Spectrum Sharing Games

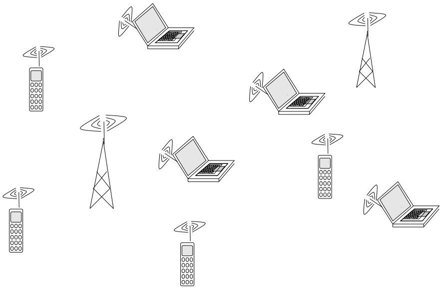
Randall Berry

Northwestern University
Deptartment of ECE

Joint work with J. Huang and M. Honig

IMA, June 29, 2005

Wireless Networks



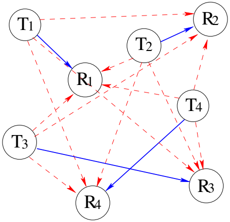
- Increasing number of wireless technologies:
  - Cellular, wireless LANs, wireless MANs, sensor networks, etc.
- Need to share limited wireless spectrum.
  - Many new applications use unlicensed bands (e.g. 802.11).
  - Interest at FCC in developing more *flexible* spectrum usage models (e.g. open access, secondary markets, etc.)

Spectrum Sharing

Key issue: *How to efficiently share spectrum.*

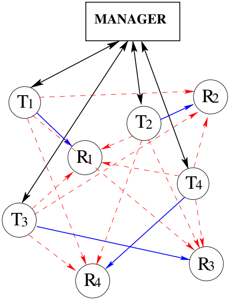
- Traditional (cellular) approach - use re-use/channel assignment/power control.
- Works well when spectrum licensed to single provider.
- In unlicensed/open spectrum can be more difficult.

Spectrum Sharing



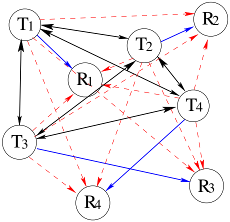
- Focus on sharing a *single* band of spectrum in a given geographic area among competing *users*.
- Each user is a single transmitter/receiver pair.
- Want to utilize spectrum *efficiently* and *fairly* with limited information exchange (*scalability*).

Two Approaches:



Managed Sharing:

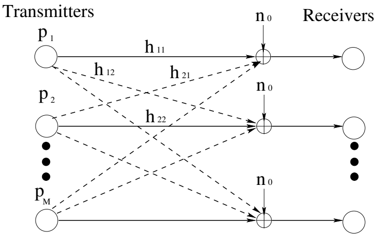
- Manager determines spectrum usage.



Unmanaged Sharing:

- Distributed algorithm determines usage.

Basic Channel Model

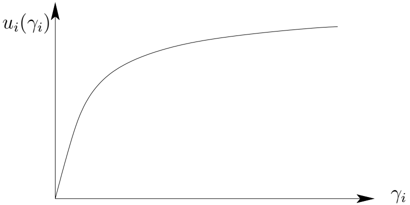


- Static channels.
- Each user spreads signal over entire bandwidth of BHz.
- User i's QoS depends on received SINR,

$$\gamma_i = \frac{h_{ii} p_i}{n_0 + \frac{1}{B} \sum_{j \neq i} h_{ji} p_j}$$

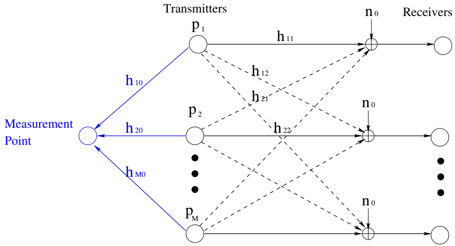
Spectrum sharing = power allocation.

User Preferences



- All users are rate-adaptive with *elastic* demands.
- User's QoS preferences given by utility $u_i(\gamma_i)$.
  - Increasing, twice differentiable, strictly concave function of γ_i.

Managed Spectrum Sharing



- Manager allocates spectrum to users.
  - e.g., secondary market.
- Constraint on total received power at *measurement point* ("interference temperature").

$$\sum_{i=1}^{M} h_{i0} p_i \leq P$$

Spectrum allocation

- Spectrum allocation = allocation of received power at MP.
- Can view this as divisible good.
- Consider auction-based approach:
  - Users bid for power, manager allocates power based on bids.
- Need to specify auction mechanism.
  - Due to interference, negative externalities between users.
  - Complicates some auction mechanisms (e.g. VCG)

Divisible (Share) Auction

- Manager announces reserve bid β and unit price π^x.
- Users submit one dimensional bids, b_i $(i = 1, .., M)$
- Power allocated in proportion to bids:

$$h_{i0} p_i = \frac{b_i}{\beta + \sum_{i=1}^{M} b_i} = \frac{b_i}{\beta + b_i + b_{-i}}$$

- User pays unit price $\times$ allocation (not bid value).
  - Improves efficiency of auction's outcome.

Pricing Schemes

- Two pricing schemes:
  - **SINR-based:** user i pays $C_i = \pi^s \gamma_i$
  - **Power-based:** user i pays $C_i = \pi^p h_{i0} p_i$
- Each user's goal is to maximize surplus,

$$S_i(b_i, b_{-i}) = u_i(\gamma_i) - C_i$$

- View user's a playing non-cooperative game.
  - Players = users (pairs);
  - Player i's action = b_i;
  - Player i's pay-off = $S_i(b_i, b_{-i})$

Definitions

- Given b_{-i}, user i's *best response* is

$$\mathcal{B}_i(b_{-i}) = \arg\max S_i(b_i; b_{-i})$$

- A set of bids $\{b_i^*\}$ is a *Nash equilibrium (N.E.)* if for all i, $b_i^* \in \mathcal{B}_i(b_{-i}^*)$.
  - No user has incentive to deviate unilaterally.
- Want to know does N.E. exist? Is it unique? What properties does it have?
  - First consider one-shot complete information game.

SINR Auction

Prop. In an SINR auction with $\beta > 0$ there exists a threshold price $\pi^s_{th} > 0$ such that a unique NE exists if $\pi^s > \pi^s_{th}$ and no NE if $\pi^s \leq \pi^s_{th}$.

SINR Auction

Prop. In an SINR auction with $\beta > 0$ there exists a threshold price $\pi^s_{th} > 0$ such that a unique NE exists if $\pi^s > \pi^s_{th}$ and no NE if $\pi^s \leq \pi^s_{th}$.

Proof idea:

- Each user has a unique best response that satisfies:

$$\mathcal{B}_i(b_{-i}) = \sum_{j \neq i} k_{ij}(\pi^s) b_j + k_{i0}(\pi^s)\beta.$$

- If auction has a unique N.E. $\mathbf{b}^*$ it must satisfy:

$$(\mathbf{I} - \mathbf{K}(\pi^s))\mathbf{b}^* = \mathbf{k_0}\beta$$

- Requires spectral radius of $\mathbf{K}(\pi^s)$, $\rho_K < 1$.
- Show ρ_K is decreasing in π^s and must be less than 1 for large enough π^s.

Pareato Optimality

- An outcome of the auction is *Pareato optimal* if no user's utility can be improved without decreasing another's.
- Due to reserve bid β, outcome of SINR auction is not Pareato optimal.
- But can be made arbitrarily close, as $\pi^s \rightarrow \pi^s_{th}$.
- Requires manager to know global information - if not can adaptively set π^s.

Iterative bidding

- Suppose each user iteratively updates their bids according to *Myopic best response* updates:

$$\mathbf{b}^{(t)} = \mathbf{K}\mathbf{b}^{(t-1)} + \mathbf{k}_0\beta$$

- For each user i this only depends on h_{ii}, h_{i0} and γ_i^{t-1}.
- **Prop.** If there exists a unique NE in the SINR auction, these updates globally converge to it.

NE with log utilities

- **Logarithmic utilities:** $U_i(\gamma_i) = \theta_i \log(\gamma_i)$.
- **Prop.** If a unique NE exists in an SINR auction with logarithmic utilities, the equilibrium SINR allocation satisfies:

$$\frac{\gamma_i}{\theta_i} = \frac{\gamma_j}{\theta_j}$$

(weighted max-min fair).

Power auction

Assumption A: Assume that for all i,

$$\min_{\gamma_i \in [0, P/n_0]} \frac{|U_i''(\gamma_i)|}{U_i'(\gamma_i)} > 0$$

Assumption B: Assume that for each i, $\frac{h_{i,j}}{h_{i,0}} = K_i$ for all $j \neq i$.

Power auction

Assumption A: Assume that for all i,

$$\min_{\gamma_i \in [0, P/n_0]} \frac{|U_i''(\gamma_i)|}{U_i'(\gamma_i)} > 0$$

Assumption B: Assume that for each i, $\frac{h_{i,j}}{h_{i,0}} = K_i$ for all $j \neq i$.

Prop. In a power-based auction under assumptions A and B with large enough (finite) bandwidth given any $\epsilon > 0$, there exists a price such that the system has a NE with a total utility that is within ϵ of the socially optimum.

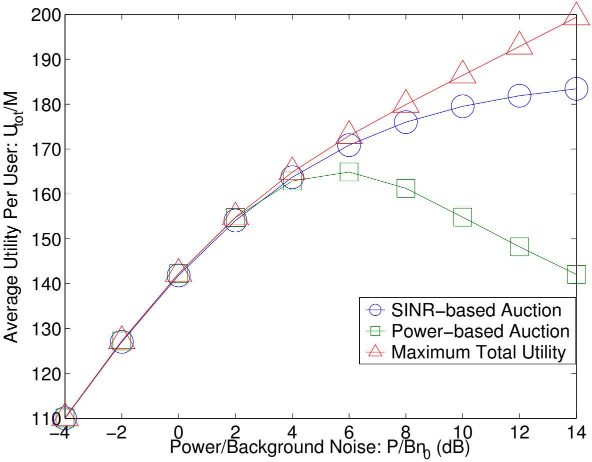
- e.g., all receiver's co-located.

Power Auction

proof idea:

- Can formulate social optimum as solution to problem that is separable across users.
- With large enough bandwidth objective becomes concave.
- Identify needed price with Lagrange multiplier and first order conditions with best responses.

Power Auction



log utilities, co-located receivers, M=10 users.

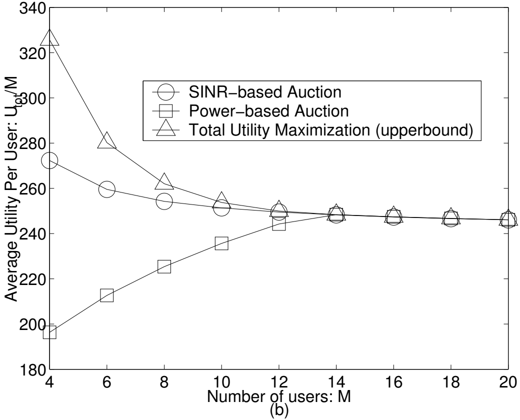
Large System Analysis

- Let P,B and $M \to \infty$ with fixed ratios.
- Assume $U_i(\gamma_i) = \theta_i U(\gamma_i)$ with θ_i i.i.d. and channel gains i.i.d., and $U(\gamma)$ asymptotically sub-linear, i.e.

$$\lim_{\gamma \to \infty} \frac{1}{\gamma} U(\gamma) = 0.$$

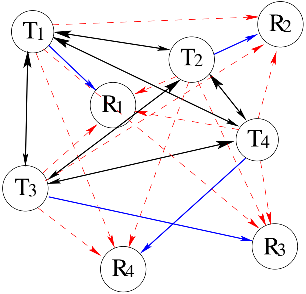
- **Prop.** In limiting system both the SINR-based and power-based auctions have a NE that corresponds to the socially optimal allocation.

Large System Convergence



log utilities, co-located receivers.

Unmanaged Spectrum Sharing



- No manager to allocate power.
- Users must coordinate power allocation with each other.
- Each user has power constraints: $p_i \in [P_i^{min}, P_i^{max}]$.

Asynchronous Distributed Pricing (ADP) Algorithm

- Each user i announces "price" (per unit interference),

$$\pi_i = -\frac{\partial u_i(\gamma_i)}{\partial I_i} = \frac{\partial u_i(\gamma_i)}{\partial \gamma_i} \frac{\gamma_i^2}{p_i h_i i}.$$

- User i updates power p_i to maximize surplus:

$$s_i = u_i(\gamma_i) - \sum_{j \neq i} \pi_j h_{ij} p_i.$$

- Repeat these steps asynchronously.

- Only need to know "adjacent" channel gains (h_{ij}) and announce single price.

ADP Algorithm

Want to know:

- When does this converge?
- If it convergence what is the resulting allocation?
- When is the allocation (socially) optimal, i.e. it maximizes the total utility, $\sum_i u_i(\gamma_i)$?

Convergence

- Define the *coefficient of risk aversion (CRA)* of a utility $U(\gamma)$ to be

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

- larger CRA $\Rightarrow$ "more concave" U.

Convergence

- Define the *coefficient of risk aversion (CRA)* of a utility $U(\gamma)$ to be

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

- larger CRA $\Rightarrow$ "more concave" U.

Prop: If for all i:

a) $P_i^{\min} > 0$, and

b) $CRA(\gamma_i) \in [1, 2]$ for all feasible γ_i;

then there is a unique optimal allocation and the ADP algorithm globally converges to this point.

- e.g. above condition is always satisfied with log utilities.
- proof based on relating this algorithm to a "fictitious game."

Fictitious Game

- Can view ADP algorithm in terms of a "fictitious game" .
- Split each user into two fictitious players
  - one sets prices and one determines power allocations.
- Formulate game so that each players best response corresponds to steps in the ADP algorithm.
  - $\Rightarrow$ NE of this games corresponds to fixed points of the ADP algorithm.
- Any fixed-point also corresponds to a solution to Kuhn-Tucker conditions of global utility maximization problem.
- Additionally with the restrictions on the utility, this game is supermodular.

Supermodular Games

- A class of games with "strategic complementarities."
  - Strategy sets are compact subsets of $\mathbb{R}$; and each users pay-off s_i has "increasing differences":

$$\frac{\partial^2 s_i}{\partial x_i \partial x_j} > 0.$$

- Key properties:

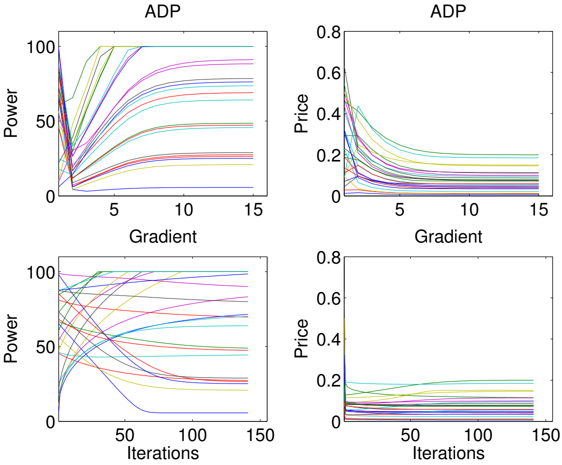
(1) a N.E. exists

(2) if the N.E. is unique then the asynchronous best response updates will globally converge to this point.

Supermodular Games

- A class of games with "strategic complementarities."
  - Strategy sets are compact subsets of $\mathbb{R}$; and each users pay-off s_i has "increasing differences":
$$\frac{\partial^2 s_i}{\partial x_i \partial x_j} > 0.$$
- Key properties:
  (1) a N.E. exists
  (2) if the N.E. is unique then the asynchronous best response updates will globally converge to this point.
- Under given assumptions, can show that with a logarithmic change of variable, the total utility maximization problem has concave objective over convex set.
  - Similar to [Chiang 05].
  - Implies there must be a unique N.E.

Convergence



log utilities

Multi-channel Model

- Assume each user can transmit over K independent channels (e.g. multi-carrier system).
- Received SINR in channel k for user i

$$\gamma_i^k = \frac{h_{ii}^k p_i^k}{n_0^i + \sum_{j \neq i} h_{ji}^k p_j^k}$$

- Can allocate power across channels subject to total power constraint:

$$\sum_k p_i^k \leq P_i^{max}.$$

- User's total utility is "carrier separable"

$$U_i = \sum_k U_i^k(\gamma_i^k).$$

Modified ADP algorithm

- Each user still announce an interference price π_i^k on each channel k.
- Each user also keeps a local "power price," μ_i, to model total power constraint.
- User chooses power p_i^k to maximize:

$$U_i^k(\gamma_i^k) - \sum_{j \neq i} h_{ij} \pi_j^k p_i^k - \mu_i.$$

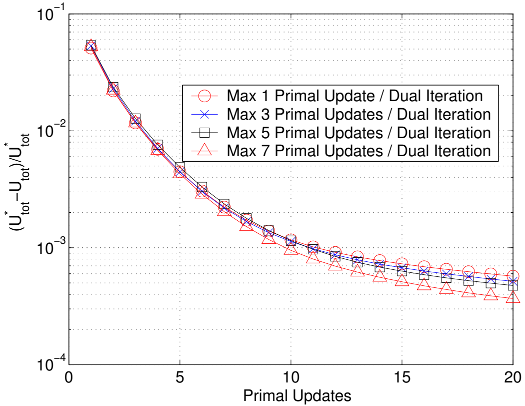
- Power price updated by:

$$\mu_i(t) = \left[\mu_i(t^-) + \kappa \left(\sum_{k \in \mathcal{K}} p_i^k - P_i^{max} \right) \right]^+.$$

Convergence

- Under similar conditions to single channel case, can show this globally converges to unique optimal power allocation.
- Difficulty here is that power price update can not be incorporated into supermodular game framework.
- Instead consider "separation of time-scales" and view power price update as distributed gradient projection algorithm for optimizing dual of utility maximization problem.
- Under given conditions, this converges globally with fixed step-size
  - Need to show Lipschitz condition for gradient of dual function.

Multi-channel Example



log utilities, 16 channels, 50 users

Conclusions

- Presented some simple models for spectrum sharing with and without a manager.
- In each case, analyzed performance, convergence using game theoretic ideas.
- Many issues not addressed e.g. multiple spectrum bands, dynamics, multiple providers, etc.

References

- J. Huang, R. Berry, and M. Honig, "Auction-based Spectrum Sharing," to appear in *ACM/Kluwer MONET* special issue on WiOpt'04.
- J. Huang, R. Berry, and M. Honig, "Distributed Interference Compensation for Wireless Networks," under submission.