Scheduling High Speed Data in (Adversarial) Wireless Networks

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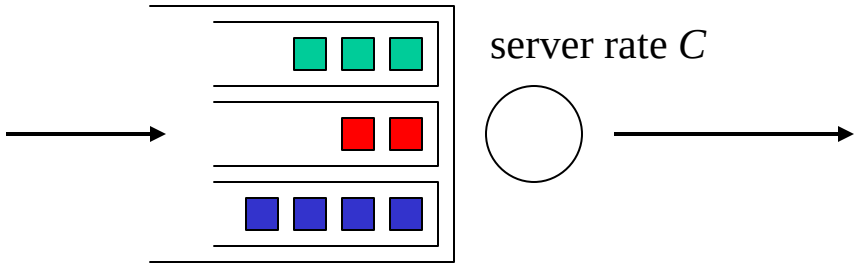
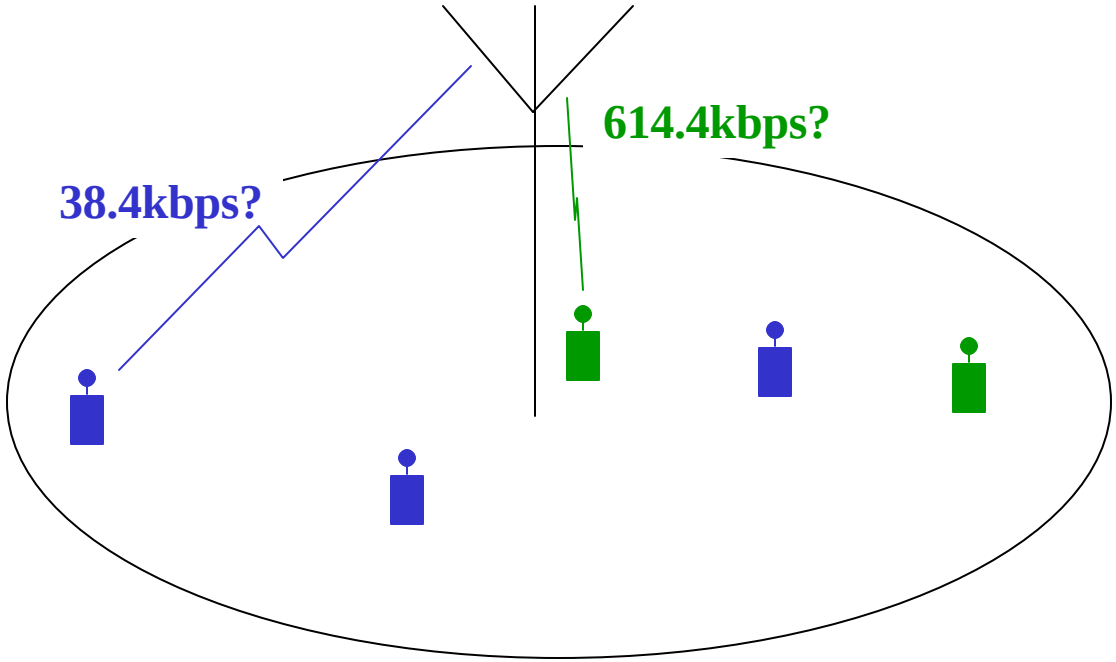
Standard single server scheduling problem



- **Choose queue/user to serve**
  - **Serve at rate *C***
  - **Service rate constant in time & user independent**

- **Many scheduling algorithms**
  - **Weighted Fair Queueing, Earliest Deadline First, First In First Out**

Basic wireless data scheduling problem

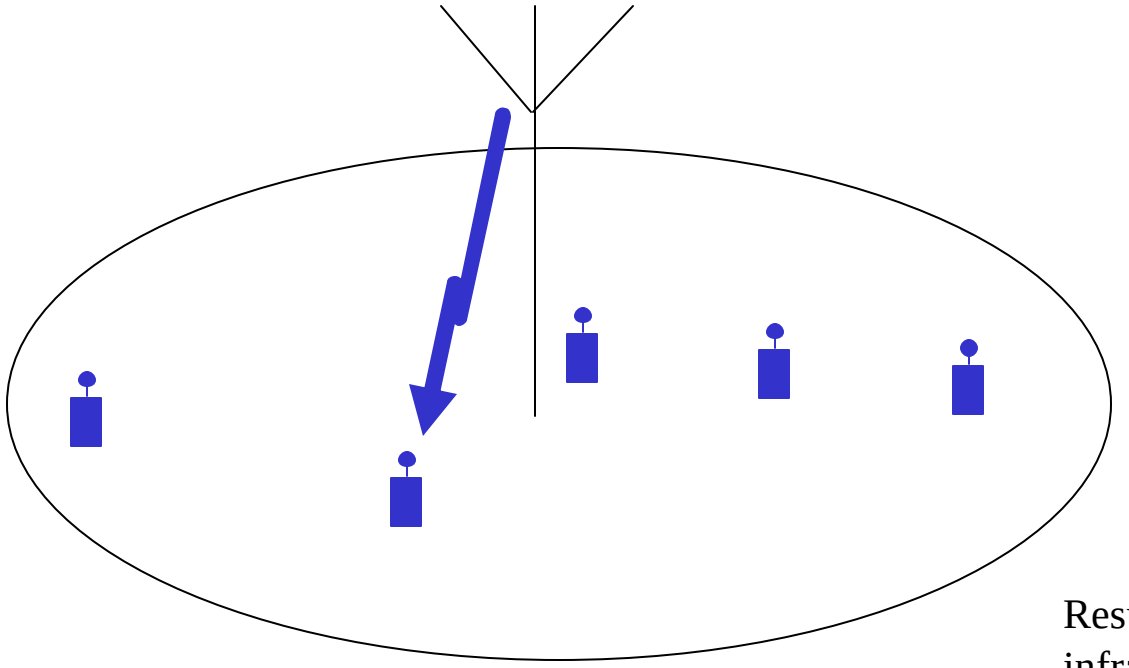


- **Choose queue/user**
  - **Serve distant user at 38.4kbps or close user at 614.4kbps**
  - **Service rates user dependent!!!!**
  - **Service rates change over time!!!!**
    (user mobility, channel fading)

Talk outline

- **Example wireless technology**
  - **CDMA1x-EVDO**
- **Models**
  - **Infinitely backlogged queues vs external arrival process**
  - **Stochastic channels vs adversarial channels**
- **Scheduling results**
  - **How to achieve fairness?**
  - **How to maintain quality-of-service?**
  - **How to keep queues and delays small?**

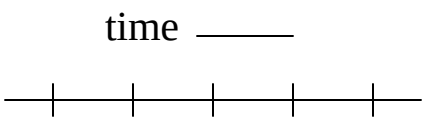
Example Technology: CDMA1x-EVDO (Data Only)



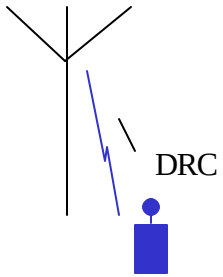
Results apply to most infrastructure based wireless data systems

- **Data only**
  - **One user at a time.**
  - **Chosen user gets all system resources (power etc.)**

EVDO continued



Time divided into 1.667ms slots
(600 slots per second)



• Mobile measures downlink signal-to-noise ratio (SNR)

•Calculates rate at which it can receive data

• Informs basestation in a Data Rate Control msg (DRC)

• Scheduler chooses user based on DRC values

Potential values for *DRC*

0kbps
38.4kbps
76.8kbps
153.6kbps
307.2kbps
614.4kbps
921.6kbps
1.2Mbps
1.8Mbps
2.4Mbps

Some issues:-

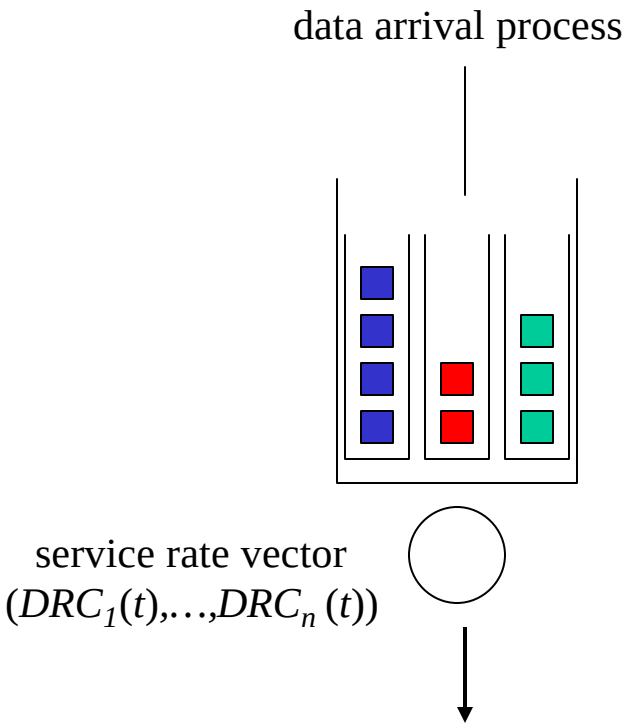
• For low DRC values, must reserve multiple slots

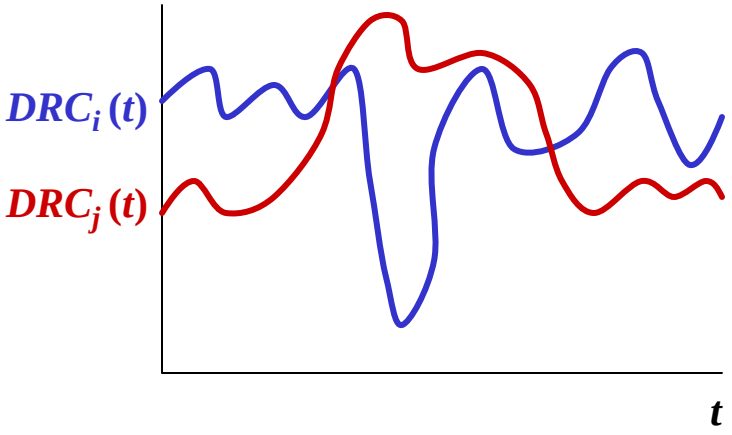
• Actual transmission rate may be higher than DRC (due to lucky decoding)

• EVDO Rev A
  – multiple users per slot
  – multiple flows per user

Wireless models

Wireless data scheduling model (models EV-DO)





- **Choose a queue/user to serve at time *t***
  - **If we choose user *i*, serve at rate $DRC_i(t)$**

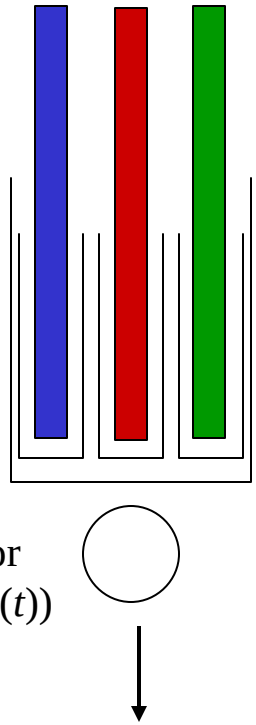
- **Opportunistic scheduling: Serve user when DRC is high**

Scheduling research: should we bother?

- **Wireline: gigantic literature on end-to-end user-level scheduling**
  - **Only the simplest schemes get implemented**
- **Wireless: is it any different?**
  - **Probably…**

Wireline Router	Wireless Basestation
• Deals with thousands of flows	• Deals with tens of flows
• Am I the bottleneck?	• I'm the bottleneck!!
• Easy to add capacity	• Hard to add capacity
• Scheduling interval ~1ms	• Scheduling interval ~1ms

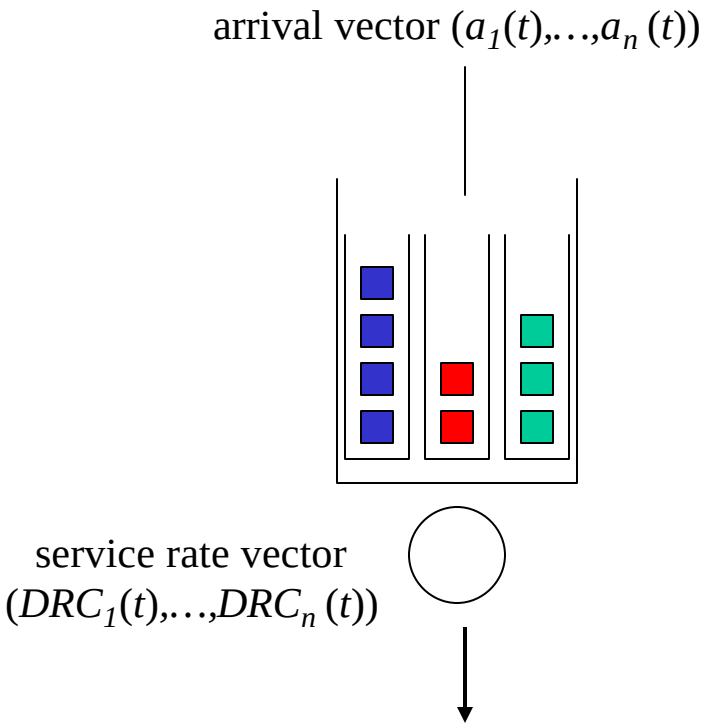
Arrival model 1: Infinitely backlogged queues



service rate vector
($DRC_1(t)$,…,$DRC_n(t)$)

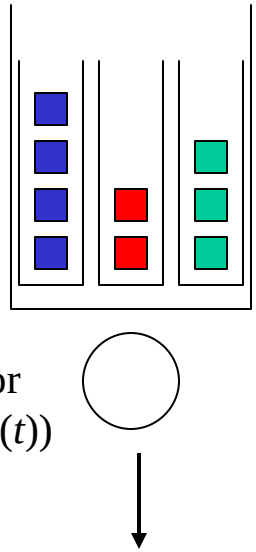
- **Each user always has data to serve**
- **R_i = long-term throughput of user *i***
- **Want to optimize $F(R_1,\ldots,R_n)$**
- **Proportional Fair**
  - **Optimize $\mathrm{S}_i \log R_i$**

Arrival model 2: External arrival process



- **Arrival vector $(a_1(t),\ldots,a_n(t))$**
- **Want stability**
  - **Want bounded queues whenever possible**
- **Want small queues**
- **Want small delays**

Channel model 1: Stationary stochastic process



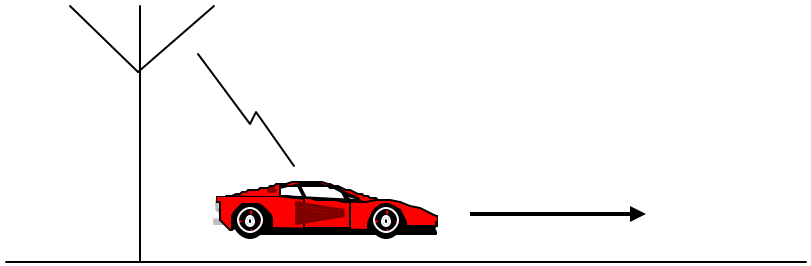
service rate vector
($DRC_1(t),\ldots,DRC_n(t)$)

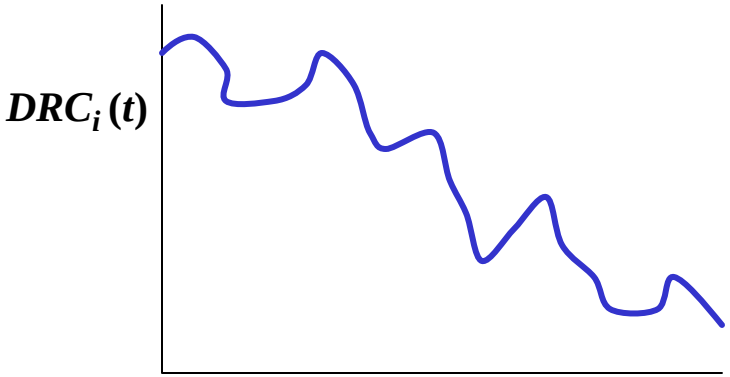
- **Service rate vector determined by state of ergodic Markov chain *M***

- $M(t) \rightarrow (DRC_1(t),\ldots,DRC_n(t))$

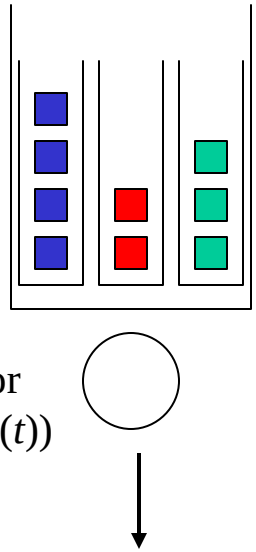
Mobility

- **Mobility destroys stationarity**





Channel model 2: Adversarial process



service rate vector
($DRC_1(t)$,…,$DRC_n(t)$)

- **Service rate vector determined by adversary**
- **Adversary wants scheduler to do bad things**
- **Adversary enables worst-case analysis**

Arrival model 1: Infinitely backlogged queues

Channel model 1: Stationary stochastic process

Proportional Fair Metric

- **Common objective**
  - **R_i = long-run throughput of user *i***
  - **Maximize** $\sum_i \log R_i$
  - **"Doubling throughput of user *i* has same effect on metric as doubling throughput of user *j* "**

Scheduler used in practice…

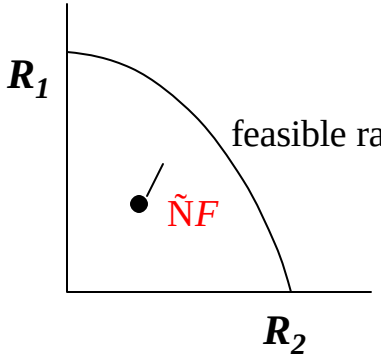
- **EV-DO scheduler - Proportional Fair (Tse)**
  - **$r_i(t)$ = service to user *i* at time *t*, (either 0 or $DRC_i(t)$)**

$$R_i(t+1) = \left(1 - \frac{1}{t}\right) R_i(t) + \frac{r_i(t)}{t} \qquad (\text{typically } t = 1024)$$

$$\text{At time } t \text{ serve } \arg\max_i \frac{DRC_i(t)}{R_i(t)}$$

  - **Prop fair maximizes $\sum_i \log R_i(t)$**

Intuitive analysis for Proportional Fair



Serve $\arg\max_i \frac{DRC_i(t)}{R_i(t)}$

$$F = \sum_i \log R_i(t)$$

$$\nabla F = \left(\frac{1}{R_1(t)}, \ldots, \frac{1}{R_n(t)} \right)$$

$$\text{Change in } F \approx \nabla F \cdot \left(\frac{1}{t}\left(r_1(t), \ldots, r_n(t)\right) - \frac{1}{t}\left(R_1(t), \ldots, R_n(t)\right) \right) = \frac{DRC_i(t)}{tR_i(t)} - \frac{n}{t} \text{ if user } i \text{ served}$$

Prop Fair maximizes change in F

System converges to optimumsolution as $t \to 0$

(Rigorous analysis by Agrawal-Subramanian, Kushner-Whiting, Stolyar)

Drawback with Proportional Fair

- **Maximizing $\sum_i \log R_i$ gives no control to rates of individual users**

- **Sometimes want to provide minimum/maximum rate constraints to individual users**

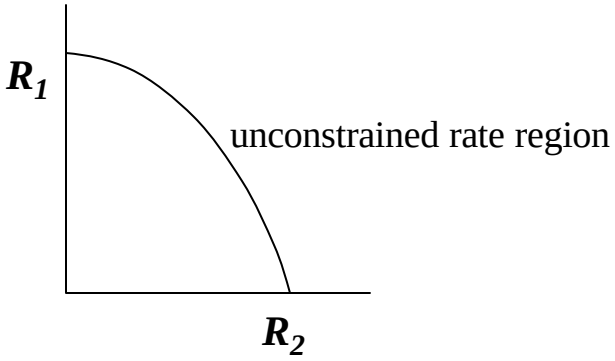
- **Objective becomes**

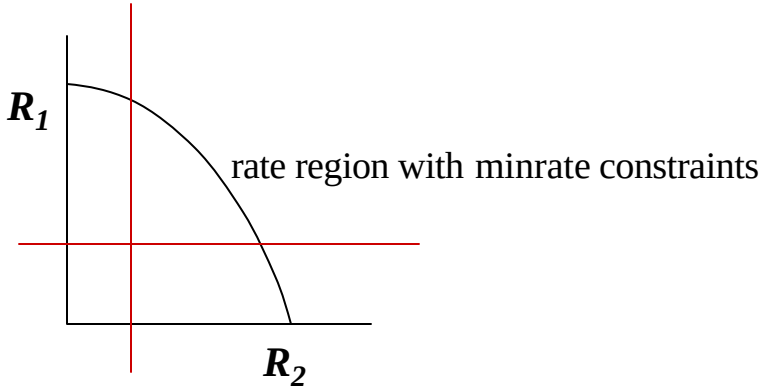
$$\text{Maximize } \sum_i \log R_i$$

Subject to

$$R_i \geq R_i^{min}$$

$$R_i \leq R_i^{max}$$

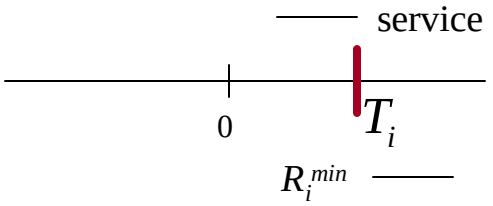




Prop Fair with rate constraints

- **PFMR (Prop Fair with min/max rate constraints)**

$$\text{serve} \quad \arg\max_i \frac{DRC_i(t)}{R_i(t)} e^{a_i T_i(t)}$$



  - Variables and Parameters
    - $R_i(t)$ – average service rate (exponentially filtered)
    - $T_i(t)$ – "token counter"
      - decremented by pckt size whenever pckt served
      - incremented at rate R_i^{min} when $T_i \geq 0$
      - incremented at rate R_i^{max} when $T_i < 0$

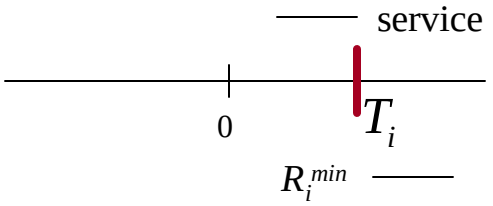
A-Qian-Stolyar

Similar objectives considered by Liu-Chong-Shroff

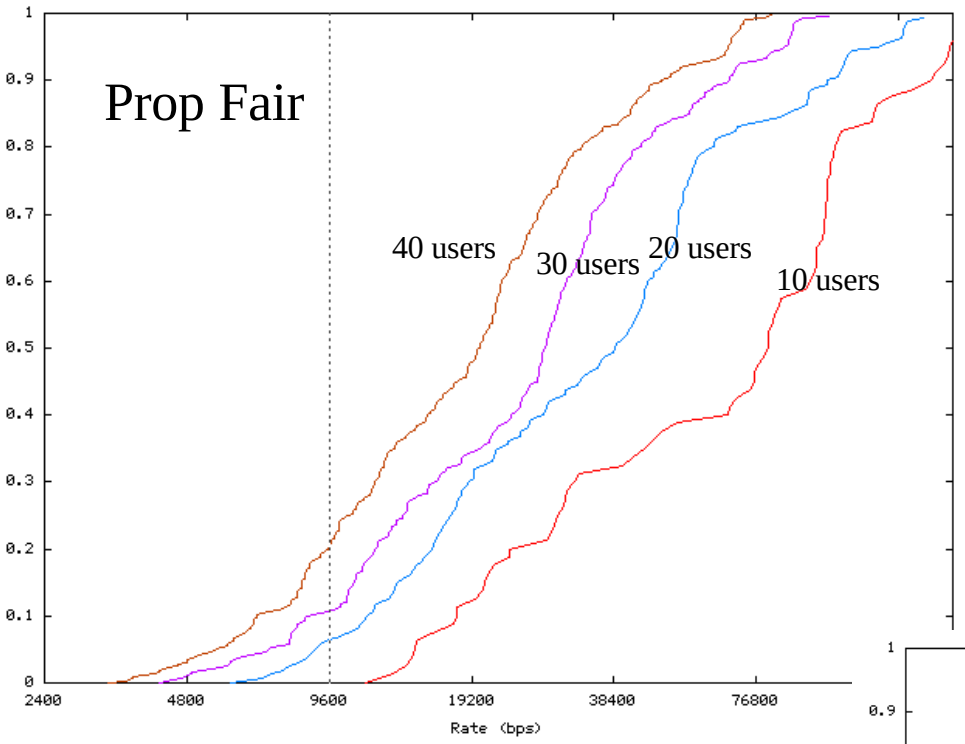
Performance of PFMR

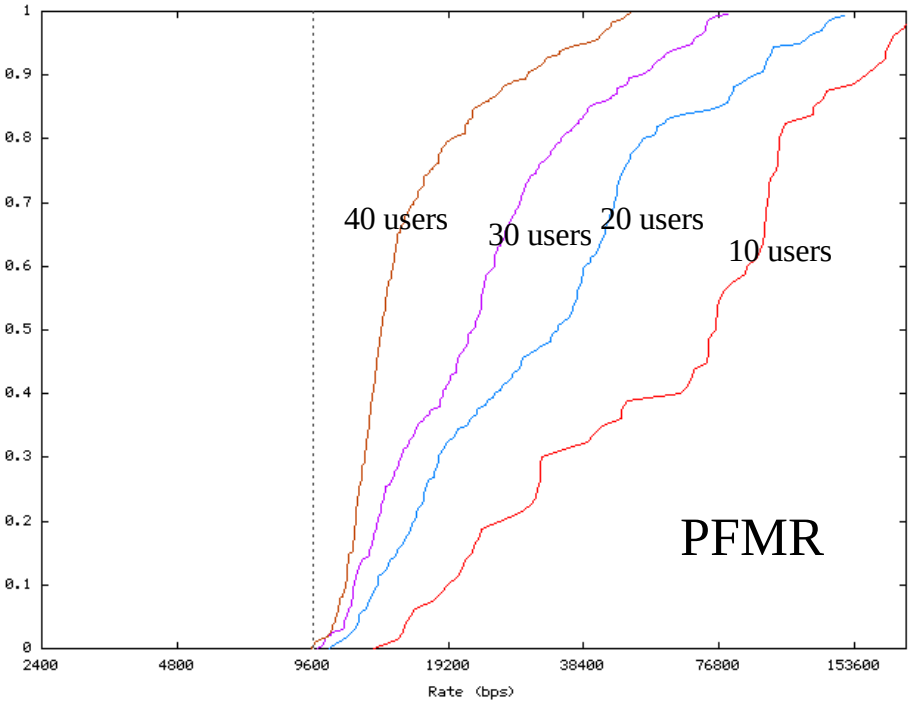
- **PFMR (Prop Fair with min/max rate constraints)**

$$\text{serve} \quad \arg\max_i \frac{DRC_i(t)}{R_i(t)} e^{a_i T_i(t)}$$



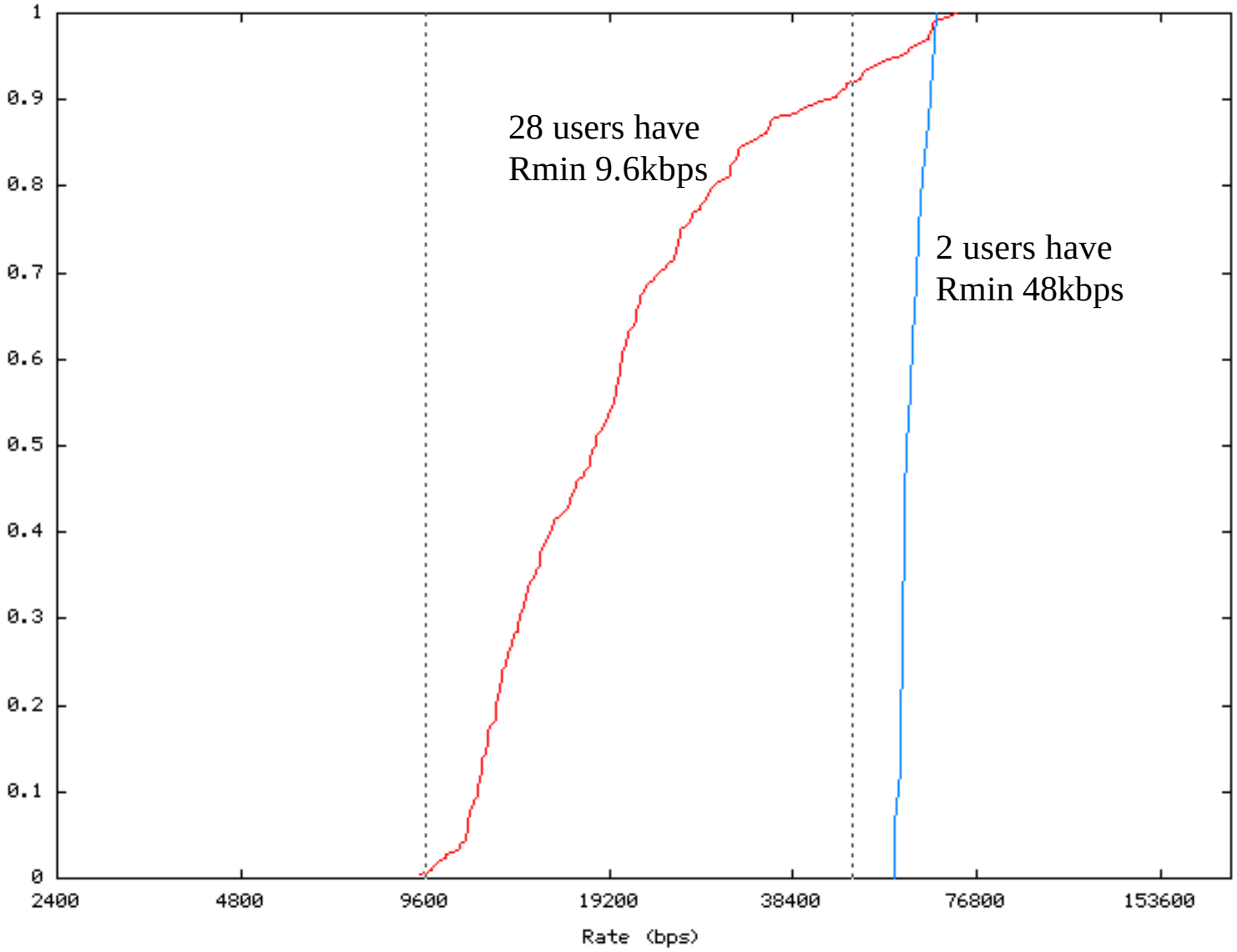
- If $R_i^{min} \le R_i(t) \le R_i^{max}$
  - Token counter stays near zero, sched metric reverts to Prop Fair
- If $R_i(t) < R_i^{min}$
  - Token counter drifts positive until user *i* satisfies min rate constraint
- If $R_i(t) > R_i^{max}$
  - Token counter drifts negative until user *i* satisfies max rate constraint
- If $t \to 0$, $a_i \propto t$ and system converges, then it converges to optimal soln





Rmin 9.6kbps for all users

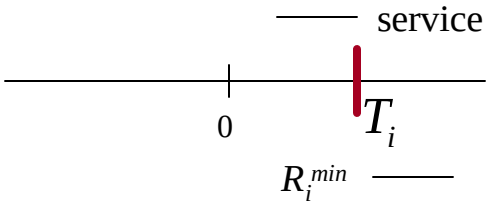
(In practice set token counter to 1.2*R_{min})


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2400
4800
9600
19200
38400
76800
153600
Rate (bps)
28 users have
Rmin 9.6kbps
2 users have
Rmin 48kbps

Comparison with "obvious" solution

- **PFMR (Prop Fair with min/max rate constraints)**

$$\text{serve} \quad \arg\max_i \frac{DRC_i(t)}{R_i(t)} e^{a_i T_i(t)}$$



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      - incremented at rate R_i^{max} when $T_i < 0$

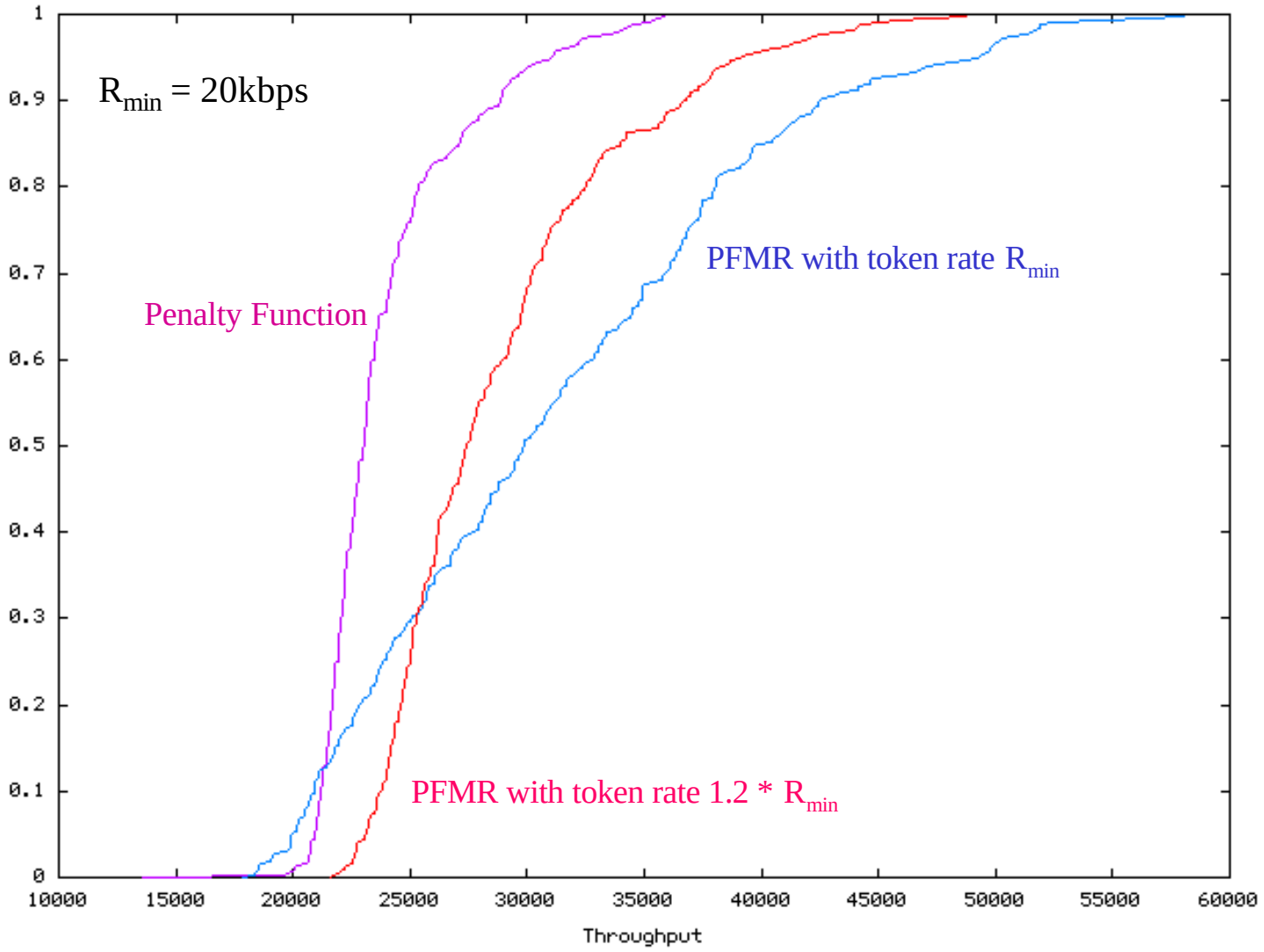
- **Penalty function method**

$$\text{serve} \quad \arg\max_i \frac{r_i}{R_i - R_{\min}}$$

PFMR provides min rate with less throughput loss !!

e.g. If Prop Fair, *by itself*, provides min rate then PFMR acts same as Prop Fair

Penalty function method has lower throughput than Prop Fair


R_{min} = 20kbps
Penalty Function
PFMR with token rate R_{min}
PFMR with token rate 1.2 * R_{min}
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10000
15000
20000
25000
30000
35000
40000
45000
50000
55000
60000
Throughput

Arrival model 1: Infinitely backlogged queues

Channel model 2: Adversarial process

Proportional Fair Metric

- **Definition different for adversarial channels**
  - **$R_i(t)$ = total throughput of user *i* by time *t***
  - **Objective: Maximize $\sum_i \log R_i(t)$ for all *t***
  - **Recall: for stochastic channels, optimality possible**

Results

- **For adversarial channels with n users:**
  - **For any algorithm (online or offline), there exists rate vector sequence and time T s.t.**

$$\sum_i \log R_i(T) \leq \sum_i \log R_i^*(T) - \Omega(n)$$

  - **For any online algorithm, there exists rate vector sequence and time T s.t.**

$$\sum_i \log R_i(T) \leq \sum_i \log R_i^*(T) - \Omega(n \log n)$$

  - **There exists online algorithm s.t. for all t and all rate vector sequences,**

$$\sum_i \log R_i(t) \geq \sum_i \log R_i^*(t) - O(n \log n)$$

Lower Bound (1)

- **For adversarial channels with *n* users:**
  - **For any algorithm (online or offline), there exists rate vector sequence and time *T* s.t.**

$$\sum_i \log R_i(T) \le \sum_i \log R_i^*(T) - \Omega(n)$$

Time	$DRC_0(t)$	$DRC_1(t)$		$DRC_{n/2-1}(t)$	$DRC_{n/2}(t)$	$DRC_{n/2+1}(t)$		$DRC_{n-1}(t)$
0	n	n	…	n	1	1	…	1
1	n	n	…	n	1	1	…	1
2	n	n	…	n	1	1	…	1
				:				
$n^2/2-2$	n	n	…	n	1	1	…	1
$n^2/2-1$	n	n	…	n	1	1	…	1

Optimal schedule for time $n^2/2-1$: assign each user n/2 slots

$n^2/2$	0	0	…	0	n	n	…	n
$n^2/2+1$	0	0	…	0	n	n	…	n
$n^2/2+2$	0	0	…	0	n	n	…	n
				:				
n^2-2	0	0	…	0	n	n	…	n
n^2-1	0	0	…	0	n	n	…	n

Optimal schedule for time n^2-1: for first $n^2/2$ slots, assign service equally among users 0,1,…,n/2-1
for remaining $n^2/2$ slots, assign service equally among users n/2,n/2+1,…,n-1

Lower Bound (2)

- **For adversarial channels with *n* users:**
  - **For any online algorithm, there exists rate vector sequence and time *T* s.t.**

$$\sum_i \log R_i(T) \leq \sum_i \log R_i^*(T) - \Omega(n \log n)$$

Time	$DRC_0(t)$	$DRC_1(t)$	$DRC_2(t)$	$DRC_3(t)$
0	1	1	1	1
1	1	1	1	1
User 2 has received least amount of service from online alg				
2	1	1	0	1
3	1	1	0	1
4	1	1	0	1
5	1	1	0	1
User 0 has received least amount of service from online alg				
6	0	1	0	1
7	0	1	0	1
8	0	1	0	1
9	0	1	0	1
10	0	1	0	1
11	0	1	0	1
12	0	1	0	1
13	0	1	0	1
User 3 has received least amount of service from online alg				
14	0	1	0	0
15	0	1	0	0

OPT serves
- user 2 in time slots 0-1
- user 0 in time slots 2-5
- user 3 in time slots 6-13

Upper Bound

- **There exists online algorithm s.t. for all t and all rate vector sequences,**

$$\sum_i \log R_i(t) \geq \sum_i \log R_i^*(t) - O(n \log n)$$

  – Randomized algorithm: at each time step serve user chosen uniformly at random

    - $\mathbf{E}[R_i(T)] \geq \sum_t DRC_i(t) / n \geq R_i^*(t) / n$

    - $\log \mathbf{E}[R_i(T)] \geq \log R_i^*(t) - \log n$

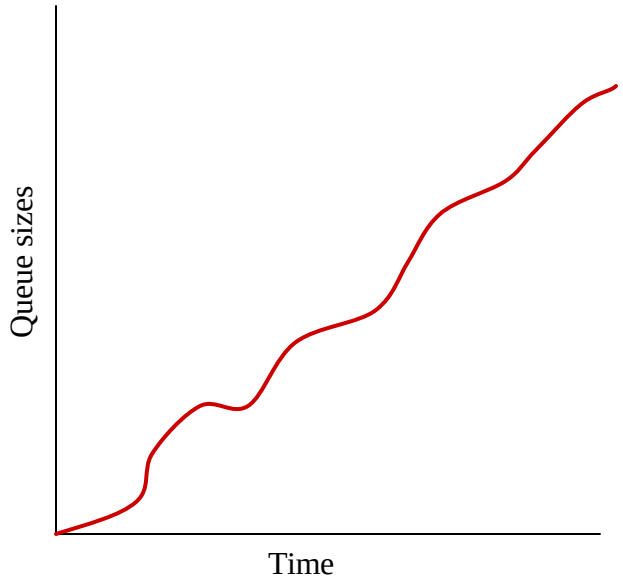
    - $\sum_i \log \mathbf{E}[R_i(T)] \geq \sum_i \log R_i^*(t) - n \log n$

  – Algorithm can be derandomized

Arrival model 2: External arrival process

System Stability

- **Want online algorithm that keeps system stable**
- **Proportional Fair is unstable!!!**



  - **Basic problem - Prop Fair does not consider queue sizes!!!!!**

Instability of Proportional Fair

$DRC_A(t)$	$DRC_B(t)$	
100	100	
100	100	
100	1000	——— B-biased slot
100	100	
100	100	
100	100	
100	100	
100	100	
100	100	
100	100	
100	100	
100	100	
100	100	
100	1000	
100	100	
100	100	
100	100	

$a_A(t)$	$a_B(t)$
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49
94	49

Stable algorithm
- Assign ½ of B-biased slots to B
- Assign remaining slots to A
- svc rate of $A \rightarrow 95$, svc rate of $B \rightarrow 50$

Proportional fair
- Assigns all B-biased slots to B
- Assigns remaining slots to A
- svc rate of $A \rightarrow 90$, svc rate of $B \rightarrow 100$

Arrival model 2: External arrival process

Channel model 1: Stationary stochastic process

Stable algorithm

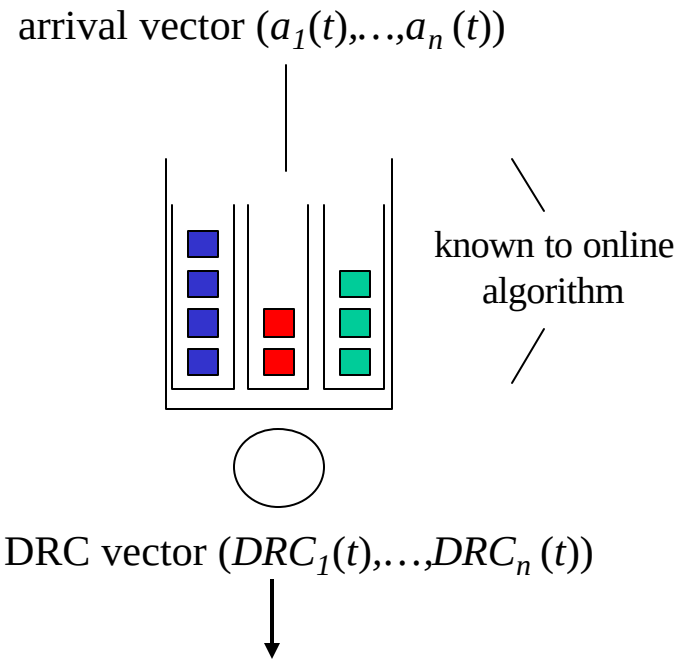
- **Max Weight** **(See Tassiulas-Ephremides, Kahale-Wright, Neely-Modiano-Rohrs, Stolyar)**
  - **$q_i(t)$ = queuesize of user *i* at time *t***
  - **Serve argmax$_i$ $q_i(t) * DRC_i(t)$**
  - **Potential function $\sum_i q_i(t)^2$ has negative drift**
  - **Queues remain stable!!!**

Arrival model 2: External arrival process

Channel model 2: Adversarial process

The adversary

- **At time *t* adversary generates**
  - **DRC vector ($DRC_1(t),\ldots,DRC_n(t)$)**
  - **Arrival vector ($a_1(t),\ldots,a_n(t)$)**
  - **Its "own" schedule $X(t)$**



- **Admissibility:**
  - **For each time window [t,t+*w*),**

$$(1\text{-}e)\sum_{t\in[t,t+w):X(t)=i} DRC_i(t) \le \sum_{t\in[t,t+w)} a_i(t)$$

  - **i.e. there exists a stable schedule**

Instability

- **No online algorithm is stable**

- ***f*(*x*) = smooth, convex, increasing, unbounded function, e.g. x^k**

- **Can force potential function**

$$\Sigma_i \; f(q_i(t))$$

to be unbounded

Instability

- **Two users, 0 and 1**
  - **DRC vector satisfies**

$$\frac{DRC_0(t)}{DRC_1(t)} = \frac{f'(q_1(t))}{f'(q_0(t))}$$

  - **If online algorithm serves user 0**

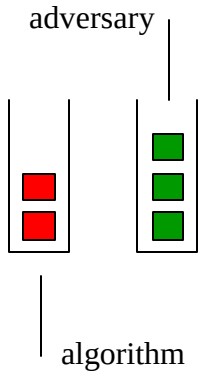
$a_0(t) = 0$ $\qquad$ $a_1(t) = (1\text{-}e)DRC_1(t)$

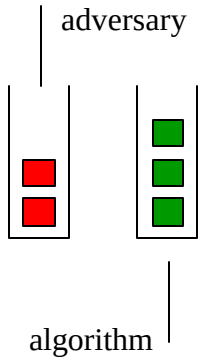
  - **If online algorithm serves user 1**

$a_0(t) = (1\text{-}e)DRC_0(t)$ $\qquad$ $a_1(t) = 0$

  - **Online algorithm always serves "wrong" user**

    ***f* always increases**





- **Remark**
  - **Rate set = set of all DRC values is infinite**

Stable algorithm

- **Is stability possible for finite rate set?**
  - **Yes**

- **Bad situation:**
  - **Online alg. keeps serving *i* but adv. keeps serving *j***

- **Intuition**
  - **Track adversary's schedule in past**
    - **Look at past DRCs and past arrivals**

  - **Try and "copy" the adversary**

Tracking Algorithms

- **Linear Tracking Algorithm**
  - **Counter $C_i(r)$ for each user i and DRC value r**
  - **Serve $j=\text{argmax}_i \; C_i(DRC_i(t))$**
  - **Decrease $C_j(DRC_j(t))$ by 1**
  - **Calculate adversary's schedule $X(s)$ for some $s \leq t$**
  - **Increase $C_{X(s)}(DRC_{X(s)}(t))$ by 1**
  - **Stable…? Don't know…**

Adv serves 1 **Trk serves 0**

DRC process: **(5,3)** **(3,10)** **(4,6)** **(1,0)** **(7,4)**

time s **time t**

$C_0(7)$ - -

$C_1(10)$ + +

- **Quadratic Tracking Algorithm**
  - **Counters for pairs of users. Stable with polynomial queues**

Stability

- **How do we compute adversary's schedule?**
  - **Need schedule $X(t)$ such that admissibility condition holds**

$$(1\text{-}e)\sum_{t \in [t,t+w):X(t)=i} DRC_i(t) \geq \sum_{t \in [t,t+w)} a_i(t)$$

  - **Problem: this is NP-hard to compute**
  - **Fix: can run tracking algorithm against a fractional schedule**

$$x(t) = (x_1(t),\ldots,x_n(t))$$

such that

$$(1\text{-}e)\sum_{t \in [t,t+w)} x_i(t)\, DRC_i(t) \geq \sum_{t \in [t,t+w)} a_i(t)$$

$$0 \leq x_i(t) \leq 1$$
$$\sum_i x_i(t) = 1$$

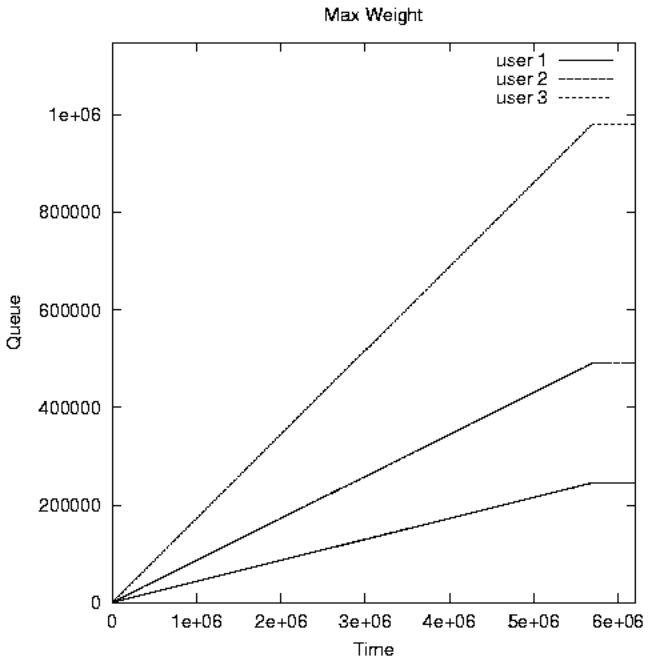
Max Weight

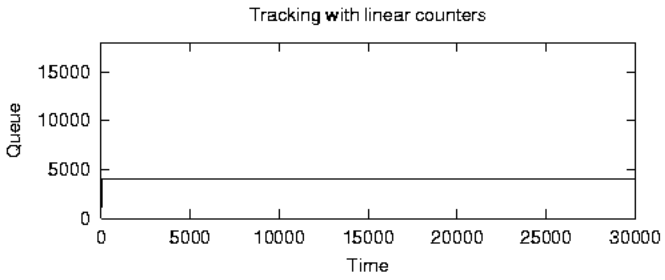
- **Max Weight**
  - **$q_i(t)$ = queuesize of user *i* at time *t***
  - **Serve $\text{argmax}_i \; q_i(t) * DRC_i(t)$**
  - **Stable with finite rate set...? Don't know...**
  - **Max Weight produces "exponential queues"**
    - **Queuesize $> 2^N$ for $N < U_{\text{rateset}}$**
    - **$U_{\text{EVDO-rateset}} = 2048$**

Max Weight

$a_0(t) = 64$ $a_1(t) = 1$
$a_2(t) = 1$ $a_3(t) = 1$

$DRC_0(t)$	$DRC_1(t)$	$DRC_2(t)$	$DRC_3(t)$	
0	0	0	0	
0	0	0	64	Build up user 1 queue
0	0	64	0	
4096	**64**	0	0	
4096	0	0	0	
0	0	0	0	
		:		
0	0	0	0	
0	0	0	64	
0	**128**	**64**	0	Build up user 2 queue
4096	64	0	0	
4096	0	0	0	
0	0	0	0	
		:		
0	0	0	0	
0	0	**128**	**64**	
0	128	64	0	Build up user 3 queue
4096	64	0	0	
4096	0	0	0	
0	0	0	0	





Summary

- **The model matters**
  - Do we assume users with large backlogs or do we want to serve an external arrival process?
  - Do we assume a smooth stationary channel process or do we assume an unpredictable adversarial channel process

- **Infinite backlogs, stationary channels**
  - Prop Fair and PFMR allocate thruputs fairly subject to bounds on user performance

- **Infinite backlogs, adversarial channels**
  - Cannot beat simple random algorithm

- **External arrival process, stationary channels**
  - Prop Fair not stable, MaxWeight maintains stability

- **External arrival process, adversarial channels**
  - Stability not always possible. Tracking algorithm gives stability when possible