Control of Multi-Node Mobile Communications Networks with Time Varying Channels via Stability Methods

Harold J. Kushner
Applied Mathematics Dept.
Brown University
Providence RI

IMA Workshop on Wireless, June 27–July 1, 2005
Unedited

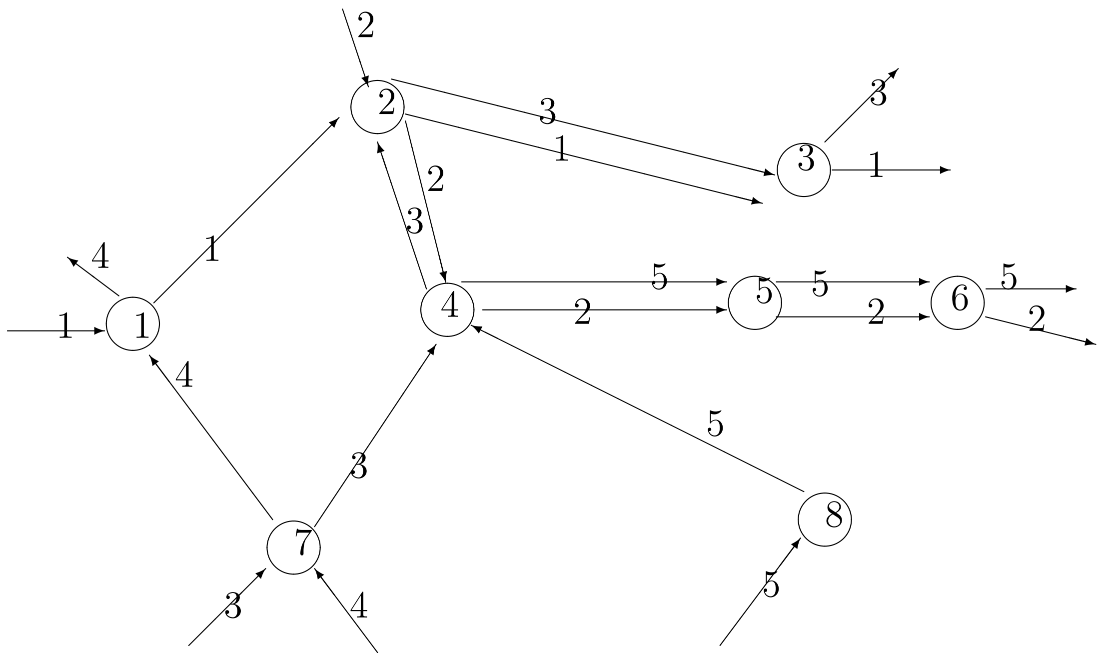

Figure 1: A Simple Network

- Bursty inputs, random channels, possibly rapidly varying.
- Current locations known.
- Unique fixed routing for each source (where enters network), and destination (where leaves network) pair.
- To allocate power, time, bandwidth, center frequency, etc., among the queues in a channel- and queue-state-dependent way.
- Scheduling intervals (20 ms or so), channel known at beginning (pilot signals).
- Decisions at the beginning of the intervals (slots)
- Ignore interuser interference, say wide band, CDMA, with orthogonal spreading sequences
- Any number of receptions, receive/transmit simult.
- To assure stability.
- Extensions include requiring acks. multicasting, non-unique routes, randomly changing routes, etc.
- A continuation of Buche-Kushner, IEEE TAC, 2004.

Stability: Definitions

Owing to the non-Markovianess of the channel and arrival and data processes, an appropriate definition of stability is a "uniform mean recurrence time" property.

Let n denote the beginning of the $(n+1)$st interval (slot). Let E_n denote expectation, given $\mathcal{F}_n$, the data to slot n, and $X(n) = \{X_{i,k}(n)\}$ the queue length vector. If node k is not on the path for source i, then $X_{i,k} = 0$.

Suppose that there are $0 < q_0 < \infty$ and real-valued $F(\cdot) \geq 0$ such that the following holds: For any n, and

$$\sigma_1 = \min\{k \geq n : |X(k)| \leq q_0\}.$$

we have

$$E_n\left[\sigma_1 - n\right] \leq F(X(n)) I_{\{|X(n)| \geq q_0\}}. \tag{2.1}$$

Then the system is said to be stable.

The definition implies "uniform" recurrence to some compact set.
If $|X(n)|$ reaches $q_1 > q_0$, then the conditional expectation of the time required to return to a value q_0 or smaller is bounded by a function of q_1, uniformly in the past history and in n.

This implies that the sequence $\{X(n)\}$ is tight or bounded in probability (see, for example, Kushner, 1984 book, Theorem 2, Chapter 6).

The proof is about the simplest possible.

Comment. An alternative stability condition requires only that return times be finite w.p.1, and that $X(n)/n$ be bounded. Where this is used, it is driven more by technical than practical reasons. Does not assure finite transit times.

The Decision Rule: Definitions

Let $f(i,k)$ = node that queue (i,k) feeds to after leaving node k.
If source i immediately goes to its final destination after leaving node k, then $f(i,k)$ is ignored.

Let $b(i,k)$ = node that queue (i,k) is fed from.
If node k is the origin node for source i, then terms involving $b(k,i)$ are ignored.

Let $d_{i,k}(n)$ = number of packets sent from queue i of node k at time n.
It will be depend on the channel state at that time and be a function of the allocated resources (e.g., power, frequency, bandwidth).
It is zero if node i is not on the path for source k.

Let $a_{i,k}(n)$ = actual random number of arrivals from the exterior, if any, from source i at node k. These will be non-zero only for the unique node $k(i)$ at which source i enters the network.

Following the idea in classical stability-control theory, the idea is to choose the $d_{i,k}(n)$ that attains

$$\min_{\{d_{i,k};i,k\}} [E_n(V(X(n+1)) - V(X(n))].$$

as well as possible.
We use

$$V(X) = \sum_{i,k} w_{i,k} X_{i,k}^p, \quad p \geq 2.$$

The powers could depend on i,k.

Appropriate strictly convex utility functions can be used instead.

The Decision Rule

To motivate the form of the decision rule, evaluate $E_n V(X(n+1)) - V(X(n))$.

We have

$$w_{i,k}\left[E_n X^p_{i,k}(n+1) - X^p_{i,k}(n+1)\right]$$
$$= w_{i,k} X^{p-1}_{i,k}(n)\left[-d_{i,k}(n) + E_n a_{i,k}(n) + d_{i,b(i,k)}(n)\right] .$$
$$+ \text{ terms of order } (p-2) \text{ in } X_{i,k}(n).$$

Summing over i, k yields, modulo terms of order $(p-2)$ in $X(n)$ and the "arrival" terms,

$$-\sum_{i,k}\left[w_{i,k} X^{p-1}_{i,k}(n) - w_{i,f(i,k)} X^{p-1}_{i,f(i,k)}(n)\right] d_{i,k}(n) \tag{2.2}$$

or, equivalently,

$$-\sum_{i,k} w_{i,k} X^{p-1}_{i,k}(n)\left[d_{i,k}(n) - d_{i,b(i,k)}(n)\right]. \tag{2.3}$$

The lower order terms in $X_{i,k}(n)$ are nonlinear functions of $d_{i,k}(n)$ and conditional moments of the number of arrivals, and would be much too hard to deal with.

It turns out, as in Buche-Kushner-IEEE-TAC-2004, that is is enough to work with the term that is first order in the decisions, which are just the terms in (2.2) and (2.3).

The Decision Rule: Continued

If the decisions are made independently at each node k, then maximize in

$$\max_{\{d_{i,k}(n):i\}} \sum_i \left[w_{i,k} X_{i,k}^{p-1}(n) - w_{i,f(i,k)} X_{i,f(i,k)}^{p-1}(n) \right] d_{i,k}(n) \tag{2.4}$$

for each k, subject to the constraints.

If there are constraints that involve coordinated decisions at a set of nodes, then the decisions for such nodes must be made together, and the decision rule is a maximizer in

$$\max_{\{d_{i,k}:i,k\}} \sum_{i,k} \left[w_{i,k} X_{i,k}^{p-1}(n) - w_{i,f(i,k)} X_{i,f(i,k)}^{p-1}(n) \right] d_{i,k}(n), \tag{2.5}$$

subject to the constraints.

By rearranging terms, this can be written as

$$\max_{\{d_{i,k}:i,k\}} \sum_{i,k} w_{i,k} X_{i,k}^{p-1}(n) \left[d_{i,k}(n) - d_{i,b(i,k)}(n) \right]. \tag{2.6}$$

Since $X(n)$ is not Markovian, classical stochastic stability theory cannot be used directly.

However the perturbed Liapunov function method [Kushner, 1984 book, SA and HT books] allows us to show, under very reasonable conditions, that maximizing (2.3), (2.4), or (2.5) yields a stable system.

Assumptions

A2.1. The $d_{i,k}(n)$ depend on the allocated resources which are denoted by $u_{i,k}(j,X)$. There are functions $g_{i,k}(j,X)$ such that

$$d_{i,k}(n) = g_{i,k}(L_k(n), X(n), u_{i,k}(L_k(n), X(n))).$$

It is always assumed that the maximizing $d_{i,k}$ exist and are Borel functions of the $X(n), L_k(n), u_{i,k}, (L_k(n), X(n)), i, k$.

A2.2. There is a constant K_1 such that $E_n|a_i(n)|^p \le K_1$.
There are $\bar{\lambda}^a_{i,k}$ such that the sums

$$\delta V^a_{i,k}(n) = \sum_{l=n}^{v} \left[E_n \alpha_{i,k}(l) - \bar{\lambda}^a_{i,k}\right], \quad i = 1, \ldots,$$

converge as $v \to \infty$, uniformly in n, ω.

The $\bar{\lambda}^a_{i,k}$, which we call the mean external data arrival rate for source i at node k, is zero if node k is not the source node for source i.

Define $\bar{\lambda}^a_i = \bar{\lambda}^a_{i,k(i)} =$"mean input rate" for source i (packets/slot).

A2.3. For each node k there are $\Pi_{k,j} \ge 0$ such that $\sum_j \Pi_{k,j} = 1$ and

$$\sum_{l=n}^{v} \left[E_n I_{\{L_k(l)=j\}} - \Pi_{k,j}\right]$$

converges as $v \to \infty$, uniformly in n, ω.

A2.4. Define

$$K_0 = \max_{i,k,j,u,X} g_{i,k}(j, X_{i,k}, u_{i,k}(j,X)).$$

There is a control $\{\tilde{u}_{i,k}\}$ such that the following holds. There are possible outputs $\{\tilde{q}^j_{i,k}\}$ such that if $X_{i,k}(n) \geq K_0$, then the number of packets sent from (i,k) per slot, under channel state j, is $\tilde{q}^j_{i,k}$, and where

$$\bar{q}_{i,k} = \sum_j \tilde{q}^j_{i,k} \Pi_{k,j} > \bar{\lambda}^a_i. \tag{2.7}$$

If k is the origin node for source i, then define $\bar{q}_{i,b(i,k)} = \bar{\lambda}^a_i$ for all X, u.

If $X_{i,k}(n) < K_0$, then the number of packets sent is no greater than $\tilde{q}^j_{i,k}$.

An alternative formulation, where $k(i)$ =origin node, $c(i)$=destination node.

$$\sum_j \tilde{q}^j_{i,k(i)} \Pi_{k(i),j} > \bar{\lambda}^a_i,$$

$$\sum_j \tilde{q}^j_{i,k} \Pi_{k,j} \leq \sum_j \tilde{q}^j_{i,f(i,k)} \Pi_{f(i,k).j}.$$

(A2.4) implies that, if the queue sizes are $\geq K_0$, then there is $c_0 > 0$ and allowable $\tilde{q}^j_{i,k}$ that can be taken to satisfy

$$\begin{aligned} &\text{Average amount into } (i,k) - \text{average amount out of } (i,k) \\ &= \bar{q}_{i,b(i,k)} - \bar{q}_{i,k} \leq -c_0, \text{ if } X_{i,k} \geq K_0. \end{aligned} \tag{2.7}$$

The Perturbed Liapunov Function Method: Introduction

The basic idea behind the perturbed Liapunov function method can be loosely summarized as follows.

Let $\{x(n)\}$ be a random process and let $V(x) \to \infty$, as $x \to \infty$.

Suppose that

$$E_n V(x(n+1)) - V(x(n)) = c_n,$$

a random quantity.

Suppose that there is a constant $\bar{c} < 0$ such that the sum

$$\delta V_n = \sum_{i=n}^{\infty} E_n[c_i - \bar{c}]$$

is well defined and (a.e.) bounded uniformly in (n, ω).

Define

$$V_n = V(x(n)) + \delta V_n.$$

Then

$$E_n \delta V_{n+1} - \delta V_n = -(c_n - \bar{c})$$

and

$$E_n V_{n+1} - V_n = c_n - [c_n - \bar{c}] = \bar{c} < 0.$$

Thus, the use of the perturbation allows the replacement of the random c_n by a "mean."

This general method is a development of this idea and is a powerful tool for treating non-Markovian systems.

See Kushner, 1984 book and SA and HT books.

The Perturbed Liapunov Function Method, Continued.

Evaluate $E_n V(X(n+1)) - V(X(n))$.

This will contain terms depending on the random arrivals, and random channel states.

Then add terms $\delta V(n)$ which are small relative to $V(X(n))$, . but such that in

$$E_n[V(X(n+1)) - V(X(n))] + E_n[\delta V(n+1) - \delta V(n)]$$

the "bad" terms are cancelled and replaced by "averages," modulo terms that are suitably dominated.

Liapunov Function Perturbations and Proof

The perturbations for the arrival terms are

$$\delta V^{a}_{i,k}(n) = w_{i,k} X^{p-1}_{i,k}(n) \sum_{l=n}^{\infty} E_n \left[a_{i,k}(l) - \bar{\lambda}^{a}_{i,k} \right]. \tag{3.1}$$

This is zero if k is not the entry node for source i.

The perturbation $\delta V^{d,+}_{i,k,j}(n)$ is concerned with the effects of the departure of packets from a queue (i,k) on the value of $X_{i,k}$, when the channel state at node k is j, and under the "reference" controls $\tilde{u}_{i,k}$.

The perturbation $\delta V^{d,-}_{i,k,j}(n)$ is concerned with the effects on $X_{i,k}$ of the inputs to (i,k) from the queue and link leading to it,

Keep in mind that the channel state j is a vector, denoting the state of the set of all channels from the relevant node k.

$$\begin{aligned} \delta V^{d,+}_{i,k,j}(n) &= -w_{i,k} X^{p-1}_{i,k}(n) \tilde{q}^{j}_{i,k} \sum_{l=n}^{\infty} E_n \left[I_{\{L_k(l)=j\}} - \Pi_{k,j} \right], \\ \delta V^{d,-}_{i,k,j}(n) &= \ w_{i,k} X^{p-1}_{i,k}(n) \tilde{q}^{j}_{b(i,k),k} \sum_{l=n}^{\infty} E_n \left[I_{\{L_{b(i,k)}(l)=j\}} - \Pi_{b(i,k),j} \right]. \end{aligned} \tag{3.2}$$

Finally, define the Liapunov function perturbation $\delta V(n)$ and the full time-dependent Liapunov function $\tilde{V}(n)$ as

$$\begin{aligned} \delta V(n) &= \sum_{i,k} \delta V^{a}_{i,k}(n) + \sum_{i,k,j,\pm} \delta V^{d,\pm}_{i,k,j}(n), \\ \tilde{V}(n) &= V(X(n)) + \delta V(n). \end{aligned} \tag{3.3}$$

Theorem 3.1. *Under* (A2.1)–(A2.4) *the system is stable.*

Proof. We need to show $\tilde{V}(n)$ has the semimartingale property for large queue state values: i.e, that there is $c < 0$ such that

$$E_n \tilde{V}(n+1) - \tilde{V}(n) \leq -c$$

when $|X(n)|$ is large enough, and then that this inequality together with the bounds on the perturbations implies

$$E_n \left[\sigma_1 - n\right] \leq F(X(n)) I_{\{|X(n)| \geq q_0\}}.$$

The first step is to evaluate

$$\begin{aligned} E_n \tilde{V}(n+1) - \tilde{V}(n) \\ &= \sum_{i,k} w_{i,k} E_n \left[X_{i,k}^{p-1}(n+1) - X_{i,k}^{p-1}(n)\right] \\ &\quad + \sum_{i,k,j,\pm} E_n \left[\delta V_{i,k,j}^{d,\pm}(n+1) - \delta V_{i,k,j}^{d,\pm}(n)\right] \\ &\quad + \sum_{i,k} E_n \left[\delta V_{i,k}^{a}(n+1) - \delta V_{i,k}^{a}(n)\right]. \end{aligned}$$

This will be done component by component, and then the results added.

In the course adding the components, many "undesirable" terms will be replaced by averages.
This is the key to the effectiveness of the method.

By "terms of order $(p-2)$" we mean terms that are bounded by

$$K|X(n)|^{p-2} + K$$

for some constant K.

A first order Taylor expansion yields

$$\begin{aligned}\sum_{i,k} w_{i,k} E_n \left[X_{i,k}^{p-1}(n+1) - X_{i,k}^{p-1}(n)\right] \\ = \sum_{i,k} w_{i,k} E_n X_{i,k}^{p-1}(n) \left[a_{i,k}(n) - d_{i,k}(n) + d_{i,b(i,k)}(n)\right] \\ + \text{terms of order } (p-2),\end{aligned} \tag{3.4}$$

Now consider the "arrival" component for node (i,k). Recall the definition

$$\delta V_{i,k}^a(n) = w_{i,k} X_{i,k}^{p-1}(n) \sum_{l=n}^{\infty} E_n \left[a_{i,k}(l) - \bar{\lambda}_{i,k}^a\right].$$

If k is the origin node for source i, then a first order expansion yields

$$\begin{aligned}E_n \delta V_{i,k}^a(n+1) - \delta V_{i,k}^a(n) \\ = -w_{i,k} X_{i,k}^{p-1}(n) \left[E_n a_{i,k}(n) - \bar{\lambda}_{i,k}^a\right] + \text{terms of order } (p-2).\end{aligned}$$

Thus,

$$\begin{aligned}\sum_{i,k} E_n \left[\delta V_{i,k}^a(n+1) - \delta V_{i,k}^a(n)\right] \\ = -\sum_{i,k} w_{i,k} X_{i,k}^{p-1}(n) \left[E_n a_{i,k}(n) - \bar{\lambda}_{i,k}^a\right] + \text{terms of order } (p-2).\end{aligned} \tag{3.5}$$

Note that by adding (3.4) and (3.5), the $w_{i,k} X_{i,k}^{p-1}(n) a_{i,k}(n)$ terms are cancelled, and the mean value term $w_{i,k} X_{i,k}^{p-1}(n) \bar{\lambda}_{i,k}^a$ term, and an "error" term of order $p-2$ appears.

The error term will be dominated by the terms of order $p-1$ for large values of the queue state.

The replacement of the random arrival term by its mean value is crucial and was the main motivation for the form of the perturbation (3.1).

Now deal with the first "departure" perturbation. This will eventually help to "average" the random $d_{i,k}(n)$ term.

$$E_n\left[\delta V^{d,+}_{i,k,j}(n+1) - \delta V^{d,+}_{i,k,j}(n)\right] =$$
$$-w_{i,k}E_nX^{p-1}_{i,k}(n+1)\tilde{q}^j_{i,k}\sum_{l=n+1}^{\infty}E_{n+1}\left[I_{\{L_k(l)=j\}} - \Pi_{k,j}\right] \tag{3.6}$$
$$+w_{i,k}X^{p-1}_{i,k}(n)\tilde{q}^j_{i,k}\sum_{l=n}^{\infty}E_n\left[I_{\{L_k(l)=j\}} - \Pi_{k,j}\right].$$

By splitting off the lowest summand from the last sum, we get

$$w_{i,k}X^{p-1}_{i,k}(n)\tilde{q}^j_{i,k}\left[I_{\{L_k(n)=j\}} - \Pi_{k,j}\right]$$
$$-w_{i,k}E_nX^{p-1}_{i,k}(n+1)\tilde{q}^j_{i,k}\sum_{l=n+1}^{\infty}E_{n+1}\left[I_{\{L_k(l)=j\}} - \Pi_{k,j}\right] \tag{3.7}$$
$$+w_{i,k}X^{p-1}_{i,k}(n)\tilde{q}^j_{i,k}\sum_{l=n+1}^{\infty}E_n\left[I_{\{L_k(l)=j\}} - \Pi_{k,j}\right].$$

Simplify as

$$E_n\left[\delta V^{d,+}_{i,k,j}(n+1) - \delta V^{d,+}_{i,k,j}(n)\right]$$
$$= w_{i,k}X^{p-1}_{i,k}(n)\tilde{q}^j_{i,k}\left[I_{\{L_k(n)=j\}} - \Pi_{k,j}\right] + \text{ terms of order } (p-2). \tag{3.8}$$

Analogously, one can show that for $X_{i,b(i,k)}(n) \geq 2K_0$,

$$E_n\left[\delta V^{d,-}_{i,k,j}(n+1) - \delta V^{d,-}_{i,k,j}(n)\right] =$$
$$-w_{i,k}X^{p-1}_{i,k}(n)\tilde{q}^j_{i,b(i,k)}\left[I_{\{L_{b(i,k)}(n)=j\}} - \Pi_{b(i,k),j}\right] + \text{ terms of order } (p-2). \tag{3.9}$$

Adding all terms in (3.4), (3.5), (3.8) and (3.9), and cancelling where possible. yields

$$
\begin{aligned}
&E_n\tilde{V}(n+1)-\tilde{V}(n)=\sum_{i,k} w_{i,k}X_{i,k}^{p-1}(n)\bar{\lambda}_{i,k}^{a}\\
&+\sum_{i,k}\left[-w_{i,k}X_{i,k}^{p-1}(n)d_{i,k}(n)+w_{i,k}X_{i,k}^{p-1}(n)d_{i,b(i,k)}(n)\right]\\
&+\sum_{i,k,j} w_{i,k}X_{i,k}^{p-1}(n)\tilde{q}_{i,k}^{j}\left[I_{\{L_k(n)=j\}}-\Pi_{k,j}\right]\\
&-\sum_{i,k,j} w_{i,k}X_{i,k}^{p-1}(n)\tilde{q}_{i,b(i,k)}^{j}\left[I_{\{L_{b(i,k)}(n)=j\}}-\Pi_{b(i,k),j}\right]\\
&+\text{ terms of order } (p-2).
\end{aligned}
\tag{3.10}
$$

Until further notice, let us suppose that all $X_{i,k}(n)\geq K_0$ for all sources i that use node k, and all k.

Then since, for each k, the $d_{i,k}(n)$ are chosen by the maximization rule and the $\tilde{q}_{i,k}^{j}$ are not necessarily maximizers, the sum

$$
\begin{aligned}
&-\sum_{i,k} w_{i,k}X_{i,k}^{p-1}(n)\left[d_{i,k}(n)-d_{i,b(i,k)}(n)\right]\\
&+\Big\{\sum_{i,k} w_{i,k}X_{i,k}^{p-1}(n)\sum_{j}\left[\tilde{q}_{i,k}^{j}I_{\{L_k(n)=j\}}\right]-\sum_{i,k} w_{i,k}X_{i,k}^{p-1}(n)\sum_{j}\left[\tilde{q}_{i,b(i,k)}^{j}I_{\{L_{b(i,k)}(n)=j\}}\right]\Big\}
\end{aligned}
\tag{3.11}
$$

is non-positive. Using this fact in (3.10) yields the upper bound:

$$
\sum_{i,k} w_{i,k}X_{i,k}^{p-1}(n)\left[\bar{\lambda}_{i,k}^{a}-\bar{q}_{i,k}+\bar{q}_{i,b(i,k)}\right]+\text{ terms of order } (p-2). \tag{3.12}
$$

By assumption, the terms in the brackets are $\leq -c_0<0$.
Thus we have proved that

$$
E_n\tilde{V}(n+1)-\tilde{V}(n)\leq -c_0\sum_{i,k} w_{i,k}X_{i,k}^{p-1}(n)+O(|X(n)|^{p-2}). \tag{3.13}
$$

$$E_n\tilde{V}(n+1) - \tilde{V}(n) \leq -c_0 \sum_{i,k} w_{i,k} X_{i,k}^{p-1}(n) + O(|X(n)|^{p-2}). \tag{3.13}$$

We also have

$$|\delta V(n)| = O(|X(n)|^{p-1}), \tag{3.14}$$

$$E_n\tilde{V}(n+1) - \tilde{V}(n) \to -\infty, \text{ uniformly in } n \text{ as } X(n) \to \infty. \tag{3.15}$$

By (3.15), there are $c_1 > 0$ and $q_0 > 0$, such that, for $|X(n)| \geq q_0$,

$$E_n\tilde{V}(n+1) - \tilde{V}(n) \leq -c_1. \tag{3.16}$$

Given small $\delta > 0$, (3.14) implies that for q_0 sufficiently large,

$$|V(X(n)) - \tilde{V}(n)| \leq \delta(1 + V(X(n)).$$

Let σ_0 be a stopping time for which $|X(\sigma_0)| = c_2 > q_0$.
Define the stopping time

$$\sigma_1 = \min\{n > \sigma_0 : |X(n)| \leq q_0\}.$$

Then, by (3.16),

$$E_{\sigma_0}\tilde{V}(\sigma_1) - \tilde{V}(\sigma_0) \leq -c_1 E_{\sigma_0}[\sigma_1 - \sigma_0]. \tag{3.17}$$

Using (3.17) and the bound (3.14) on $\tilde{V}(n) - V(X(n))$ to bound $\tilde{V}(\sigma_i) - V(X(\sigma_i))$, $i = 0, 1$, yields

$$E_{\sigma_0}(\sigma_1 - \sigma_0) \leq \frac{2\delta + V(X(\sigma_0))(1+\delta) + \delta E_{\sigma_0} V(X(\sigma_1))}{c_1}$$

which implies that the definition of stability (2.1) holds since

$$V(X(\sigma_1)) \leq \sup_{|x| \leq q_0} V(x).$$

We have neglected to account for the possibility that some component $X_{i,k}(n) < K_0$. The adjustment to the computations is simple to do. The differences are dominated by the effects of the large terms.

2. Acknowledgments of packet receipt required.

We supposed that a lost packet is not retransmitted, a common assumption in ad-hoc networks.

Suppose that lost packets must be retransmitted after a delay.
Owing to the difficulty of analyzing the loss process when it depends strongly on the magnitude of the controlled transmission processes, it is often assumed that losses are largely a consequence of uncontrolled traffic.

Take the following common approach.
Acks of received packets at the destination are sent to the source node.
If an ack for a source i packet originating at node k is not received there within $W_{i,k}$ scheduling intervals, all of the packets sent in the same scheduling interval are retransmitted.

Suppose that the process of non-receipt of an ack is iid and independent of the channel states, decisions, and arrivals.

Define $p_{i,k}$= probability that some transmission in a scheduling interval is not acknowledged.
Let $\zeta_{i,k}(n)$ indicate that the transmissions from (i, k) $W_{i,k}$ intervals ago must be retransmitted.
If (i, k) is an intermediate or terminal node for source i, then $\zeta_{i,k}(n) = p_{i,k} \equiv 0$.

We assumed that acks go to the origin node.
An alternative is for transmissions on all links to be acknowledged. Or for only the lost packets to be retransmitted.Similar development.

The queue dynamics are

$$X_{i,k}(n+1) = X_{i,k}(n) + a_{i,k}(n) - d_{i,k}(n) + d_{i,b(i,k)}(n) + d_{i,k}(n - W_{i,k}))\zeta_{i,k}(n),$$

$$X^p_{i,k}(n+1) = X^{p-1}_{i,k}(n)\left[-d_{i,k}(n) + d_{i,b(i,k)}(n) + d_{i,k}(n - W_{i,k})\zeta_{i,k}(n)\right] + X^{p-1}_{i,k}(n)a_{i,k}(n)$$
$$+\text{terms of order } (p-2).$$

The new consideration is the "return" term.

The additional component of the perturbation is

$$\delta V^W(n) = \sum_{i,k} p_{i,k} w_{i,k} X^{p-1}_{i,k}(n) \sum_{m=n-W_{i,k}}^{n-1} d_{i,k}(l). \tag{4.5}$$

This component will deal with averaging the increase in the source queue due to the losses.

We can write

$$\begin{aligned} &E_n[V(X(n+1)) - V(X(n))] + E_n[\delta V^W(n+1) - \delta V^W(n)] \\ &\quad = \sum_{i,k} w_{i,k} X^{p-1}_{i,k}(n)\left[E_n a_{i,k}(n) - d_{i,k}(n) + d_{i,b(i,k)}(n) + p_{i,k} d_{i,k}(n - W_{i,k})\right] \\ &\qquad + \sum_{i,k} p_{i,k} w_{i,k} X^{p-1}_{i,k}(n)\left[d_{i,k}(n) - d_{i,k}(n - W_{i,k})\right] \\ &\qquad + \text{terms of order } (p-2). \end{aligned} \tag{4.6}$$

The terms with $d_{i,k}(n - W_{i,k})$ in the second and third lines cancel each other.

Recall that, if k is the origin node for source i, then $d_{i,b(i,k)}(n) = 0$.

The appropriate decision rule is, subject to the constraints, and for each k,

$$\max_{\{d_{i,k}(n):i\}} \sum_i \left[w_{i,k} X_{i,k}^{p-1}(n) d_{i,k}(n)(1-p_{i,k}) - w_{i,f(i,k)} X_{i,f(i,k)}^{p-1}(n) d_{i,k}(n) \right].$$

The perturbed Liapunov function is

$$\begin{aligned} \tilde{V}^W(n) = V(X(n)) + \delta V^W(n) + \textstyle\sum_{i,k} \delta V_{i,k}^a(n) \\ + \sum_{i,k,j} (1-p_{i,k}) \delta V_{i,k,j}^{d,+}(n) + \sum_{i,k,j} \delta V_{i,k,j}^{d,-}(n). \end{aligned} \tag{4.7}$$

Then, using previous expansions

$$\begin{aligned} & E_n \tilde{V}^W(n+1) - \tilde{V}^W(n) = \sum_{i,k} w_{i,k} X_{i,k}^{p-1}(n) \bar{\lambda}_i^a \\ & + \sum_{i,k} \left[-(1-p_{i,k}) w_{i,k} X_{i,k}^{p-1}(n) d_{i,k}(n) + w_{i,k} X_{i,k}^{p-1}(n) d_{i,b(i,k)}(n) \right] \\ & + \sum_{i,k,j} (1-p_{i,k}) w_{i,k} X_{i,k}^{p-1}(n) \tilde{q}_{i,k}^j \left[I_{\{L_k(n)=j\}} - \Pi_{k,j} \right] \\ & - \sum_{i,k,j} w_{i,k} X_{i,k}^{p-1}(n) \tilde{q}_{i,b(i,k)} \left[I_{\{L_{b(i,k)}(n)=j\}} - \Pi_{b(i,k),j} \right] \\ & + \text{terms of order } (p-2). \end{aligned}$$

The second line is due to (the non-arrival parts of) the second and third lines of (4.6).
The third line is due to $\delta V_{i,k,j}^{d,+}(n)$.
The fourth line is due to $\delta V_{i,k,j}^{d,-}(n)$.

Assuming that the queues are $\geq K_0$ and dominating terms as before yields the upper bound:

$$\sum_{i,k} w_{i,k} X_{i,k}^{p-1}(n) \left[\bar{\lambda}_i^a - (1 - p_{i,k})\bar{q}_{i,k} + \bar{q}_{i,b(i,k)}\right] \; + \; \text{terms of order } (p-2)..$$

If $p_{i,k} > 0$, then $\bar{q}_{i,b(i,k)} = 0$ and at most one of $\bar{q}_{i,b(i,k)}$ and $\bar{\lambda}_{i,k}^a$ can be non zero for any i, k.

The proof is completed as in the theorem.

The net effect of the loss if packets is that for the same input rates, the mean channel rates along the path for source i must be able to handle a flow that is increased by a factor of $1/(1 - p_{i,k(i)})$, where $k(i)$ is the origin node for source i, hardly surprising.

If the packet loss process for any source i is not i.i.d., but is correlated, then the $\zeta_{i,k}(n)$ are correlated.
We can then use another perturbation to average them.

3. Non-Unique Routes

We have supposed that the route from source to destination is unique.
Extend to the non-unique case.
The differences are largely notational.

If node k is on some route for source i, then packets in queue (i,k) might be sent to and/or received from several different nodes.

The term $d_{i,k}(n)$ previously denoted the amount to be sent from queue i at node k, to the unique upstream node $f(i,k)$.

Replace $d_{i,k}(n)$ and $f(i,k)$ by $d_{i,k,\alpha}(n)$ and $f(i,k,\alpha)$, resp., where α denotes the canonical upstream node and takes values in a set that is known at node i.

Analogously, replace $b(i,k)$ by $b(i,k,\beta)$, where β indexes the possible nodes that can transmit to queue (i,k).

Define $u_{i,k,\alpha}(\cdot), g_{i,k,\alpha}(\cdot), \ldots,$ analogously to the definitions without the α.

The local rule is, subject to the constraints, and for each k,

$$\max_{\{d_{i,k.\alpha}(n);i,\alpha\}} \sum_{i,\alpha} \left[w_{i,k} X_{i,k}^{p-1}(n) - w_{i,f(i,k,\alpha)} X_{i,f(i,k,\alpha)}^{p-1}(n) \right] d_{i,k,\alpha}(n).$$

Condition (A2.4) becomes: There are admissible levels of transmission $\tilde{q}^j_{i,k,\alpha}$ such that, for each i,

$$\sum_{j,\alpha} \tilde{q}^j_{i,k(i),\alpha} \Pi_{k(i),j} > \bar{\lambda}^a_i.$$

For $k \neq k(i)$, average input $\leq$ average output.

4. Multicasting

Suppose that some sources have multiple destinations, with a unique route for each source-destination pair.

Let the route network for each source form a tree, with the source as the root and the final destinations as the end branches.

Suppose that if the tree for some source i branches at node k, then transmissions must be done to all of the branches simultaneously.

Redefine $f(i,k,\gamma)$ to index the forward nodes for queue (i,k), where γ indexes the next nodes.
Then the local decision rule is replaced by

$$\max_{\{d_{i,k}(n):i\}} \sum_{i,\gamma} \left[w_{i,k} X_{i,k}^{p-1}(n) - \sum_{\gamma} w_{i,f(i,k,\gamma)} X_{i,f(i,k,\gamma)}^{p-1}(n) \right] d_{i,k}(n),$$

subject to the constraints at node k.

The non-local rules are modified analogously.

The mean flow condition is changed in an obvious way.

A Priori Routing

Approaches to getting the a priori routes: for example, minimal hop routes, or selecting the next node to be the one closest to the destination.

None account for the random variations in the channels nor are sensitive to the power requirements.

A useful approach for getting the routing and the $\tilde{q}^j_{i,k}$ is based on a type of fluid flow approximation.

We suppose that power only is to be allocated, that any received packet must have a given minimum signal to noise ratio.

We allow that the routing for each source is not necessarily unique.

We need only deal with flows when all the queue levels are large.

The example is intended to be illustrative of the possibilities only.

Let $q^j_{i,k,m}$ =packets/slot to be sent from (i,k) to (i,m) in channel state j.
A packet needs $p^j_{i,k,m}$ units of energy for the min S/N ratio at the receiver.

$$\text{fixed slot duration constraint} \quad \sum_{i,m} q^j_{i,k,m} \le Q_i. \tag{5.1}$$

$$\text{total energy constraint} \quad \sum_{i,m} p^j_{i,k,m} q^j_{i,k,m} \le P_k, \quad \text{each } j,k. \tag{5.2}$$

Possibly $q^j_{i,k,m} > 0$ for more than one m. So use $u_{i,k,m}(j,X)$ and $g_{i,k,m}(j,X,u_{i,k,m})$.

We need to assure that the average output for each non-source/destination node equals the average inputs. Hence

$$\overline{\text{out}} = \sum_{m,j} q^j_{i,k,m} \Pi_{k,j} \ge \sum_{l,j} q^j_{i,l,k} \Pi_{l,j} = \overline{\text{ in }}. \tag{5.3}$$

If node $k(i)$ is the input node for source i, then replace (5.3) by

$$\overline{\text{out}} = \sum_{m,j} q^j_{i,k(i),m} \Pi_{k(i),j} = \bar{\lambda}^a_i + \epsilon. \tag{5.4}$$

The (arbitrarily small) $\epsilon > 0$ assures slight overcapacity so that (A2.4) holds.

If $c(i)$ =destination node for source i, then to assure that all packets end up where they are intended, use

$$\sum_{j,k} q^j_{i,k,c(i)} \Pi_{k,j} \ge \bar{\lambda}^a_i + \epsilon. \tag{5.5}$$

Any flows $q^j_{i,k,m}$ that satisfy the constraints (5.1)–(5.5) will yield an acceptable a priori route.
But it makes sense to select one via an optimization problem.
One reasonable cost criterion is the total average power given by

$$\sum_{i,k,m,j} p^j_{i,k,m} q^j_{i,k,m} \Pi_{k,j}. \tag{5.6}$$

Minimize (5.6), subject to (5.1)–(5.5).

-

Comment. The above approach to getting the a priori routes might yield a multiple routes for some sources.

However, given these routes, the original maximization rules still work.

At any node k, we queue all of the packets for each source i together.

Replace the basic local decision rule by

$$\max_{\{d_{i,k,m}(n):i,m\}} \sum_i \left[w_{i,k} X_{i,k}^{p-1}(n) - w_{i,f(j,i,k,m)} X_{i,f(j,i,k,m)}^{p-1}(n) \right] d_{i,k,m}(n),$$

where for each $j,k,i,\ f(j,i,k,m)$ indexes the links for which $q_{i,k,m}^j > 0$ and $d_{i,k,m}(n)$ is the amount sent to node m.

The proof of Theorem 3.1 requires only a slight modification.

For multicasting, simply use the constraint

$$\sum_{k,j} q_{i,k,c(i)}^j \Pi_{k,j} \geq \bar{\lambda}_i^a + \epsilon. \tag{5.7}$$

for each destination node $c(i)$ for source i.