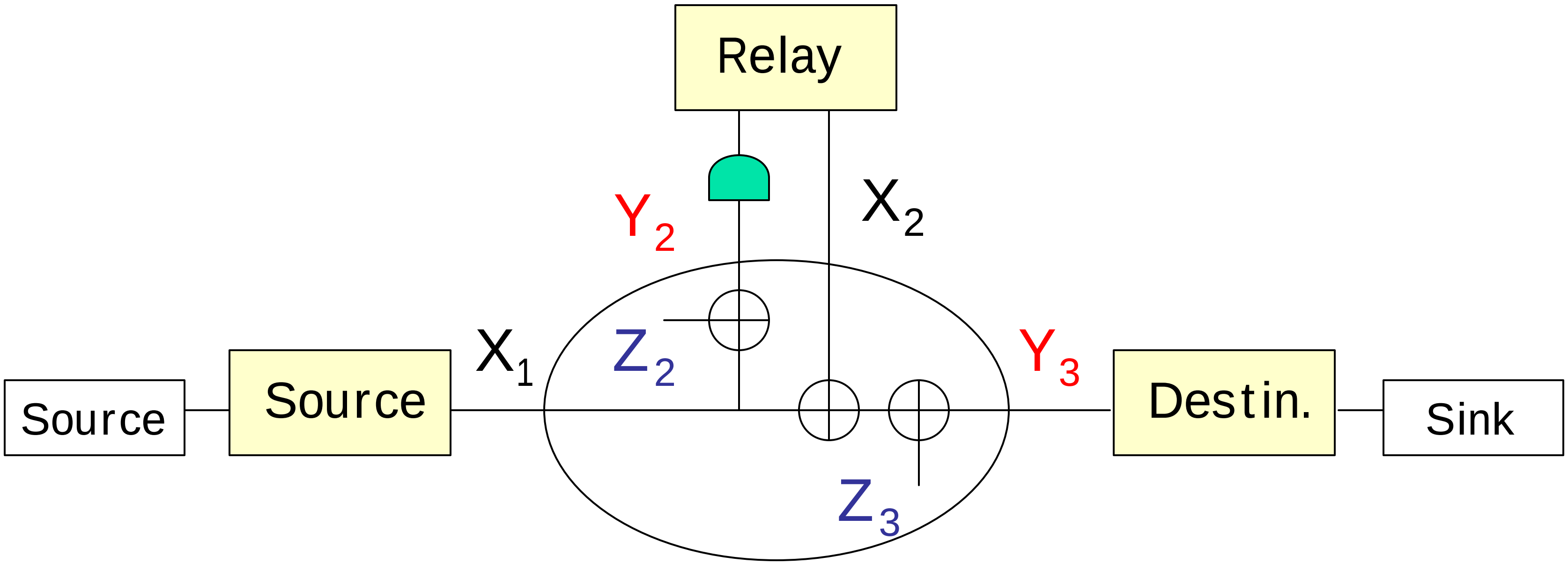


- Suppose every node has a 1-port constraint
- Basic routing throughput:
  1/2 bit/use
- Basic network coding:
  2/3 bits/use
- Relay routing:
  0.732 bits/use*

*in general, one should combine network coding and relaying

*for packets, the gains are smaller but do permit “covert” communication

3) Wireless Relaying

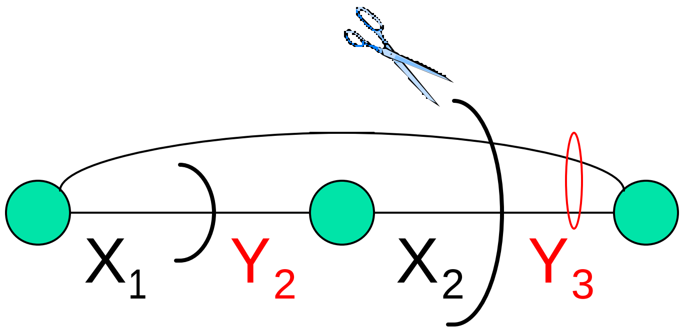


- Complex alphabets, lossless paths, full duplex (for now)
- Power: $E[|X_{ti}|^2] = P_t$, for t=1,2, all i, Gaussian noise Z_t (var. 1)
- No relay (X_2=0): the capacity is $\max_{P(x_1)} I(X_1;Y_3) = \log(1+P_1)$
- The capacity of the above problem is still open !

Coding Methods

- Various relaying strategies exist:
  - Amplify-and-forward (amplify Y_2)
  - Decode-and-forward (includes basic multi-hopping)
  - Compress-and-forward (quantize Y_2 and encode)
- The best decode-and-forward scheme achieves

$$R = \max_{P(x_1,x_2)} \min \left[I(X_1;Y_2|X_2),\ I(X_1X_2;Y_3) \right]$$



Block-Markov Coding

- Carleial's decode-and-forward strategy (1982): choose $P_1' = P_1$ and set $\beta = [(P_1 - P_1')/P_2]^{1/2}$

	Block 1	Block 2	Block 3	Block 4
$\underline{x}_1$	$\underline{x}'_1(w_1)+\beta\,\underline{x}_2(1)$	$\underline{x}'_1(w_2)+\beta\,\underline{x}_2(w_1)$	$\underline{x}'_1(w_3)+\beta\,\underline{x}_2(w_2)$	$\underline{x}'_1(1)+\beta\,\underline{x}_2(w_3)$
$\underline{x}_2$	$\underline{x}_2(1)$	$\underline{x}_2(w_1)$	$\underline{x}_2(w_2)$	$\underline{x}_2(w_3)$

- Relay: $R < I(X_1;Y_2|X_2) = \log(1+P_1')$
- Dest: $R < I(X_1X_2;Y_3) = \log(1+P_1'+(1+\beta)^2 P_2)$
- 3 additions to basic multi-hopping: Tx at same time, coherent combining (sync!), Rx with all information

Fading Channels (Random Phases)

- Phase sync. is often not possible and ß=0 is best

	Block 1	Block 2	Block 3	Block 4
$\underline{x}_1$	$\underline{x}_1(w_1)$	$\underline{x}_1(w_2)$	$\underline{x}_1(w_3)$	$\underline{x}_1(1)$
$\underline{x}_2$	$\underline{x}_2(1)$	$\underline{x}_2(w_1)$	$\underline{x}_2(w_2)$	$\underline{x}_2(w_3)$

- A recent result (KGG, 2003): DF achieves capacity if the relay is near, but not necessarily co-located with, the source (for the full-duplex case)

Half-Duplex Devices

- A natural approach (not DF in a strict sense):

	Block 1	Block 2	Block 3	Block 4
$\underline{x}_1$	$\underline{x}_1(w_1)$	$\underline{x}_1(w_2)$	$\underline{x}_1(w_3)$	$\underline{x}_1(w_4)$
$\underline{x}_2$	$\underline{0}$	$\underline{x}_2(w_1)$	$\underline{0}$	$\underline{x}_2(w_3)$

- But we can do better: we can modulate the listen/talk times. Let $M_2=0$ or 1 if the relay listens or talks, resp. We can achieve:

$$R = \min [\ I(X_1;Y_2|X_2M_2),\ I(X_1X_2M_2;Y_3)\ ]$$

$$= \min [\ I(X_1;Y_2|X_2M_2),\ I(M_2;Y_3) + I(X_1X_2;Y_3|M_2)\ ]$$

Timing Modulation

- The above explains why our wireline rate improved!
  - In that example, we had M_2=X_2 and

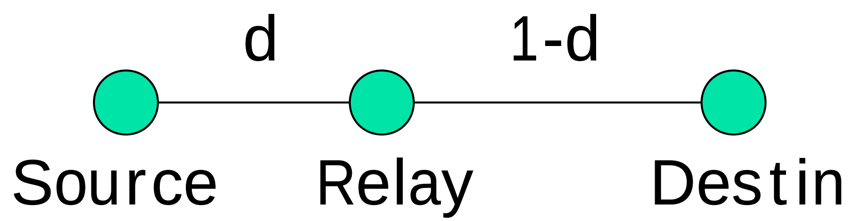
    $I(X_1;Y_2|X_2M_2) = I(M_2;Y_3) = 0.773$

    $I(X_1X_2;Y_3|M_2) = 0$

- Notes:
  - Recent result (K, 2004): DF with timing modulation achieves capacity if the relay is near, but not nec. co-located with, the source (and if duplex ratio/mode power/mod. is limited)
  - Timing modulation improves AF, DF, CF rates
  - If one cannot modulate the timing every symbol, then (d,k) constraints become interesting

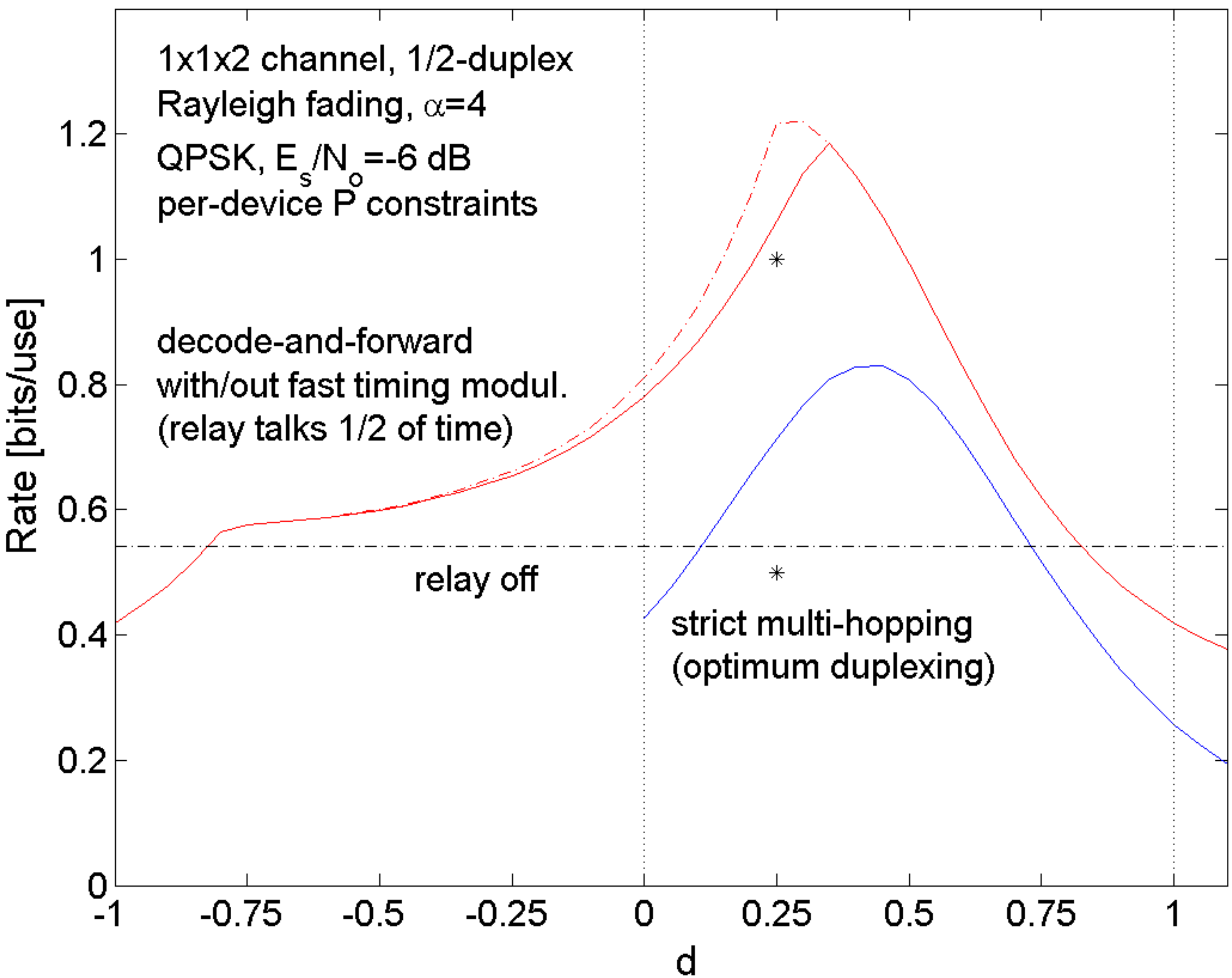
A Geometric Example

- Geometry:



- Parameters:
  - Source and relay have 1 antenna, destin. has 2 antennas
  - half-duplex relay
  - Rayleigh fading, link gains known at the receivers only
  - Attenuation exponent a=4, QPSK symbols
  - E_S/N_O=-6 dB, per-symbol & device power constraints, $P_1=P_2$
  - Fast timing modulation with $\Pr[M_2=0] = \Pr[M_2=1] = 1/2$

Achievable Rates for a Half-Duplex Relay Channel

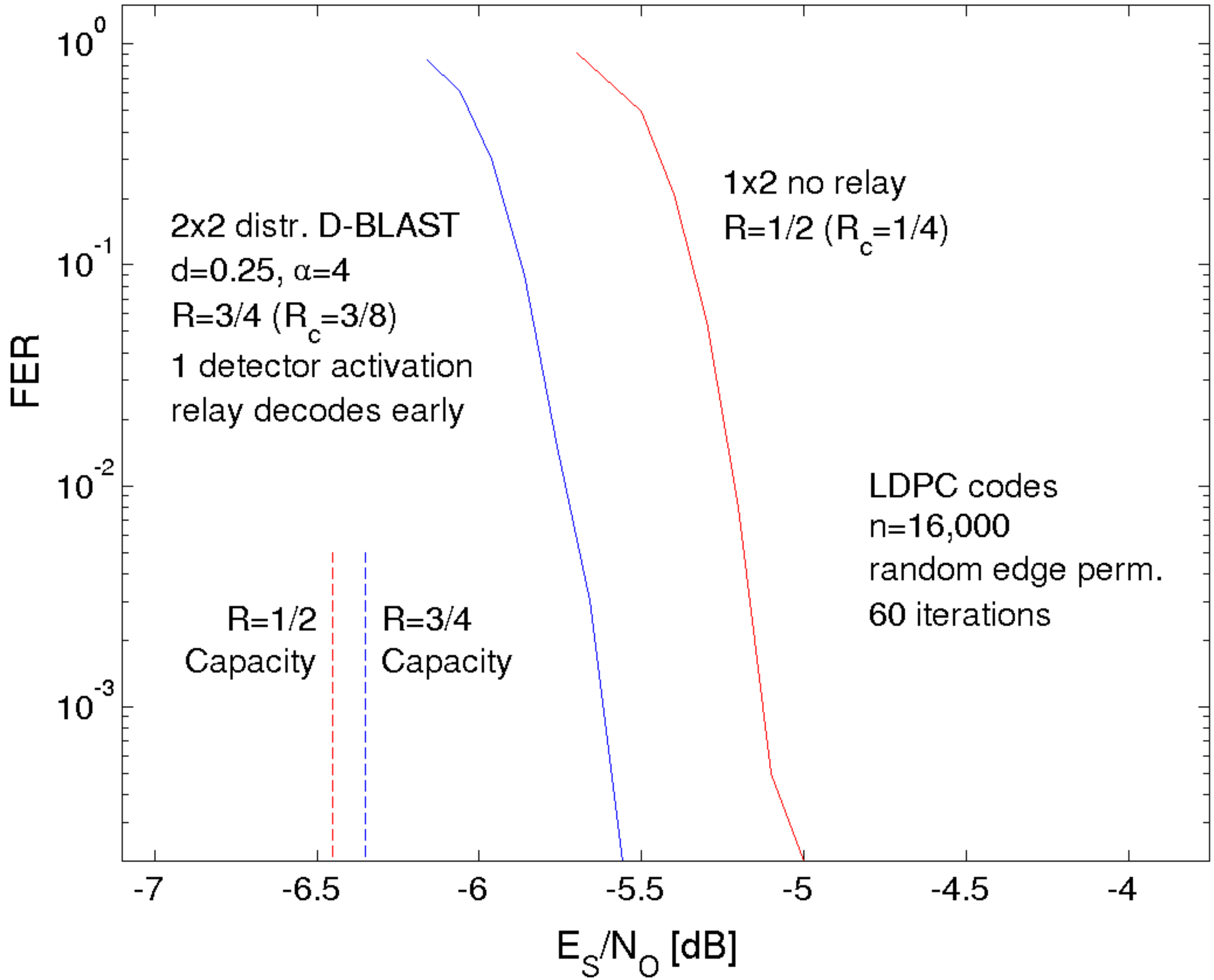


Note:
- DF with timing mod. and optim. duplexing gives capacity if d is near zero

Code Design Example

- Consider the above geometry with d=0.25
- Go for R=1/2 without relay and R=1 with relay
- Use two LDPC codes designed for AWGN channels. Length n=16,000 and design rates
  - R_c=1/4 for S? D link (code spans two blocks)
    (EXIT threshold E_b/N_O at -0.4 dB, capacity at -0.72 dB)
  - R_c=3/8 for S? D and SR? D links
    (EXIT threshold E_b/N_O at 0.1 dB, capacity at -0.35 dB)
- Receivers: 60 iterations
- Relay decodes after 4000 symbols (low error rate)
- Expect similar error rates for other two decoders

Frame Error Rates for a Half-Duplex Relay Channel with Rayleigh Fading



Notes:
- within 1.3 dB of capacity at FER of 10^{-3}
- get closer by making n larger

Comparisons

- Some advantages over
  - multi-hopping
  - distributed space-time codes (various papers, 2001-present)
  - distributed V-BLAST (Augustín et al, Barbarossa et al (2004))

1) One can approach capacity (and outage capacity)

2) One has (almost) flat detector EXIT curves.

Summary

- Information-theoretic models and insights:
  - Let one understand basic limitations of wireline and wireless relaying (in small networks for now) in a unified way
  - Lead to new coding methods (e.g. timing modulation) that improve established methods (e.g., routing, network coding, multi-hopping)
  - Let one put some important methods for relaying (DF) and multi-antenna transmission in a common framework
  - Lead to new relaying methods that can approach capacity