Estimation of DGPS Carrier-Phase Errors Using a Reference Receiver Network

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Overview

- Motivation
- Setting up the problem
- NetAdjust solution
- Implementation issues
- Putting this approach in context
- Covariance function description
- NetAdjust test results
- Covariance analysis technique
- Summary/Conclusion

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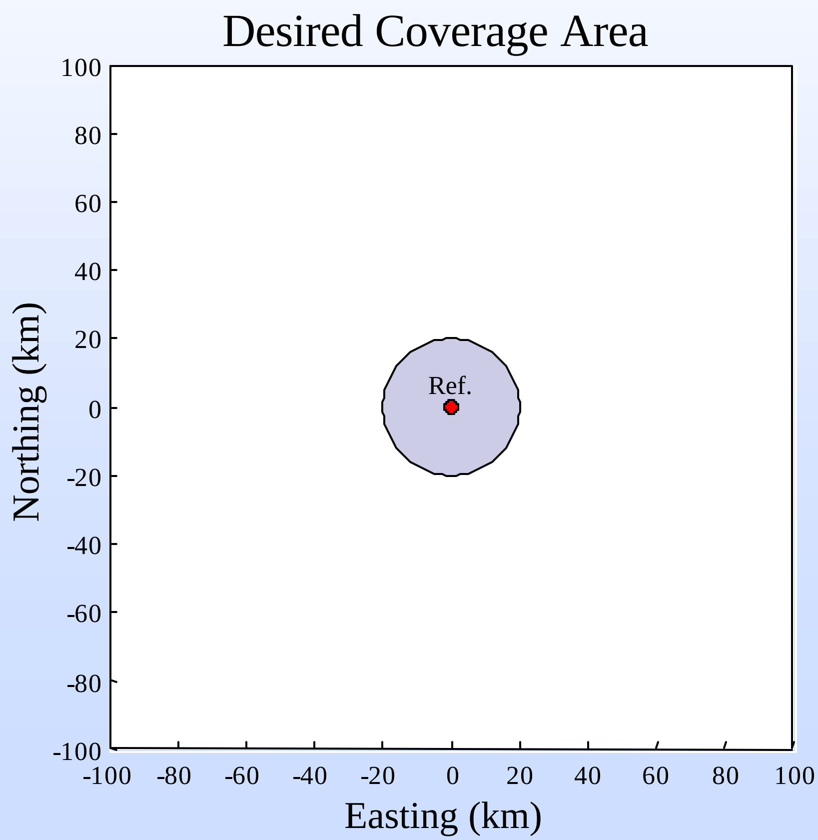
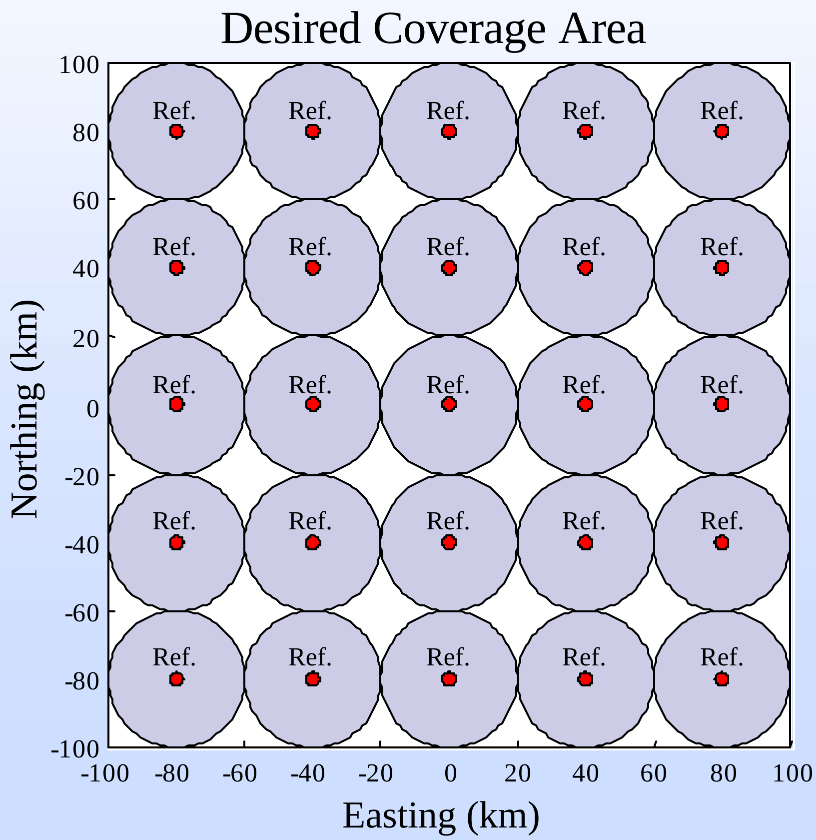
Reference Receiver Network Motivation (1/3)

- Single reference receiver coverage



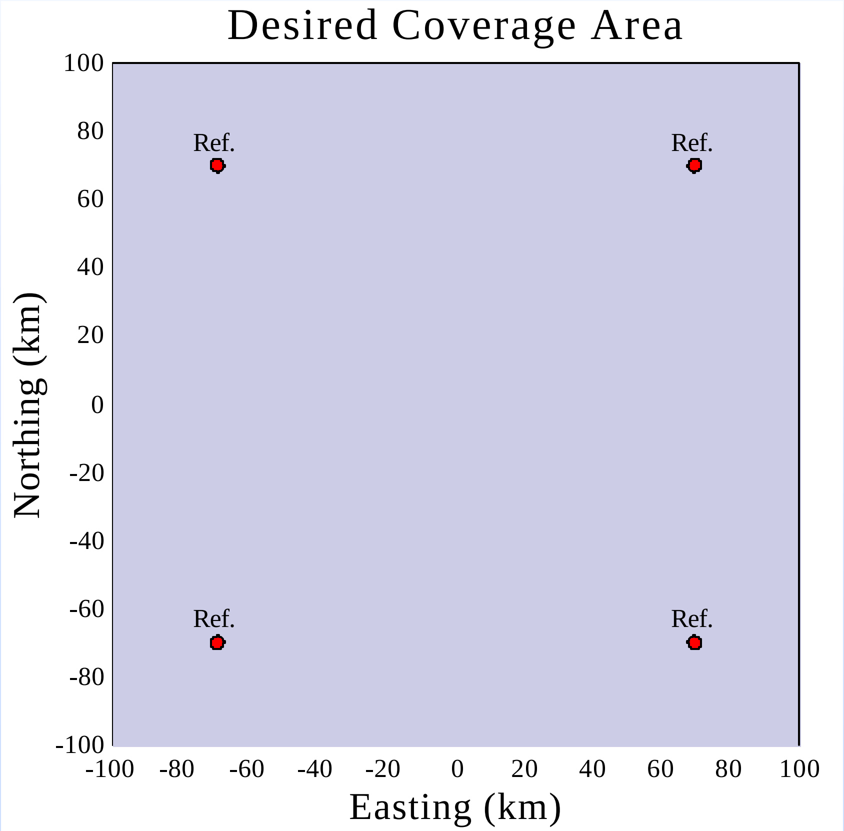
Reference Receiver Network Motivation (2/3)

- One (poor) solution



Reference Receiver Network Motivation (3/3)

- Better solution: use a network



Phase Measurements

- Measurement with errors

$$\phi = \frac{1}{\lambda}\left(r + c\delta t_u - c\delta t_{sv} + T + I + \upsilon + m\right) + N$$

λ = phase wavelength

r = true range to satellite

δt_u = receiver clock bias

δt_{sv} = satellite clock bias

T = tropospheric error

I = ionospheric error

υ = phase meas noise

m = phase multipath

N = integer ambiguity

- Double-differencing

$$\nabla\Delta\phi_{12}^{ab} \equiv \Delta\phi_{12}^{a} - \Delta\phi_{12}^{b} = \phi_1^a - \phi_2^a - (\phi_1^b - \phi_2^b)$$

$$= \frac{1}{\lambda}\left(\nabla\Delta r_{12}^{ab} + \nabla\Delta T_{12}^{ab} + \nabla\Delta I_{12}^{ab} + \nabla\Delta\upsilon_{12}^{ab} + \nabla\Delta m_{12}^{ab}\right) + \nabla\Delta N_{12}^{ab}$$

Double-Difference Phase Errors

- Highest positioning accuracy obtained by differential carrier-phase ambiguity resolution

$$\nabla\Delta\phi = \frac{1}{\lambda}\left(\nabla\Delta r + \underbrace{\nabla\Delta T}_{\text{tropo}} + \underbrace{\nabla\Delta I}_{\text{iono}} - \underbrace{\nabla\Delta m}_{\text{multipath}} + \underbrace{\nabla\Delta \upsilon}_{\text{meas noise}}\right) + \underbrace{\nabla\Delta N}_{\substack{\text{integer}\\ \text{ambiguity}}}$$

  - If "close" to reference receiver, then correlated errors are removed

$$\nabla\Delta\phi = \frac{1}{\lambda}\left(\nabla\Delta r + \underbrace{\nabla\Delta m + \nabla\Delta \upsilon}_{\text{cm-level errors}}\right) + \nabla\Delta N$$

Why Reducing Errors Helps Ambiguity Resolution

- Almost all ambiguity resolution routines use some sort of residual analysis to determine integer ambiguities

$$\underbrace{\nabla\Delta\phi - \nabla\Delta N - \frac{1}{\lambda}\nabla\Delta\hat{r}}_{\text{measurement residuals}} = \underbrace{\frac{1}{\lambda}\left(\nabla\Delta T + \nabla\Delta I - \nabla\Delta m + \nabla\Delta \upsilon + \nabla\Delta p_{sv}\right)}_{\text{differential } (\nabla\Delta) \text{ errors}}$$

	Low $\nabla\Delta$ Errors	High $\nabla\Delta$ Errors
Correct $\nabla\Delta N$	Low Residuals	Generally High Residuals
Wrong $\nabla\Delta N$	Generally High Residuals	Generally High Residuals

Why Reducing Errors Helps Ambiguity Resolution

- Almost all ambiguity resolution routines use some sort of residual analysis to determine integer ambiguities

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Goal

	Low $\nabla\Delta$ Errors	High $\nabla\Delta$ Errors
Correct $\nabla\Delta N$	Low Residuals	Generally High Residuals
Wrong $\nabla\Delta N$	Generally High Residuals	Generally High Residuals

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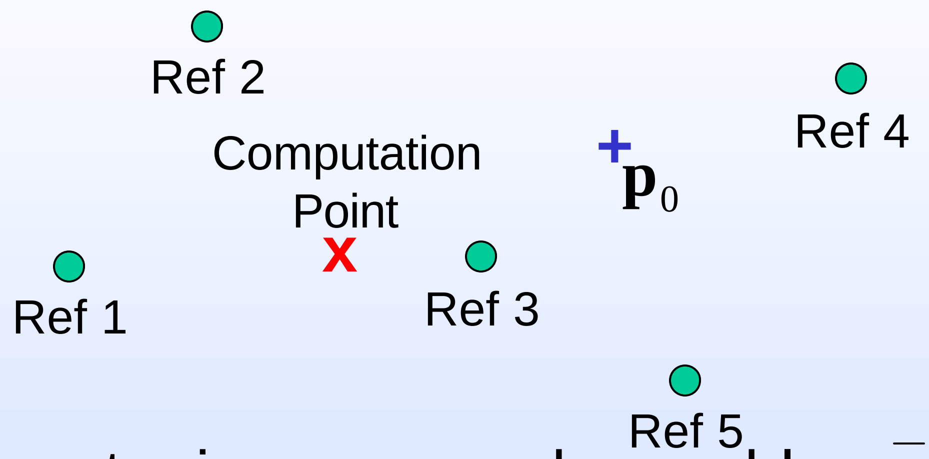
Setting Up the Problem

Sample 5-receiver network:



Measurement-minus-range observable: $\bar{\phi} = \phi - r_{calc}$

Measurements:

$$\mathbf{l}_{n_1} = \begin{bmatrix} \bar{\phi}_1^1 \\ \bar{\phi}_1^2 \\ \bar{\phi}_1^3 \\ \bar{\phi}_1^4 \end{bmatrix}, \quad \mathbf{l}_{n_2} = \begin{bmatrix} \bar{\phi}_2^1 \\ \bar{\phi}_2^2 \\ \bar{\phi}_2^3 \\ \bar{\phi}_2^4 \end{bmatrix}, \quad \mathbf{l}_{n_3} = \ldots \quad \mathbf{l}_n = \begin{bmatrix} \mathbf{l}_{n_1} \\ \mathbf{l}_{n_2} \\ \mathbf{l}_{n_3} \\ \mathbf{l}_{n_4} \\ \mathbf{l}_{n_5} \end{bmatrix}, \quad \tilde{\mathbf{l}}_{cp} = \begin{bmatrix} \tilde{\phi}_{cp}^1 \\ \tilde{\phi}_{cp}^2 \\ \tilde{\phi}_{cp}^3 \\ \tilde{\phi}_{cp}^4 \end{bmatrix}, \quad \mathbf{l} = \begin{bmatrix} \mathbf{l}_n \\ \tilde{\mathbf{l}}_{cp} \end{bmatrix}$$

Setting Up the Problem

Measurement errors:

$$\mathbf{l} = \delta\phi_{\text{clock}} + \underbrace{\delta_c\phi(\mathbf{p}_0) + d_c\phi(\mathbf{p},\mathbf{p}_0)}_{\substack{\text{Correlated Errors} \\ \text{(Iono, Tropo, SV Position)}}} + \underbrace{\delta_u\phi}_{\substack{\text{Uncorrelated} \\ \text{Errors} \\ \text{(Multipath, Noise)}}} + \underbrace{\mathbf{N}}_{\substack{\text{Integer} \\ \text{Ambiguity}}}$$

Double-difference errors:

$$\nabla\Delta\mathbf{l} = \nabla\Delta d_c\phi(\mathbf{p},\mathbf{p}_0) + \nabla\Delta\delta_u\phi + \nabla\Delta\mathbf{N}$$

Errors to be eliminated:

$$\begin{bmatrix} \nabla\Delta\delta\mathbf{l}_n \\ \nabla\Delta\delta\mathbf{l}_{cp,n} \end{bmatrix} = \nabla\Delta\delta\mathbf{l} = \nabla\Delta d_c\phi(\mathbf{p},\mathbf{p}_0) + \nabla\Delta\delta_u\phi$$

Measurements available:

$$\nabla\Delta\delta\mathbf{l}_n = \nabla\Delta\mathbf{l}_n - \nabla\Delta\mathbf{N} \qquad \text{Note}: \nabla\Delta\mathbf{l}_n = \mathbf{B}_n\mathbf{l}_n$$

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NetAdjust Solution

- Use a linear minimum variance of error estimator
  - Generic case (to estimate $\mathbf{x}$ given measurements $\mathbf{Y}$)

$$\hat{\mathbf{x}} = E[\mathbf{x}] + \mathbf{C}_{\mathbf{x},\mathbf{Y}} \mathbf{C}_{\mathbf{Y}}^{-1} (\mathbf{Y} - E[\mathbf{Y}])$$

or, if $E[\mathbf{x}] = E[\mathbf{Y}] = 0$, then

$$\hat{\mathbf{x}} = \mathbf{C}_{\mathbf{x},\mathbf{Y}} \mathbf{C}_{\mathbf{Y}}^{-1} \mathbf{Y}$$

    - Assumption: $\mathbf{x}$ and $\mathbf{Y}$ are jointly Gaussian
  - Our case (to estimate $\delta \mathbf{l}$ given measurements $\nabla \Delta \delta \mathbf{l}_n$)

$$\delta \hat{\mathbf{l}} = \mathbf{C}_{\delta \mathbf{l}, \nabla \Delta \delta \mathbf{l}_n} \left(\mathbf{C}_{\nabla \Delta \delta \mathbf{l}_n} \right)^{-1} \nabla \Delta \delta \mathbf{l}_n$$

    - Assumes $\delta \mathbf{l}$ and $\nabla \Delta \delta \mathbf{l}_n$ are zero-mean

Are the Assumptions Valid?

- Assumption 1: $\delta \mathbf{l}$ and $\nabla\Delta\delta\mathbf{l}_n$ are jointly Gaussian
  - Each individual error source tends to be Gaussian
  - Central limit theorem strengthens assumption
- Assumption 2: $\delta \mathbf{l}$ and $\nabla\Delta\delta\mathbf{l}_n$ are zero-mean
  - Reasonable for uncorrelated errors (multipath and noise)
  - Reasonable for correlated errors, if systemic biases removed by a model

Cleaner Statement of NetAdjust Solution

- Corrections to apply to measurements from reference receiver network

$$\delta \hat{l}_n = C_{\delta l_n} B_n^T \left(B_n C_{\delta l_n} B_n^T \right)^{-1} \left(B_n l_n - \Delta \nabla N_n \right) \qquad (1)$$

- Corrections to apply to mobile receiver measurements

$$\delta \hat{l}_{cp} = C_{\delta l_{cp}, \delta l_n} B_n^T \left(B_n C_{\delta l_n} B_n^T \right)^{-1} \left(B_n l_n - \Delta \nabla N_n \right) \qquad (2)$$

- Minimizes trace$\left(\mathbf{C}_{\nabla \Delta \delta \, \mathbf{l}_{cp,n}} \right)$ --the ultimate goal!

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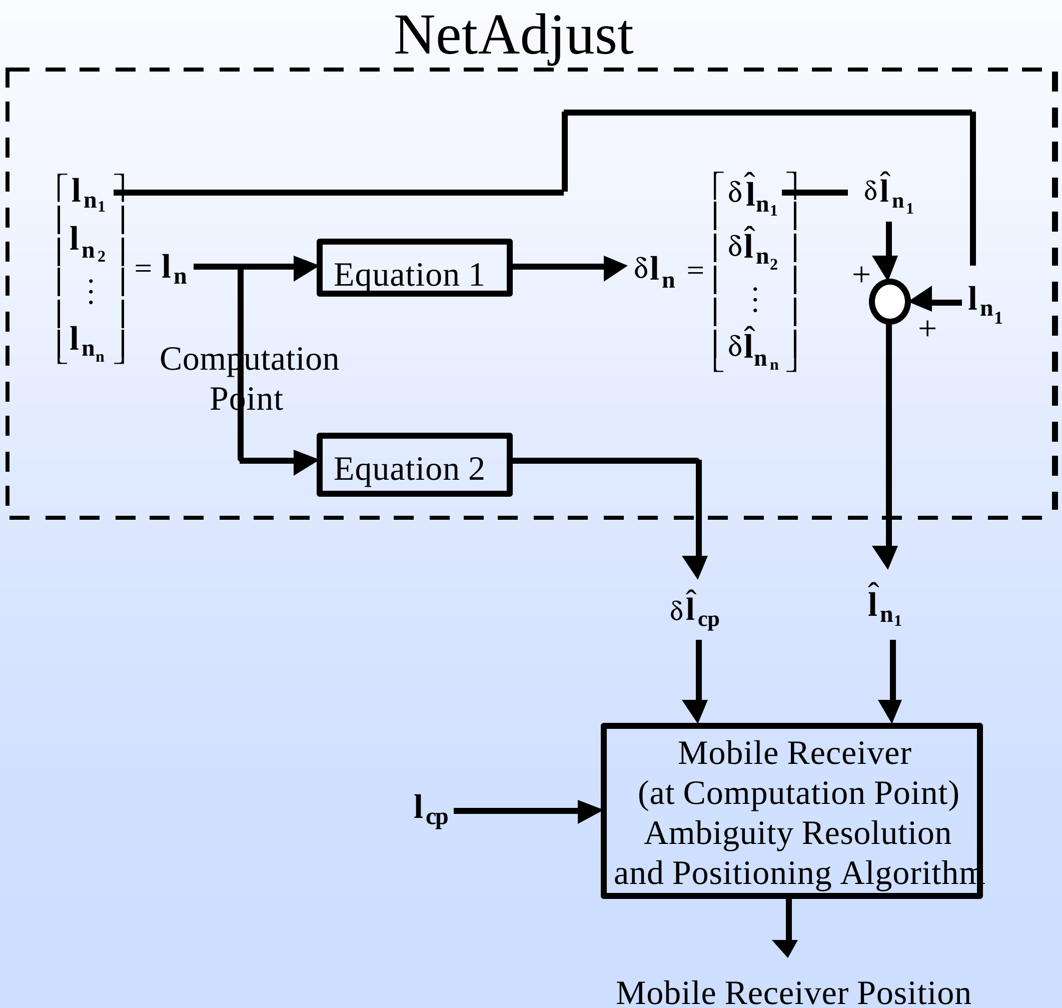
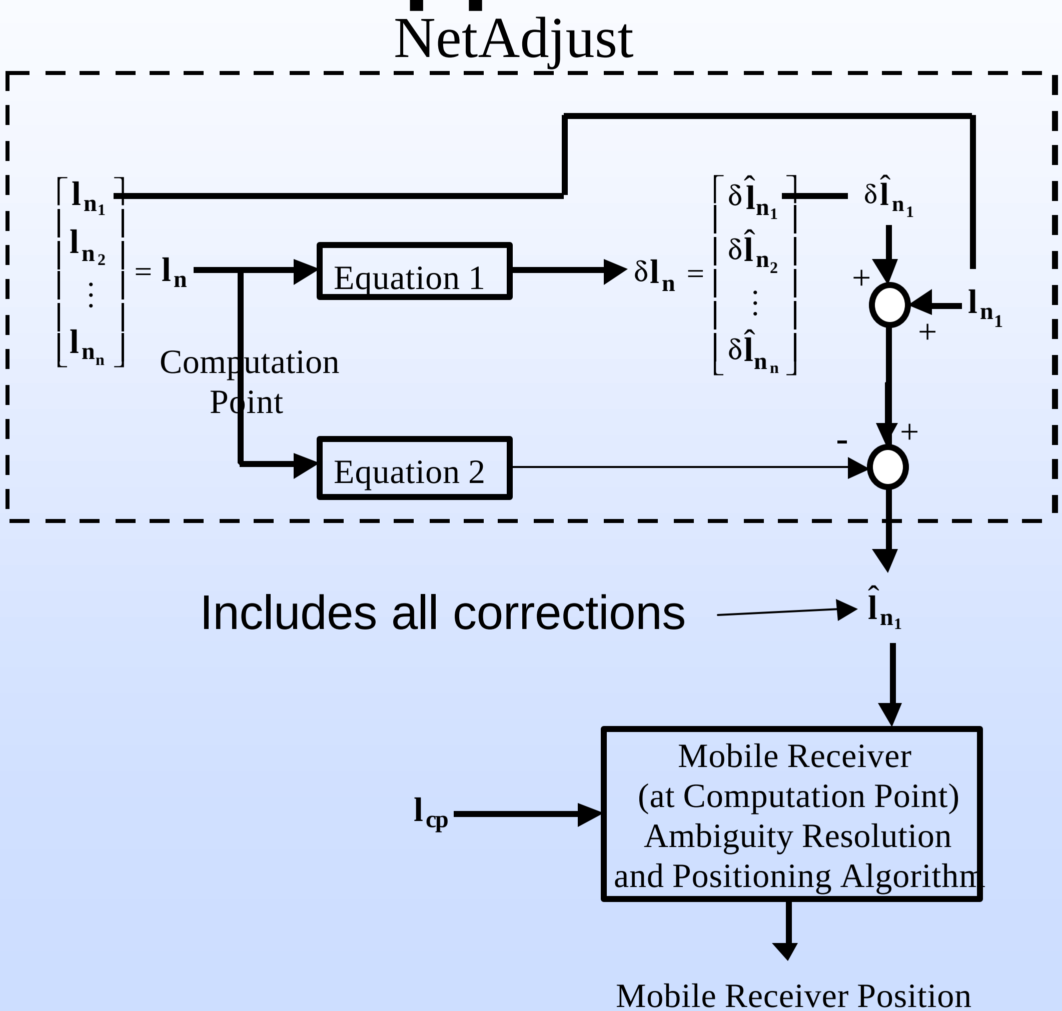
Implementation Approach



Alternate Implementation Approach



How Do You Transmit for Mobile User at Any Location?

- Question that must be answered for multi-user one-way-broadcast network
- Corrections vary with location (as they should)
- Variation is not easily modeled
- Different approaches can be taken using grid
  - Nearest point
  - Interpolation (linear, quadratic)
  - Update rates
- See ION AM 2000 paper by Fotopoulos

Calculation of Network Ambiguities

- Algorithm requires no initialization *per se*
- Ambiguities between reference receivers must be known
  - Best if all fixed
  - Will work (slightly less well) with floating ambiguity estimates
  - Can account for a mix of fixed and floating
- Real-time estimation of ambiguities between network reference stations is one of the largest implementation challenges

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Three "Views" of the NetAdjust Approach

- Linear Minimum of Variance Estimator
  - Explicitly minimizes squared error Bayes' risk
  - Estimation of one variable using observables
- Least-Squares Condition Adjustment
  - Apply condition to measurements
    - Condition is that all double-differenced measurement-minus-range observables within network are zero
    - Explains the "data encapsulation" effect
- Least-Squares Collocation
  - Interpolation
  - Use of covariance kernel

Three Classes of Approaches to This Problem

- Error Mitigation Approach
  - Explicitly estimate individual error sources
  - Gao, vanderMarel, etc.
- Polynomial Fit Approach
  - Assume differential errors can be expressed as a particular functional form of position
  - Calculate coefficients for the specified function
  - Varner, Wubbena, etc.
- Covariance Fit Approach (NetAdjust)
  - Assume error covariance can be expressed as a functional form
  - Use functionally generated covariance with NetAdjust

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Covariance Function Concept

Data from test network

Information about error characteristics (i.e., covariance matrix)

Express covariance in functional form:
Example: function of:

- Distance
- SV elevation
- Rcvr-specific multipath/noise levels

Use covariance function to generate predicted covariance matrix for new configuration

Calculate corrections

Predict performance

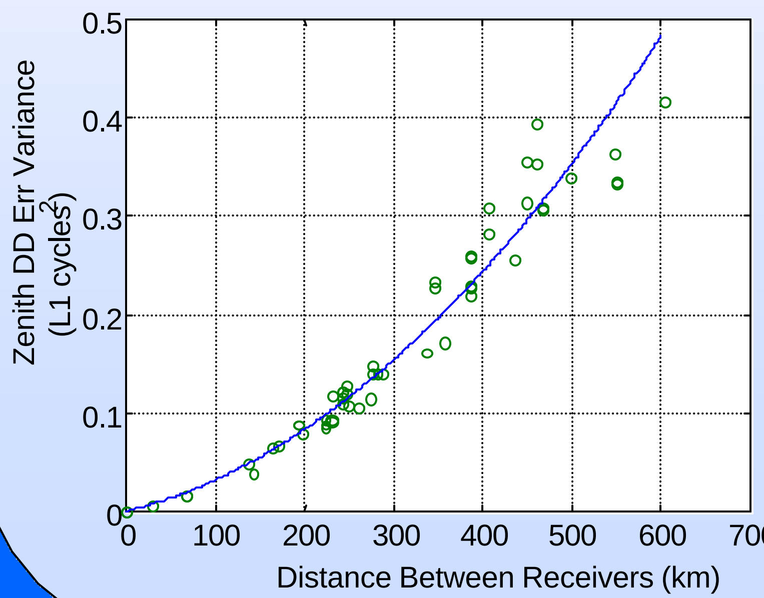
Zenith Phase Covariance Functions

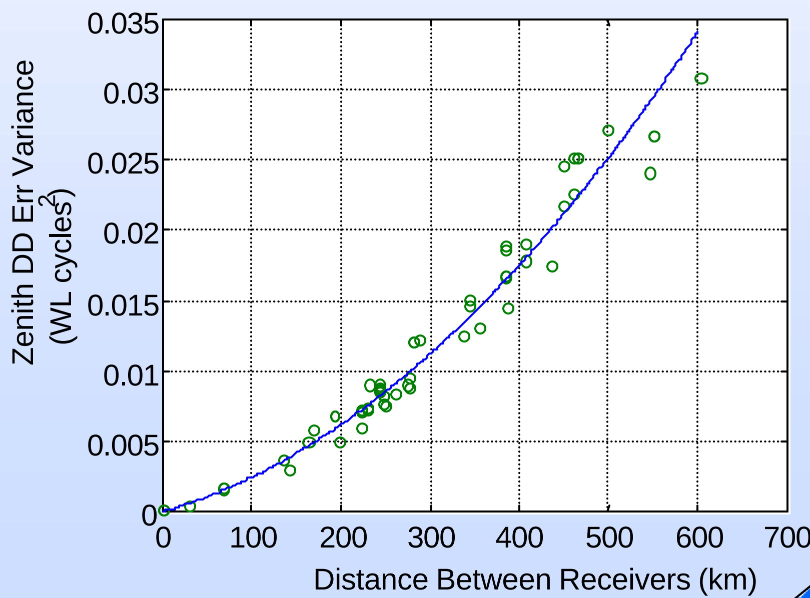
(Based on 55 Baselines Between 11 Receivers)

$$\sigma^2_{DD} = c_1 d + c_2 d^2 (+\text{multipath/noise term})$$

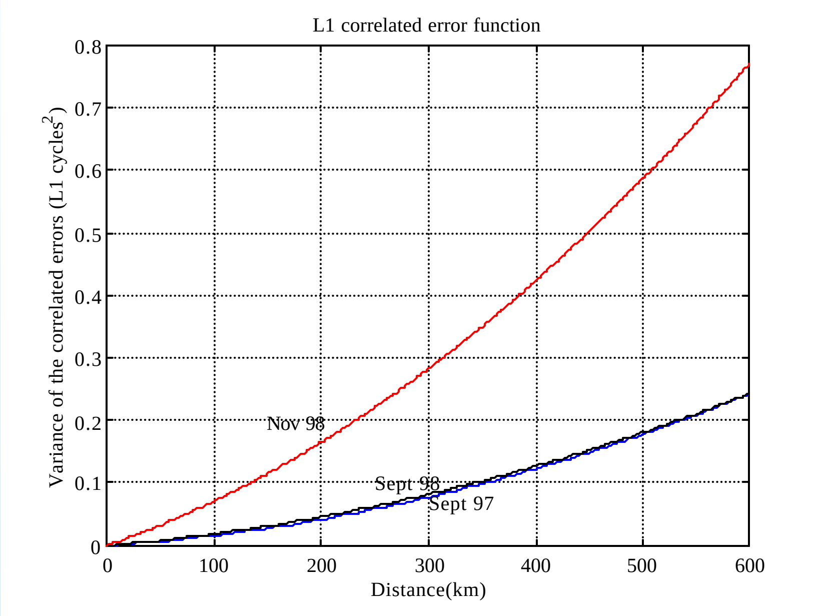
L1 $c_1 = 2.2048E-4$, $c_2 = 9.7531E-7$

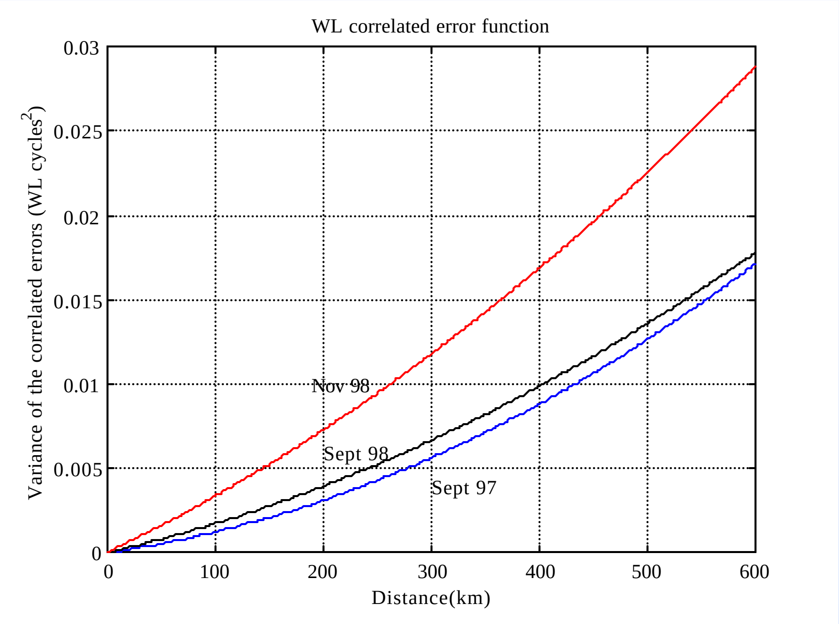
WL $c_1 = 1.7881E-5$, $c_2 = 6.5065E-8$





Example of How Covariance Function Can Change





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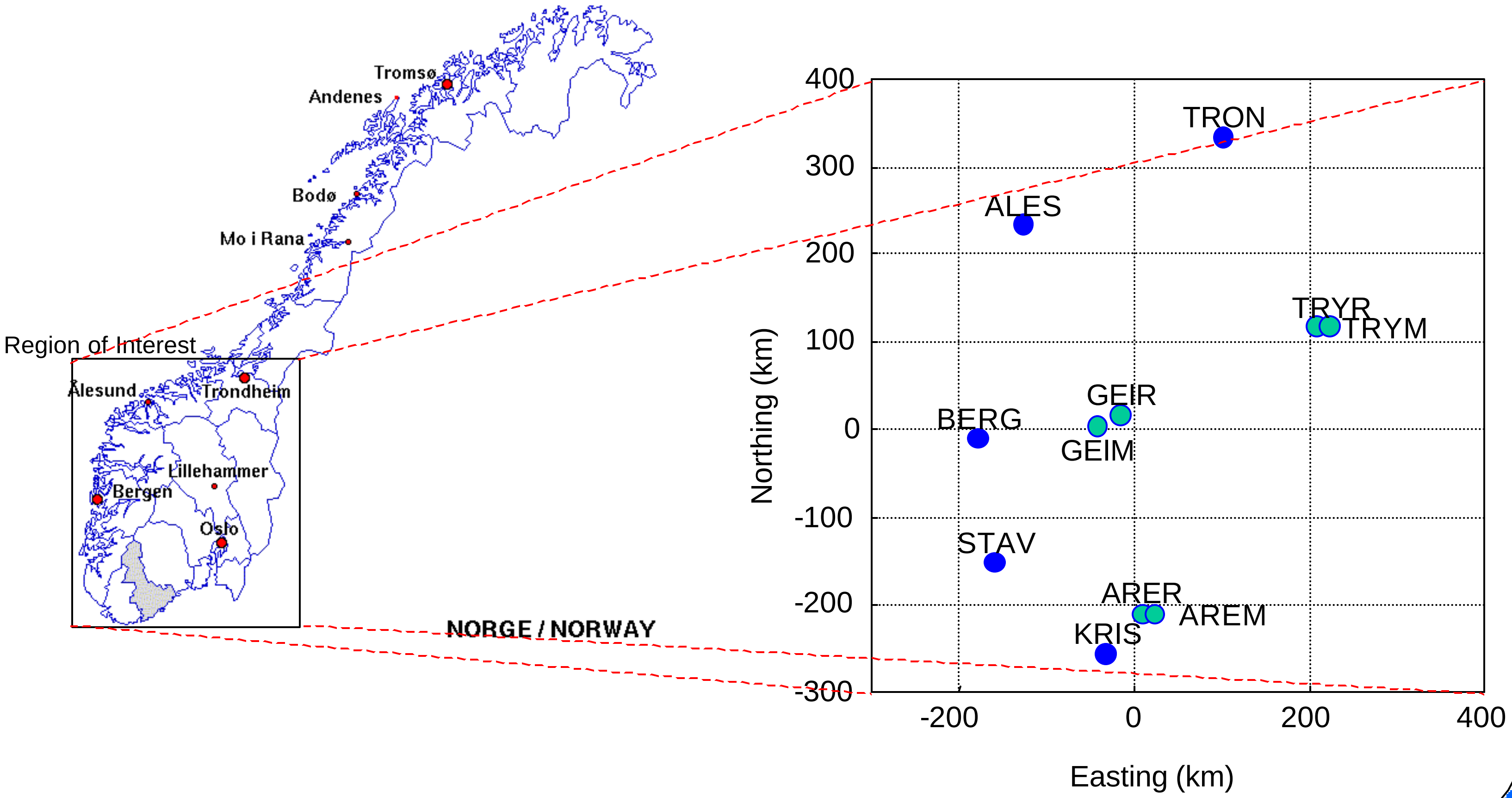
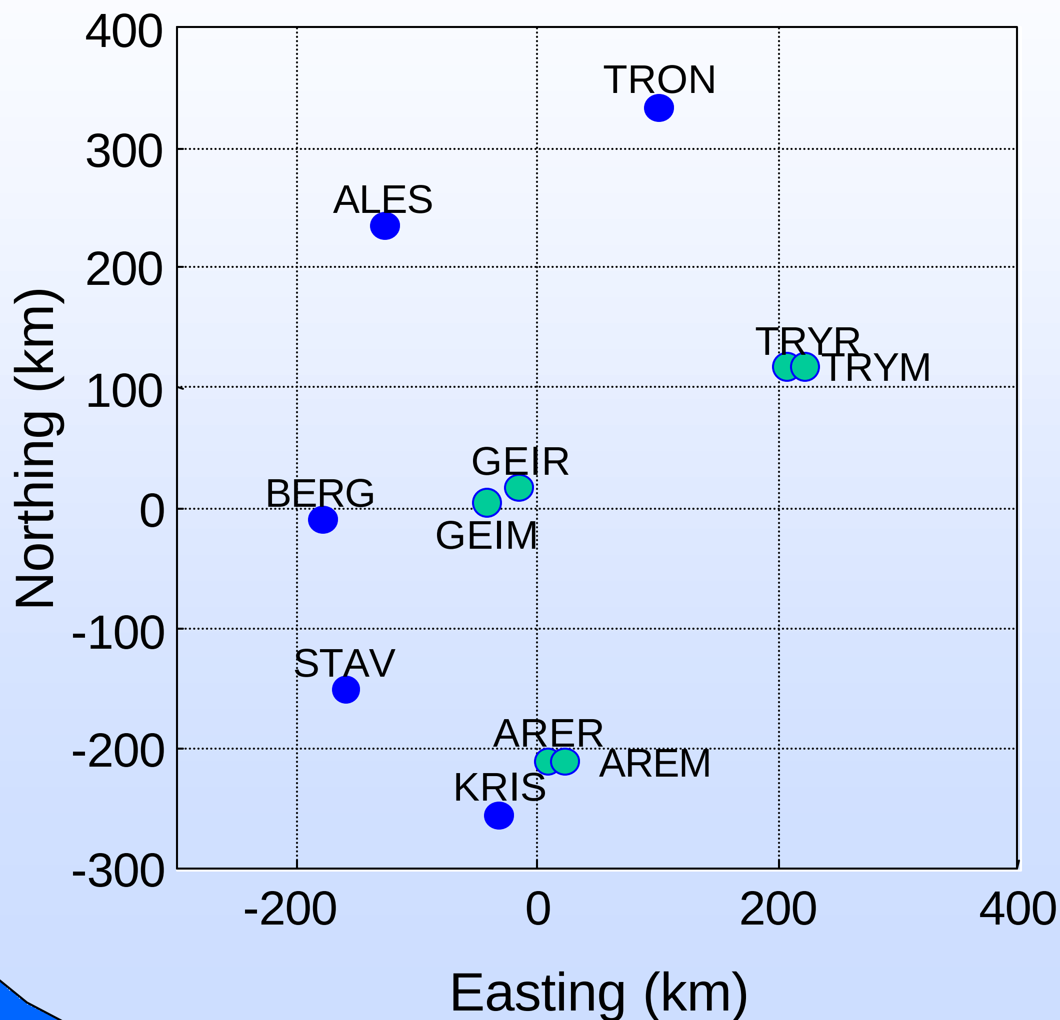
Norway Test Network



Norway Network

- 24 hours of data at 2 second intervals
- Ambiguities calculated
  - Between every pair of reference receivers
  - Over 24 hour period
- Receiver positions calculated
  - Based on ionospheric-free carrier-phase observable (requires L1 and L2 ambiguities)
  - Network adjustment procedure
  - Relative positioning accuracy: 2-3mm horizontal, 5-7mm vertical

Seven Test Networks



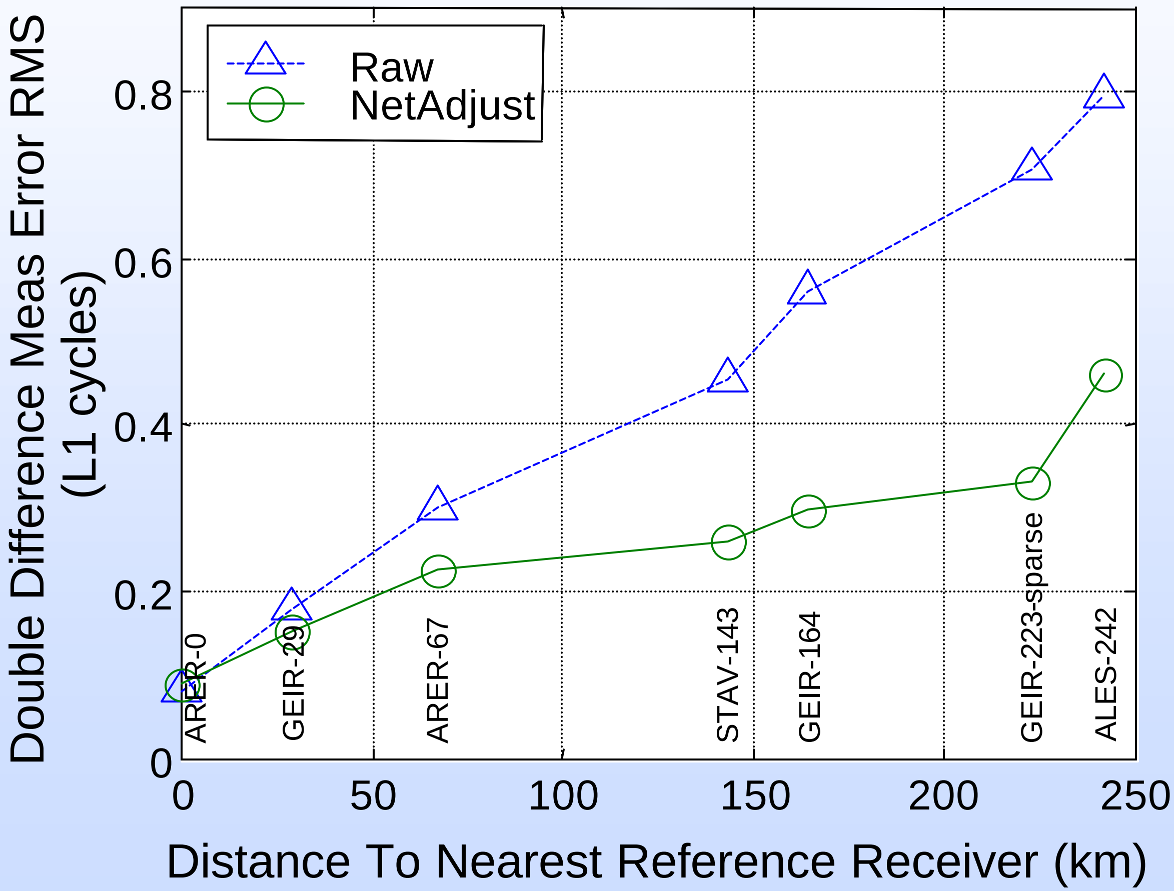
<u>Test Networks</u>

ARER-0
GEIR-29
ARER-67
STAV-143
GEIR-164
GEIR-223-sparse
ALES-242

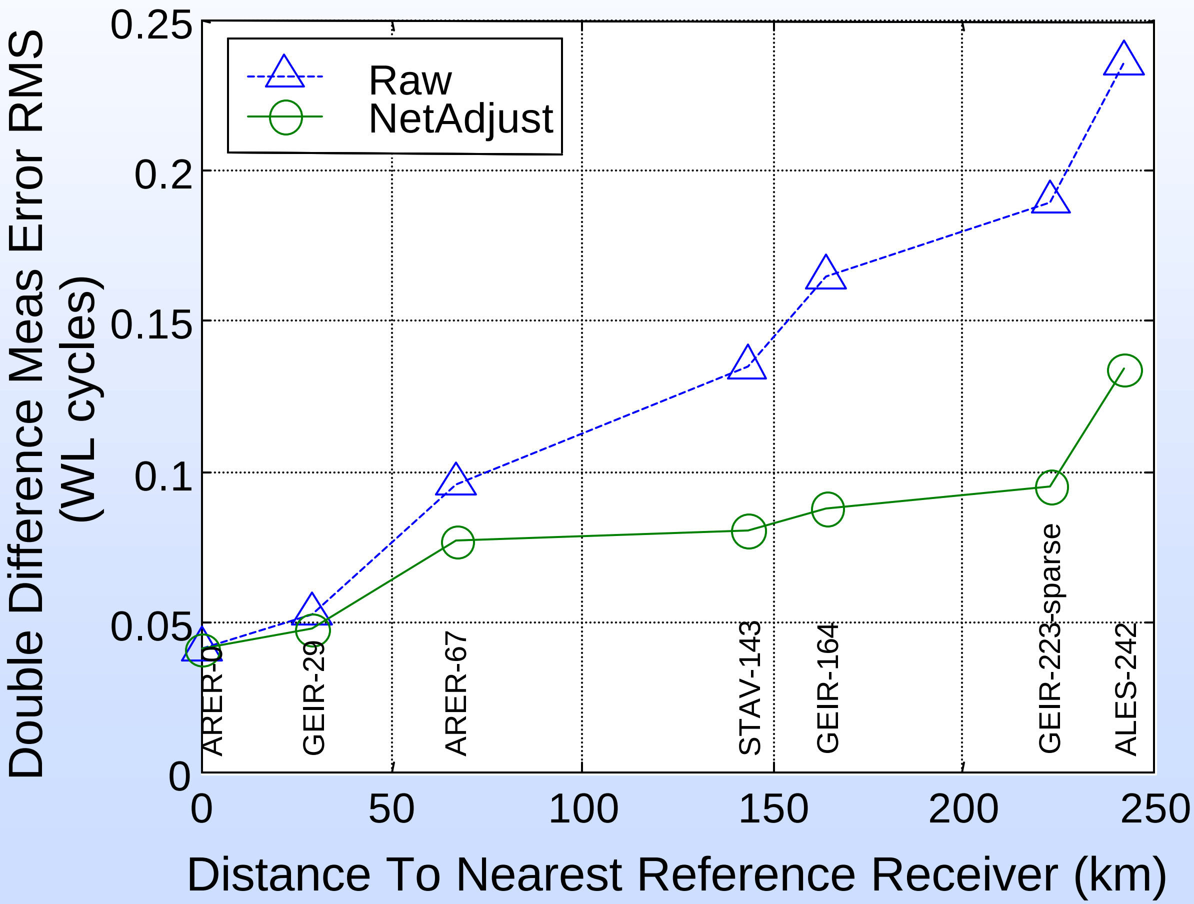
Testing NetAdjust on Norway Network

- Improvement in double-difference **measurement error**
- Improvement in differential **positioning accuracy** (using correct integer ambiguities)
- Improvement in **carrier-phase ambiguity resolution**

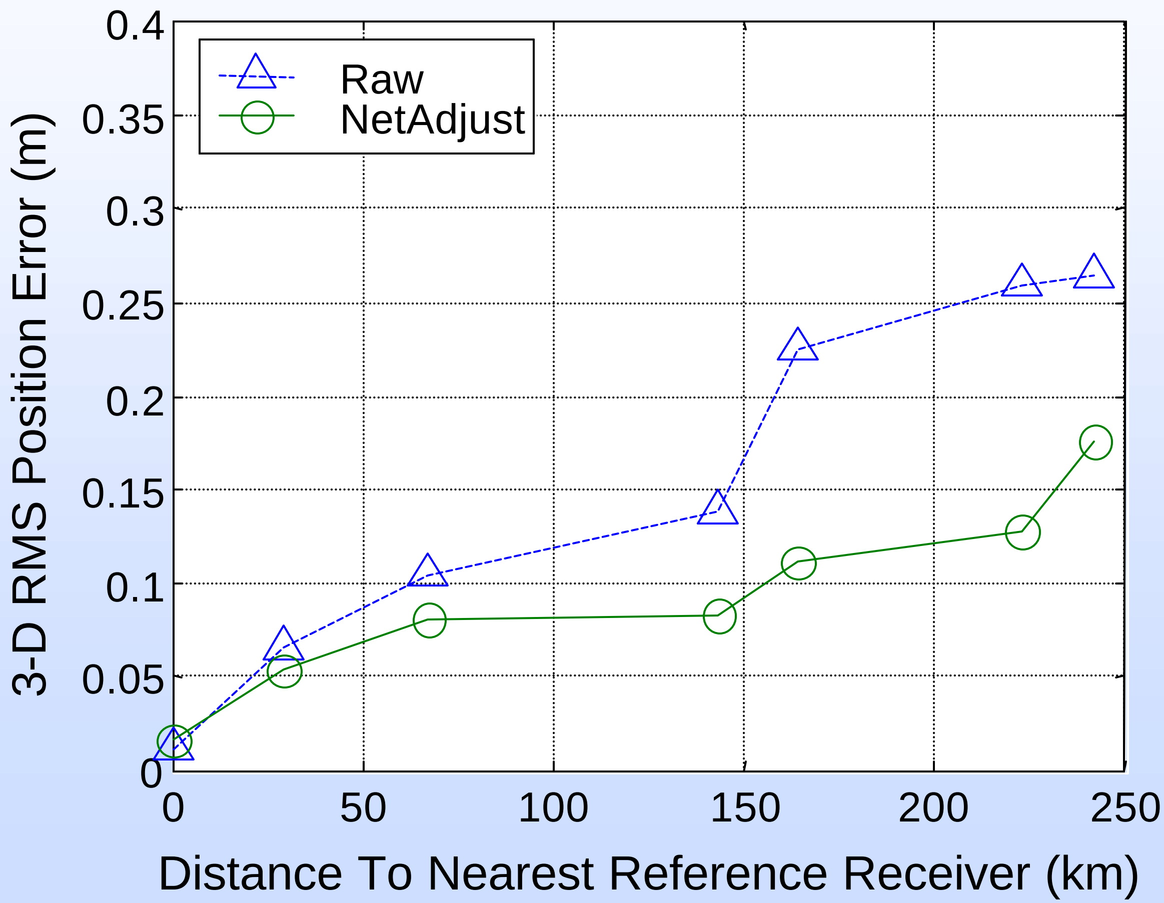
Improvement in DD Measurement Error
L1 Phase



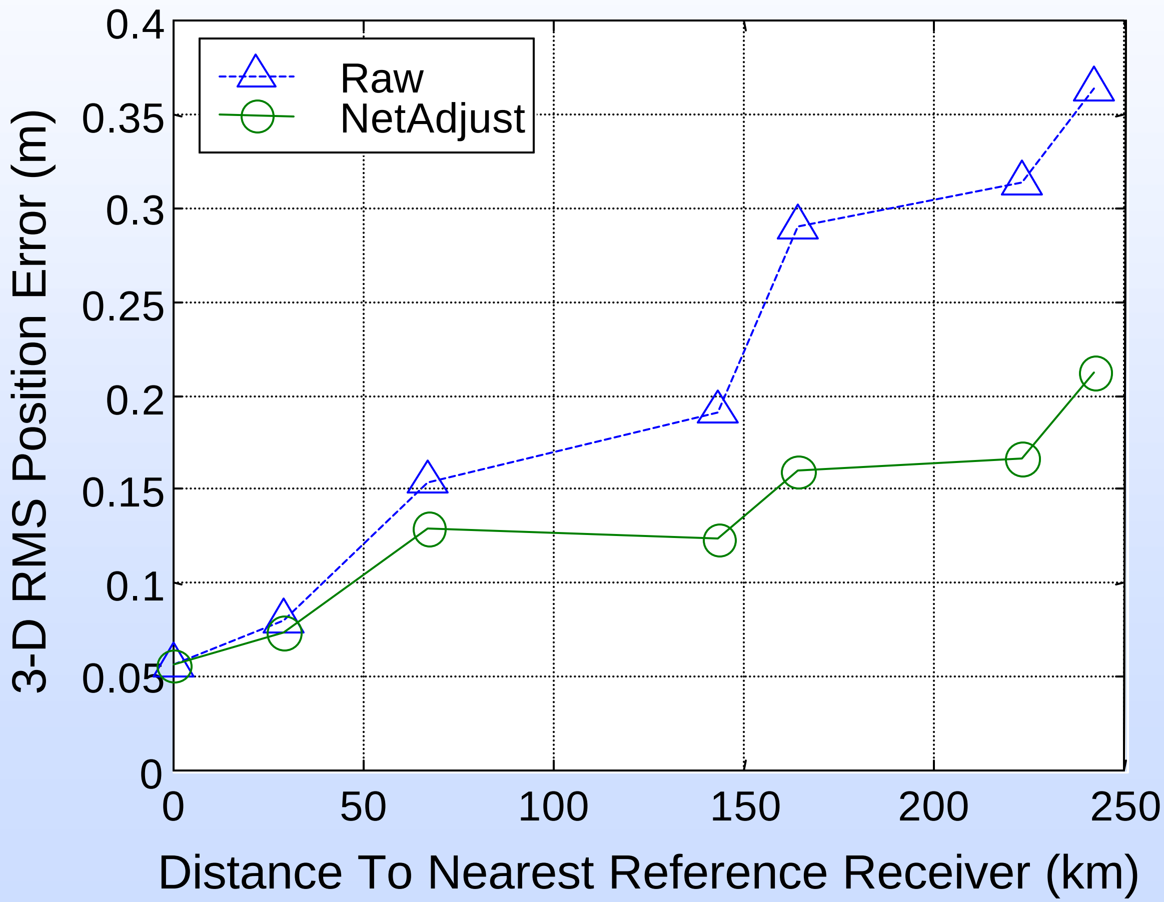
Improvement in DD Measurement Error
WL Phase



Improvement in Positioning Accuracy
L1 Phase (Fixed Integer Ambiguities)



Improvement in Positioning Accuracy
WL Phase (Fixed Integer Ambiguities)

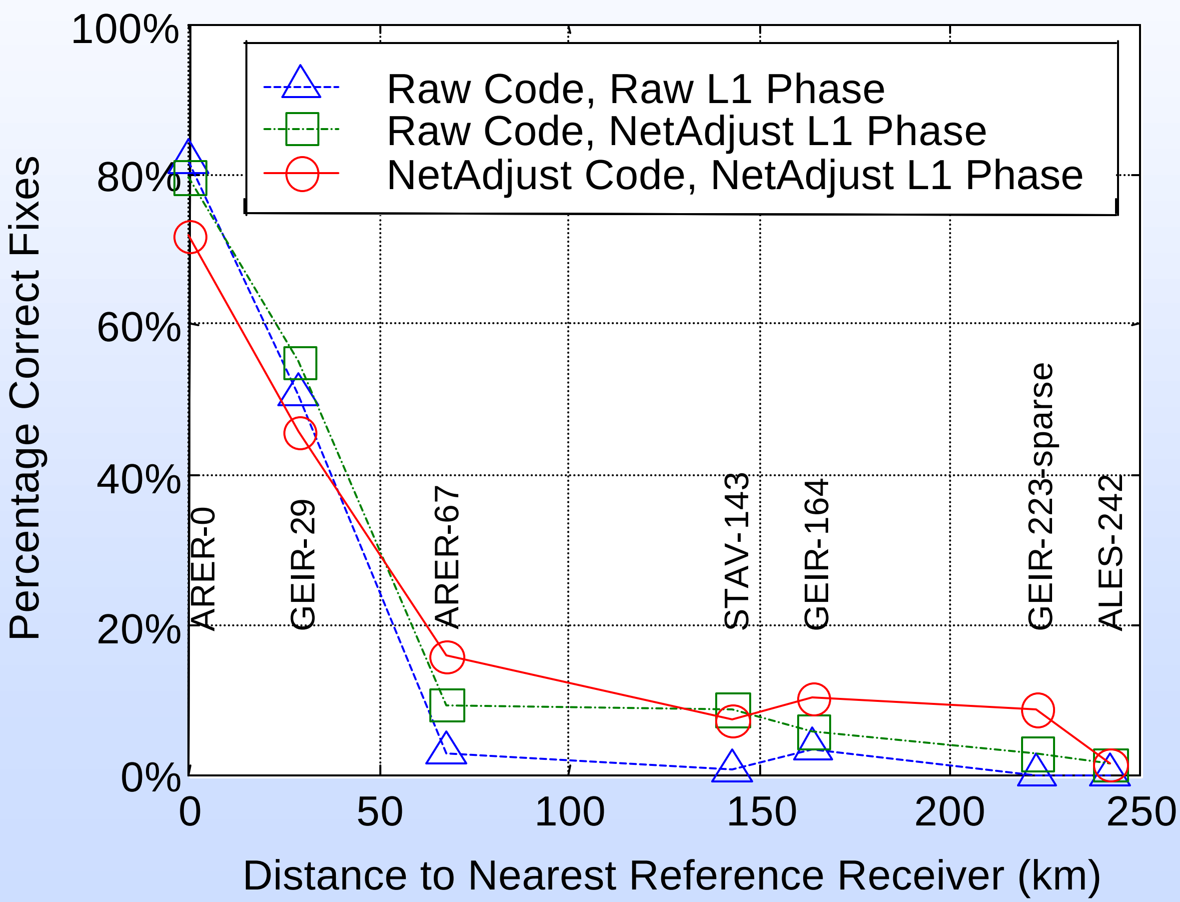


Improvement in Ambiguity Resolution

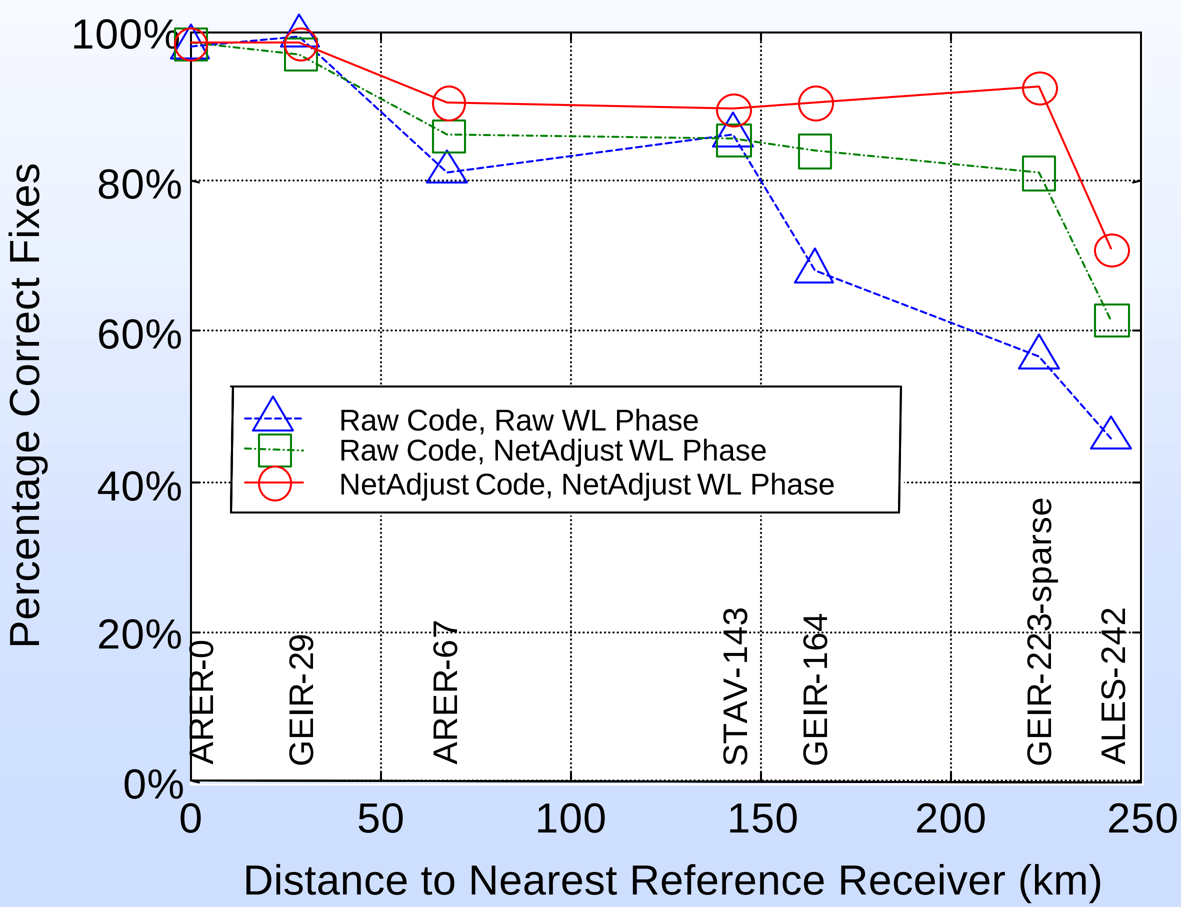
- University of Calgary's FLYKIN™ software
  - Run iteratively, start times staggered by 10 minutes (138 runs over 24 hours)
  - Stopped immediately if integer ambiguities determined
- Three performance criteria
  - Percentage of correct fixes
  - Percentage of incorrect fixes
  - Average time to resolve ambiguities

Improvement in Ambiguity Resolution

Percentage of Correct Fixes - L1 Phase

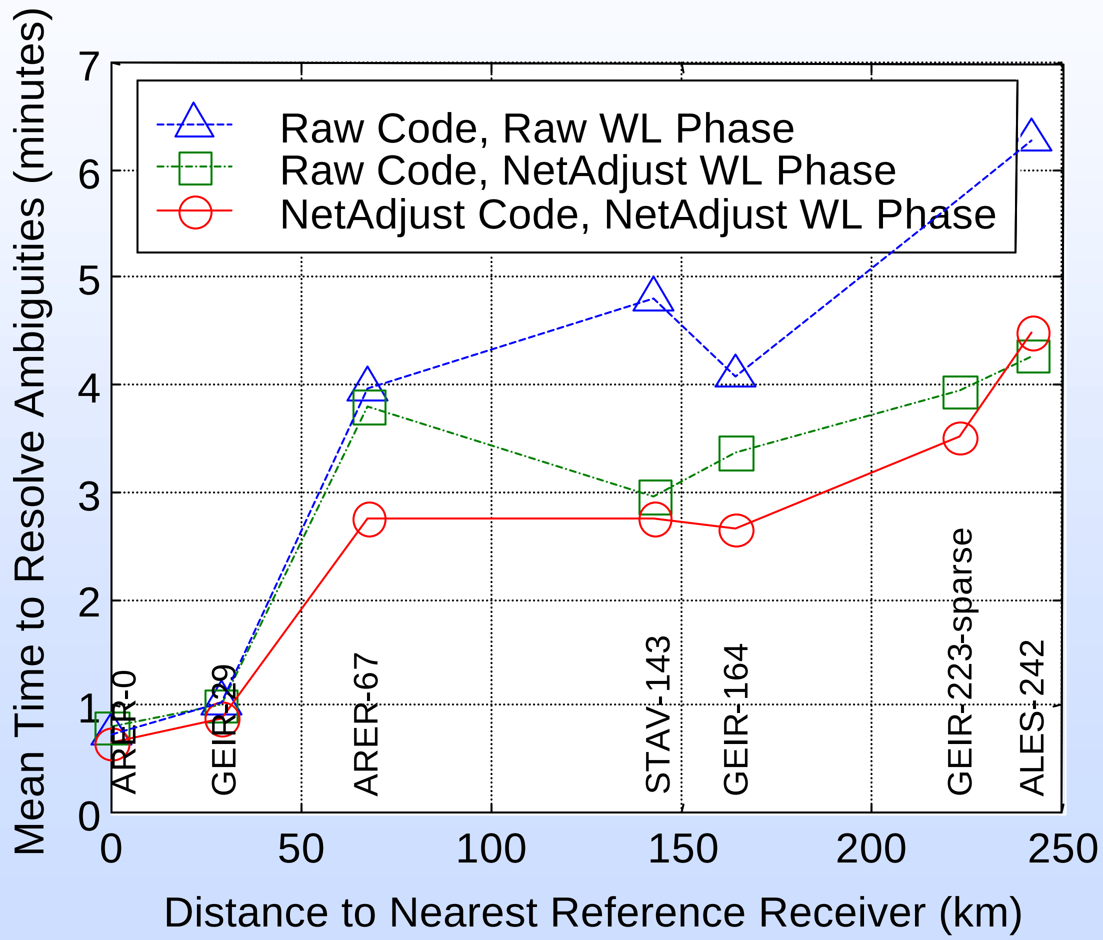


Improvement in Ambiguity Resolution
Percentage of Correct Fixes - WL Phase



Improvement in Ambiguity Resolution

Mean Time to Fix - WL Phase



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Motivation

- It's difficult and costly to deploy a reference receiver network
- Differential network performance varies with
  - Number/location of reference receivers
  - Number/geometry of visible satellites
  - Type of measurement used (e.g., L1 or WL)
  - Characteristics (especially correlations) of DGPS errors
- May be possible to test small subset of network configurations
- Desirable to **predict** performance for other (untested) network configurations
  - "What if" scenarios
  - Based upon test results
  - Critical for final network design

Covariance Analysis Procedure

- Straightforward propagation of DGPS measurement error covariance into double-difference space:

$$C_{err(\Delta\nabla l_{cp})} = B_2 C_{\delta l} B_2^T - B_2 C_{\delta l} B_1^T \left(B_1 C_{\delta l} B_1^T \right)^{-1} B_1 C_{\delta l} B_2^T$$

where

$C_{err(\Delta\nabla l_{cp})} \rightarrow$ DD error covariance between reference/mobile receivers

$B_1 \rightarrow$ DD operator (matrix) between reference receivers

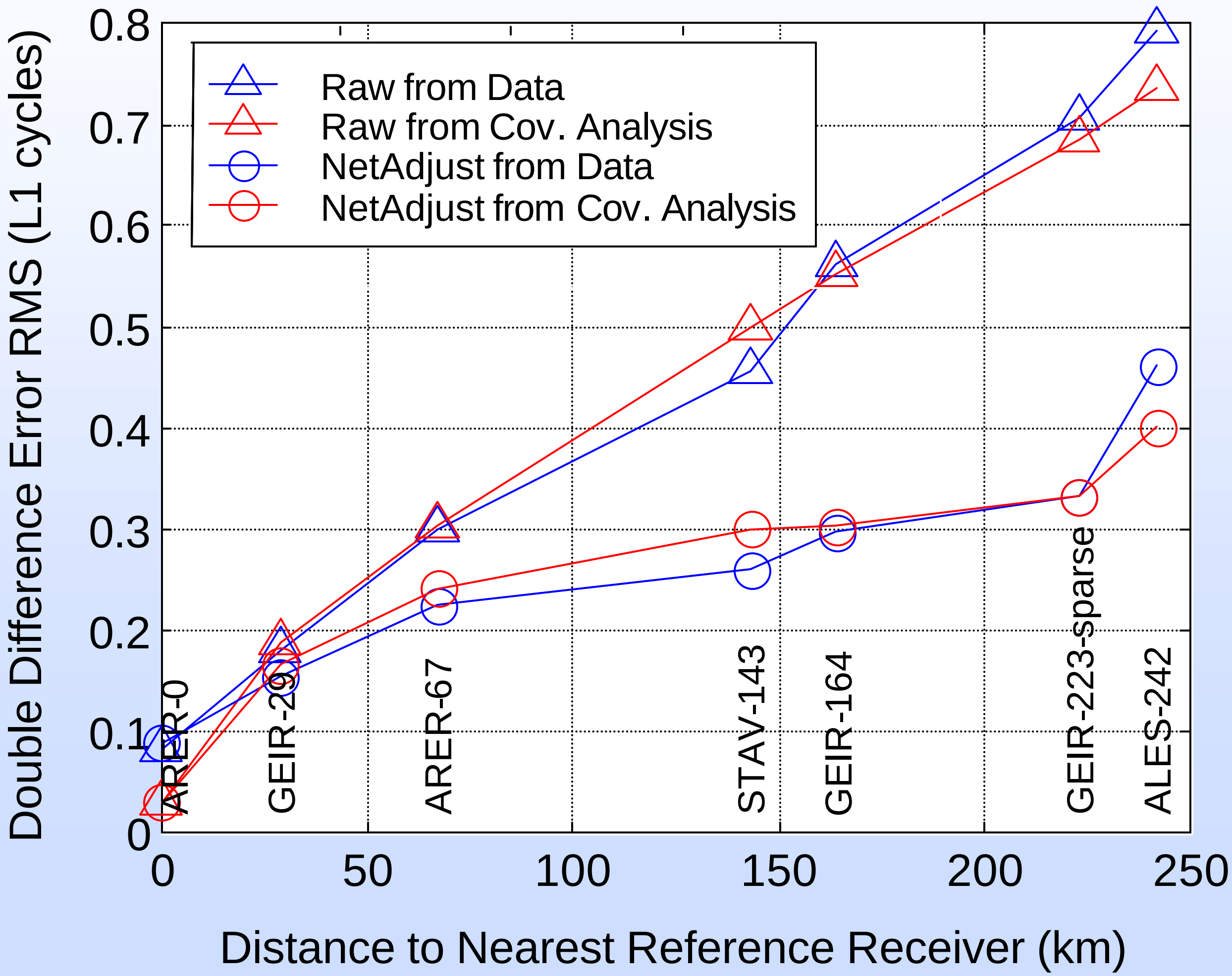
$B_2 \rightarrow$ DD operator (matrix) between reference/mobile receivers

$C_{\delta l} \rightarrow$ DGPS meas error covariance (from covariance function)

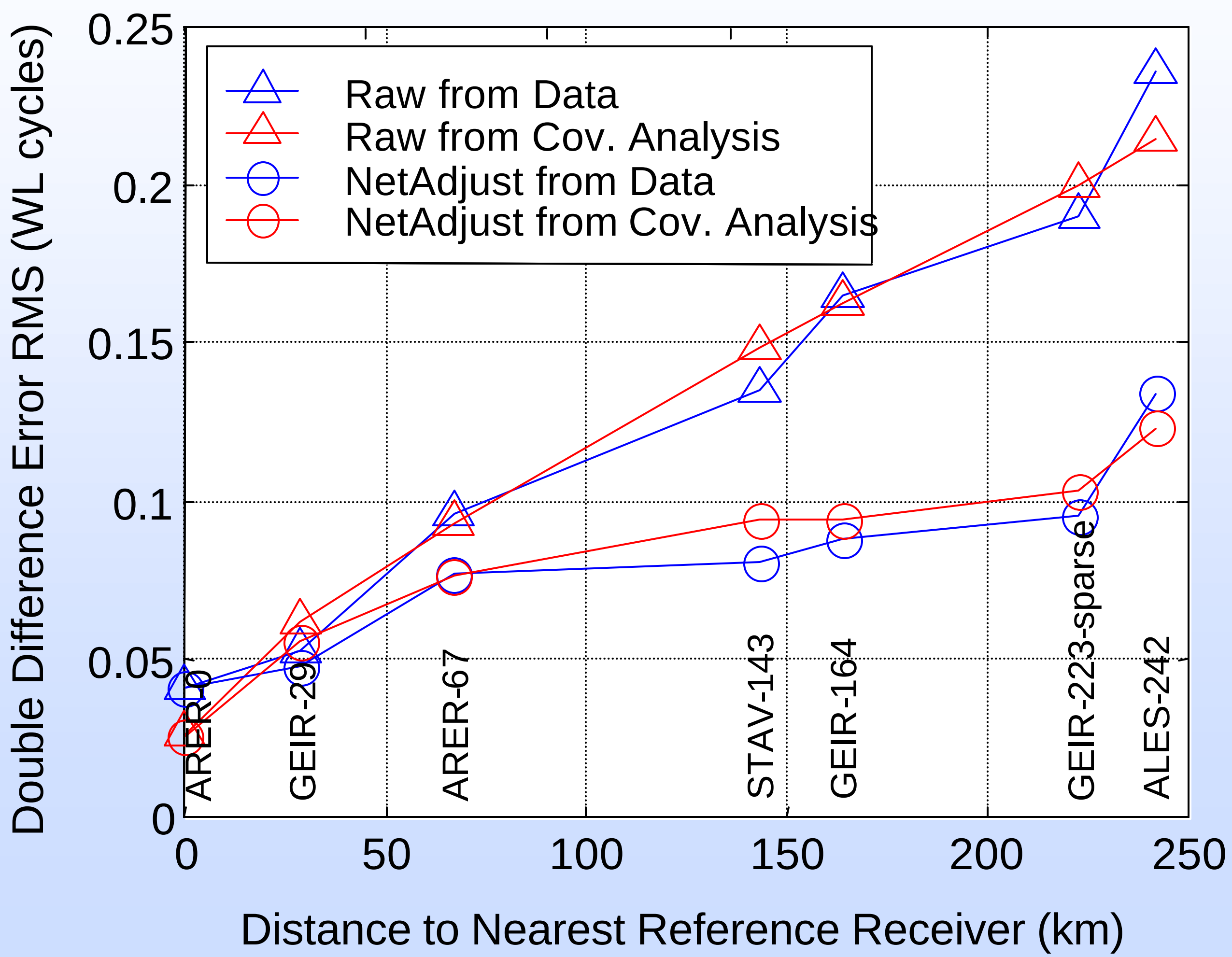
Validation of Covariance Function and Analysis Procedure

- Seven “test networks” selected
  - One receiver selected as “mobile” receiver
  - Remaining (or subset) form network
  - Closest reference receiver identified (for single reference case)
- Double difference errors predicted by covariance analysis
  - Single reference (raw) case
  - Multiple reference (NetAdjust) case
- Prediction compared with actual results

Validation: Predicted and Actual L1 Phase



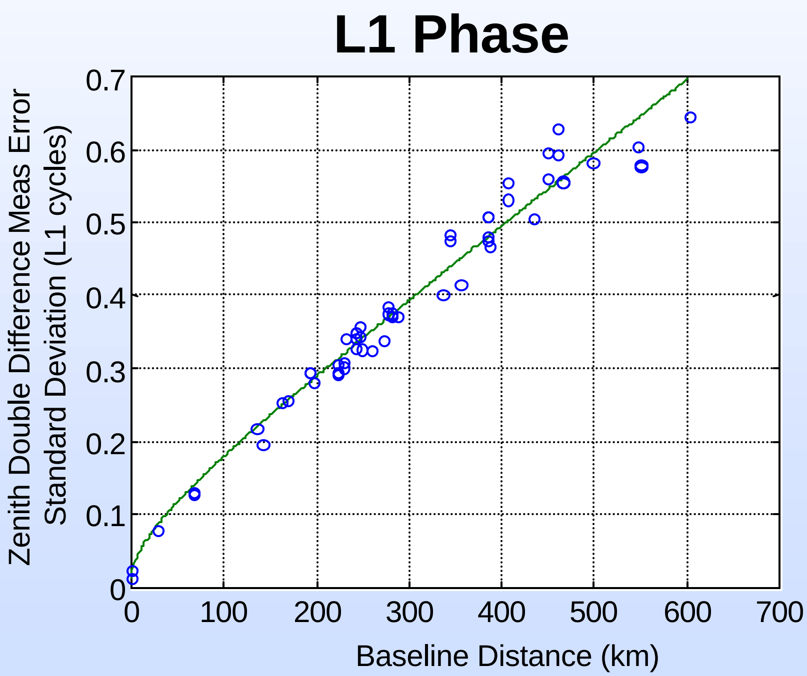
Validation: Predicted and Actual WL Phase

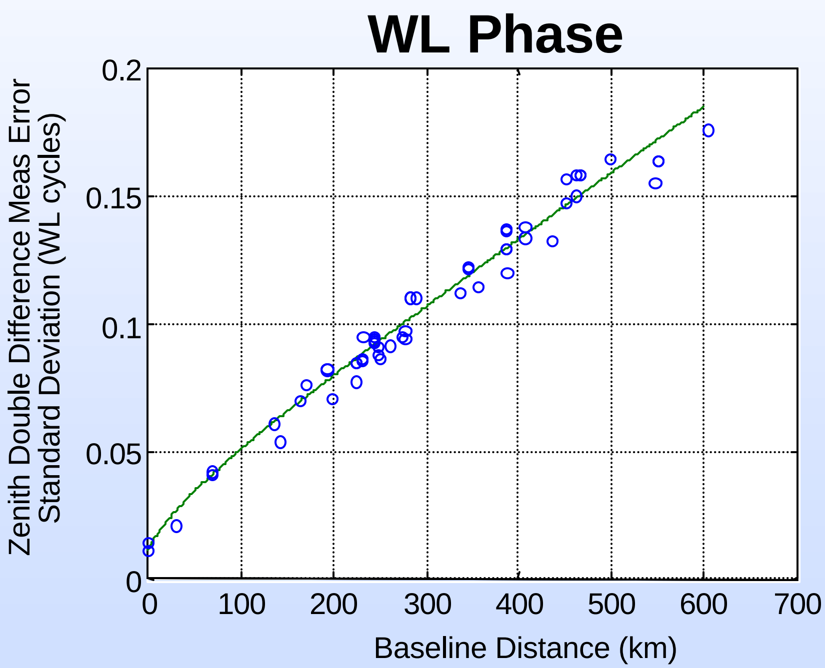


Development of Network Performance Specification

- Primary emphasis is carrier-phase ambiguity resolution
- Develop relationship between double difference measurement error and distance between mobile and reference receivers
- Specification made in terms of distance from reference receiver (under “normal” conditions)
  - More intuitive than pure error statistics
  - Typically, already is distance specification established
- Convert distance specification into measurement error specification

Zenith DD Measurement Error vs. Baseline Distance

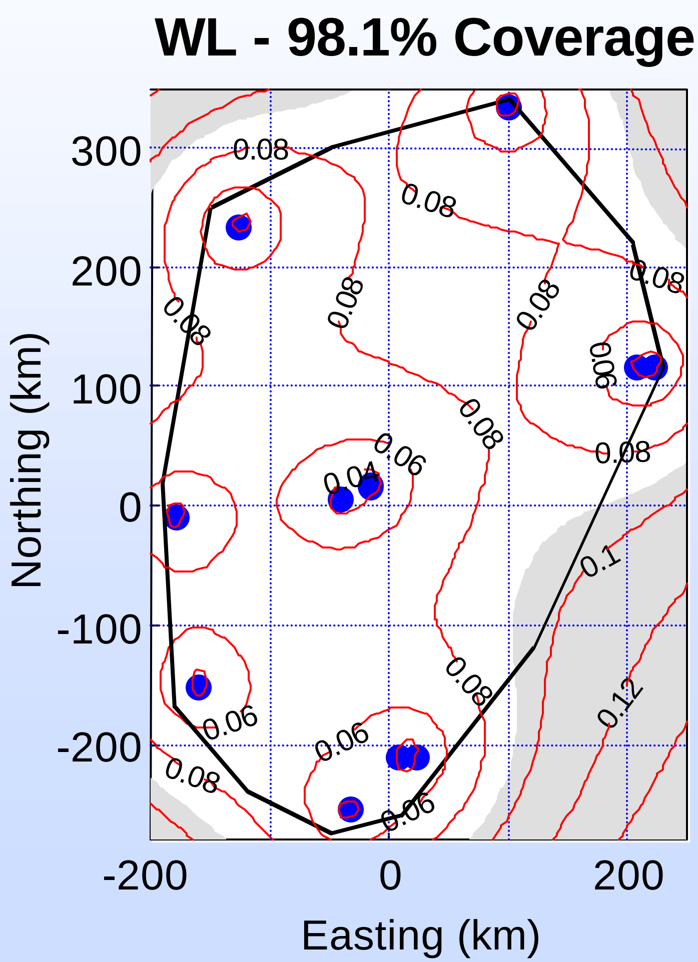




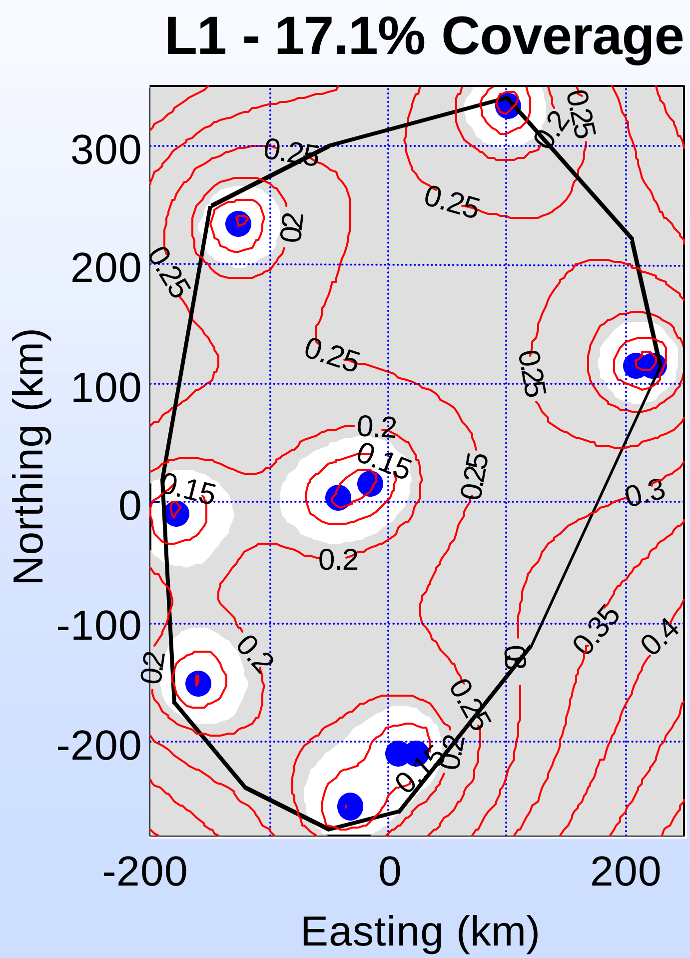
Specifications Chosen for Demonstration Purposes

- L1 Phase
  - Distance: 25 km
  - Zenith DD Meas Error Std Dev: 0.079 L1 cycles
  - DD Meas Error Std Dev: 0.182 L1 cycles
- WL Phase
  - Distance: 60 km
  - Zenith DD Meas Error Std Dev: 0.038 WL cycles
  - DD Meas Error Std Dev: 0.092
- Note: Assuming 7 SVs for plots that follow

WL Covariance Analysis Results

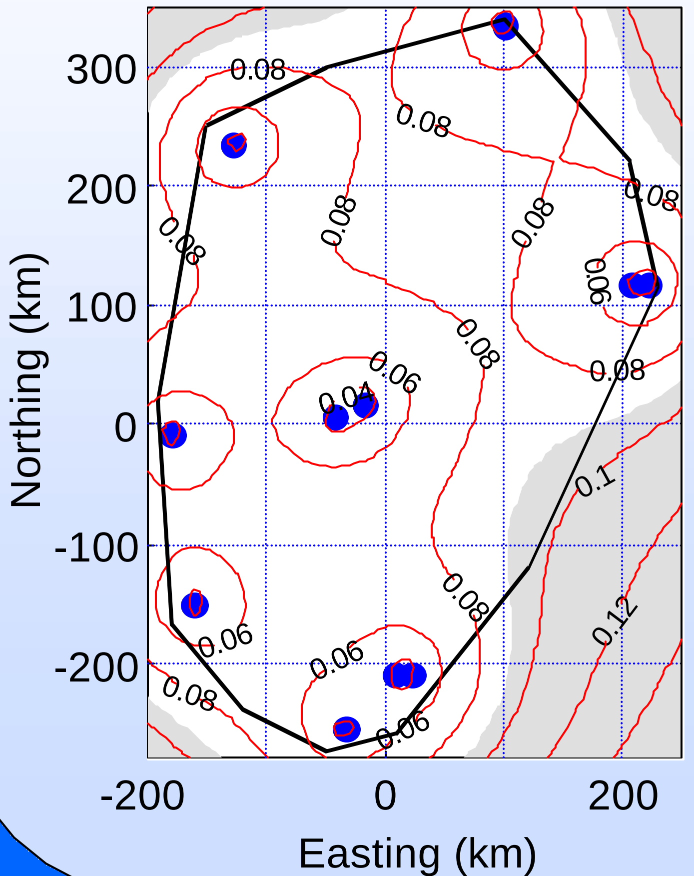


L1 Covariance Analysis Results

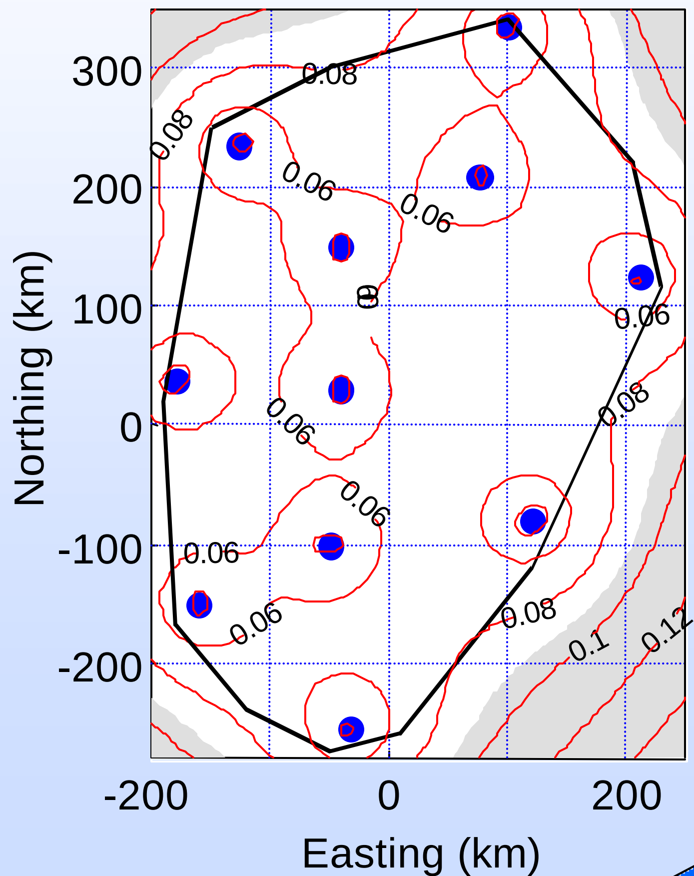


Effect of Repositioning Reference Receivers (WL)

Original - 98.1% Coverage

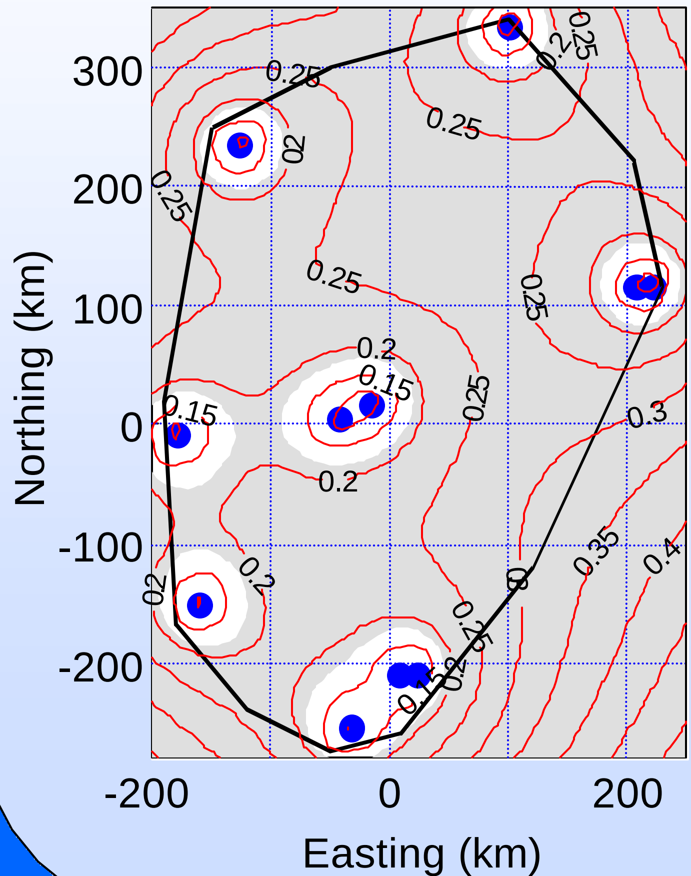


Repositioned - 100.0% Coverage

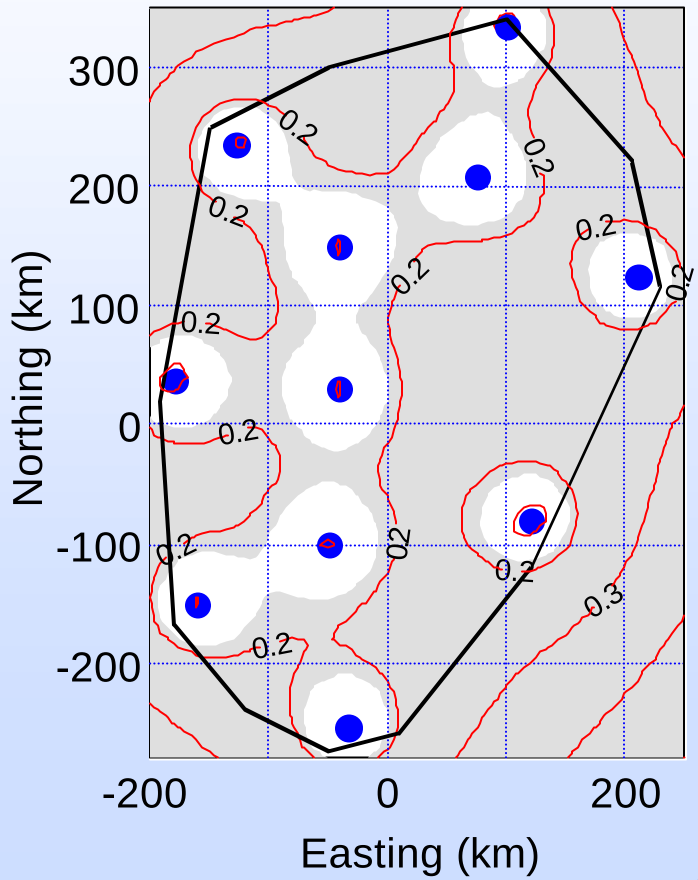


Effect of Repositioning Reference Receivers (L1)

Original - 17.1% Coverage

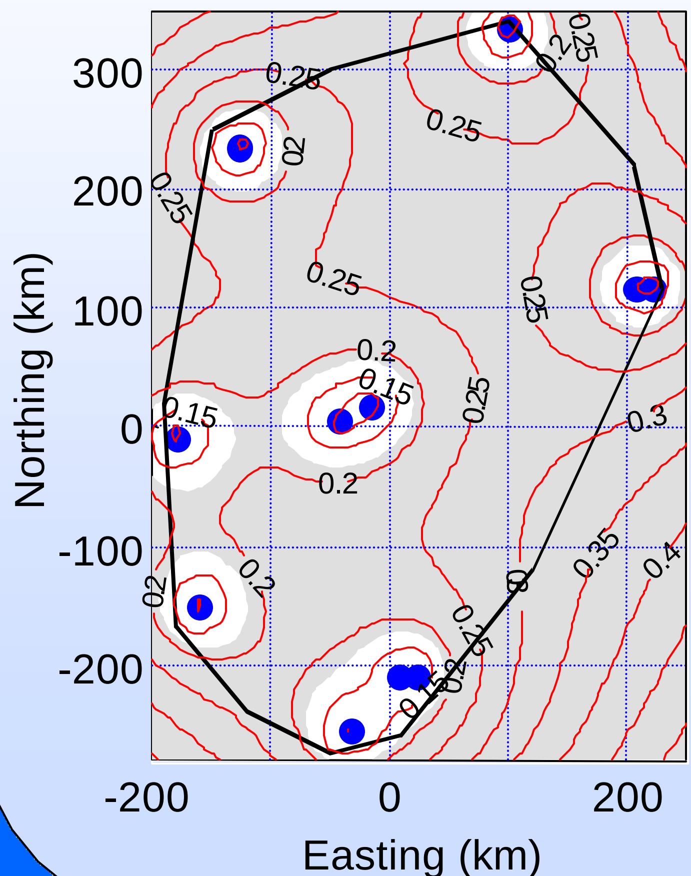


Repositioned - 30.7% Coverage

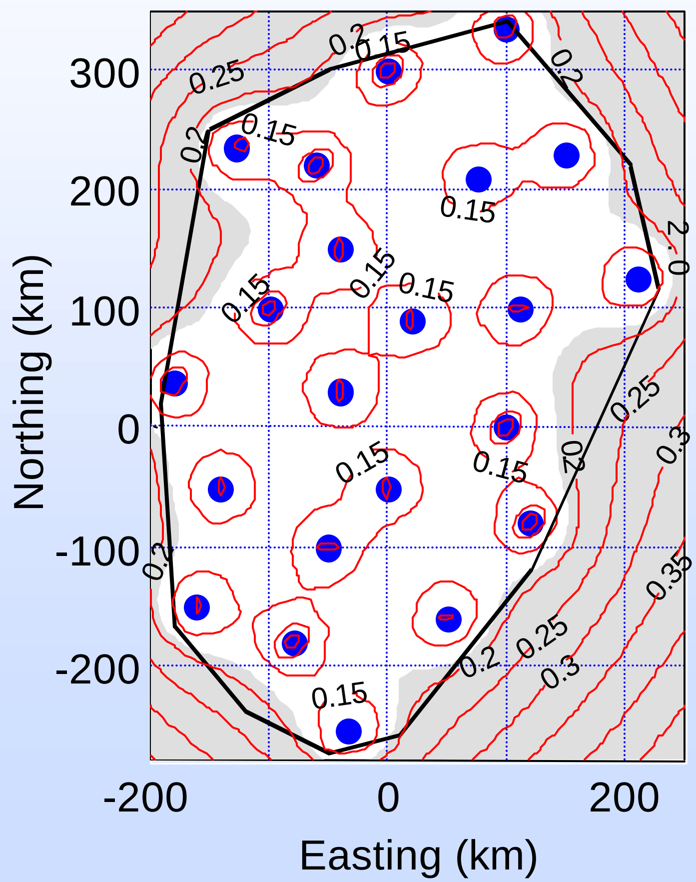


Effect of Repositioning/Adding Reference Receivers (L1)

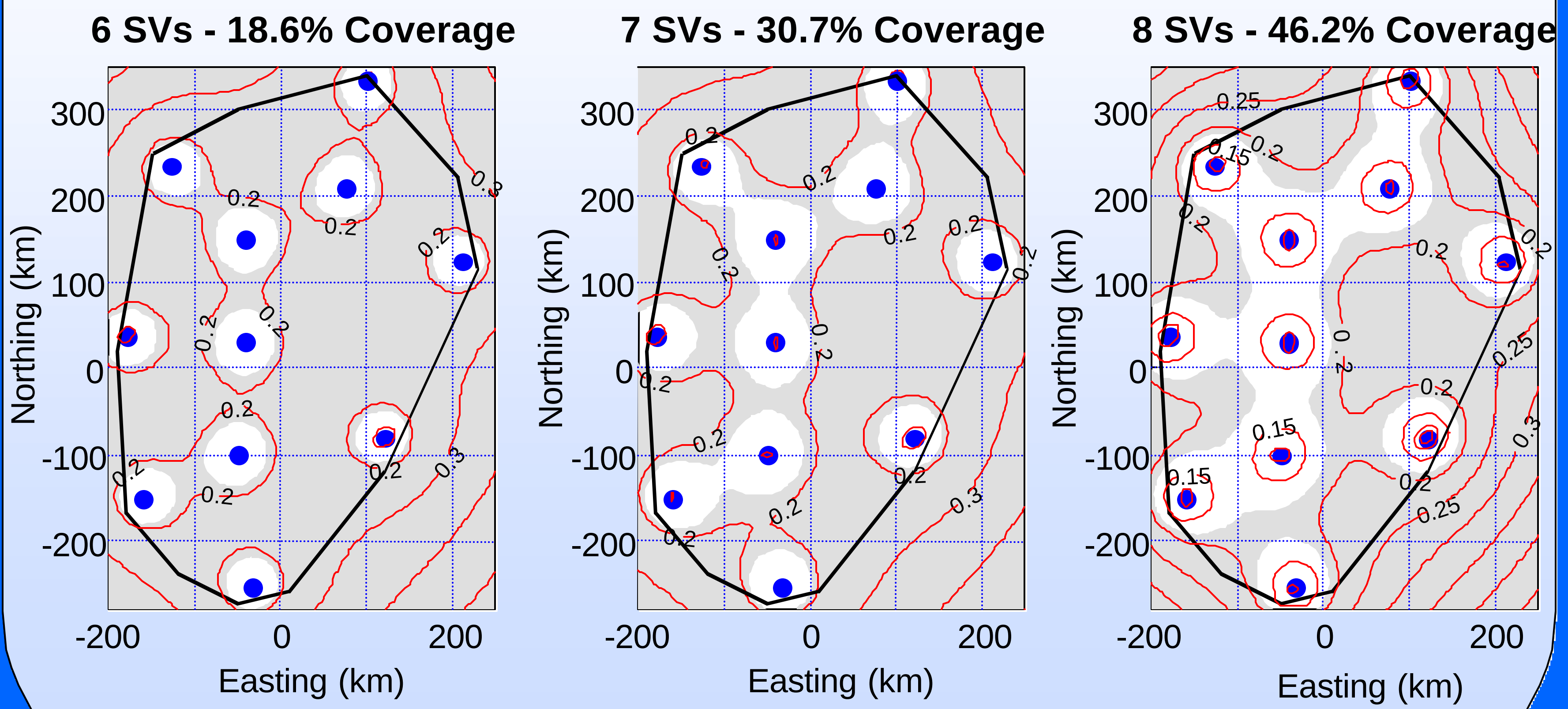
Original - 17.1% Coverage



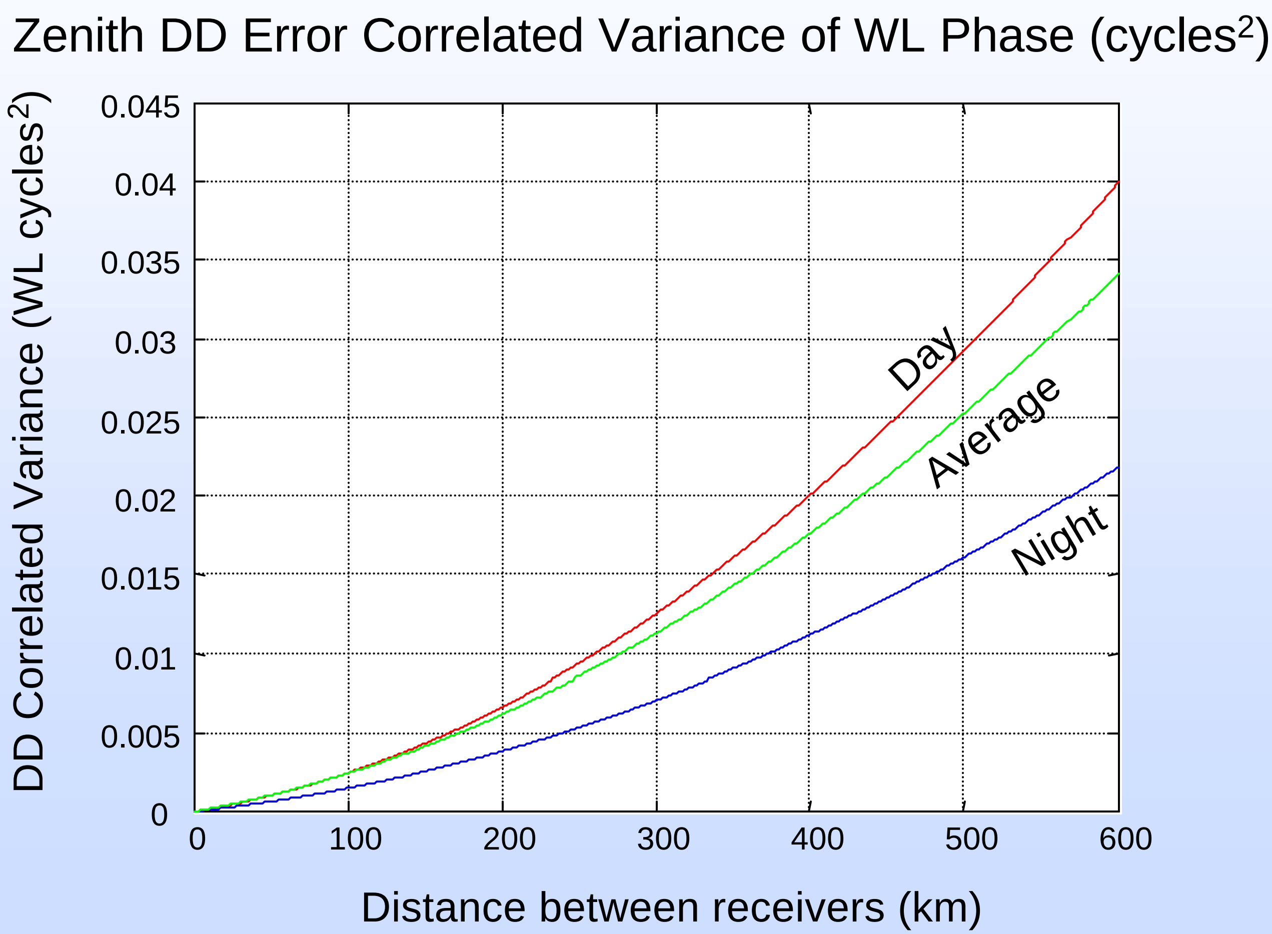
22 Ref Rcvrs - 91.4% Coverage



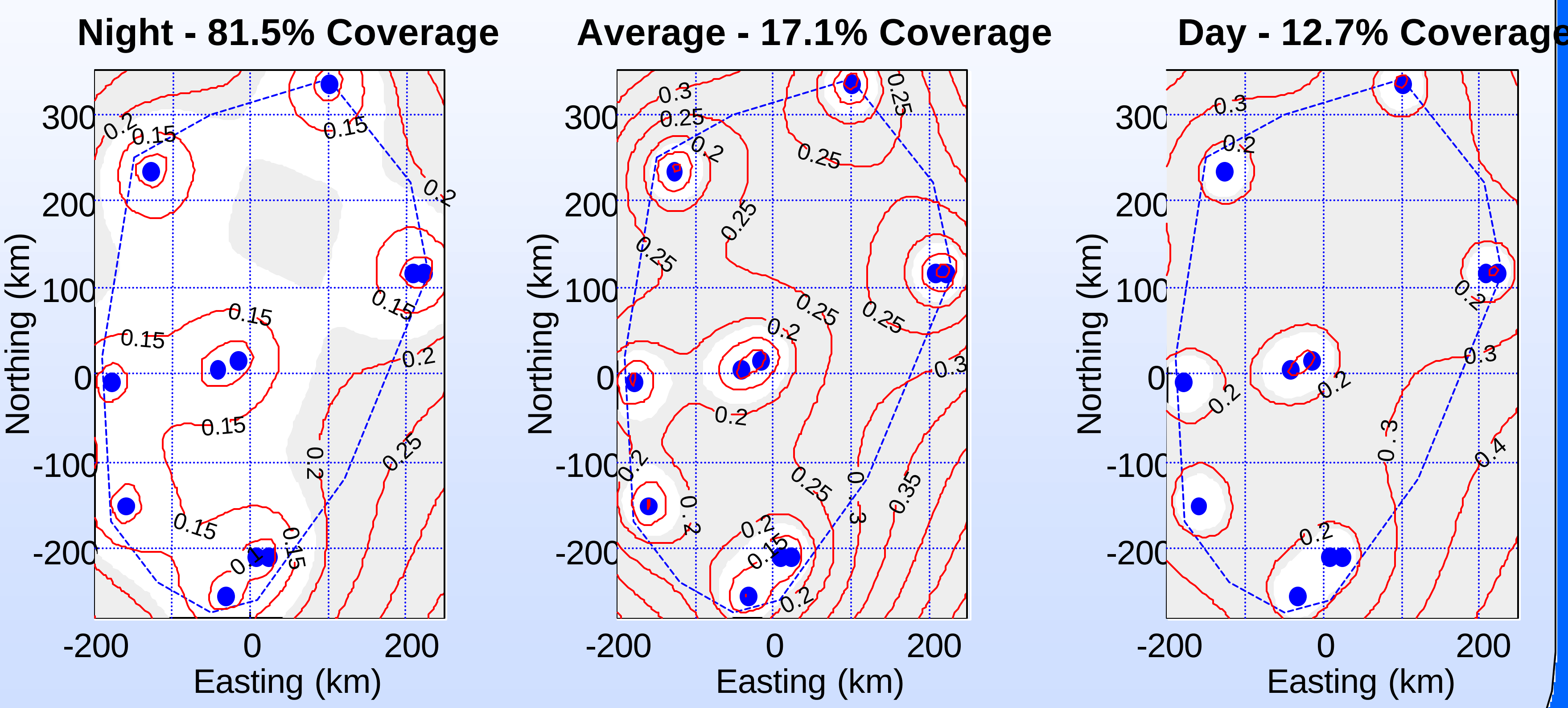
Effect of Varying Satellite Constellation (L1)



Analysis of Day/Night Variation WL Covariance Function



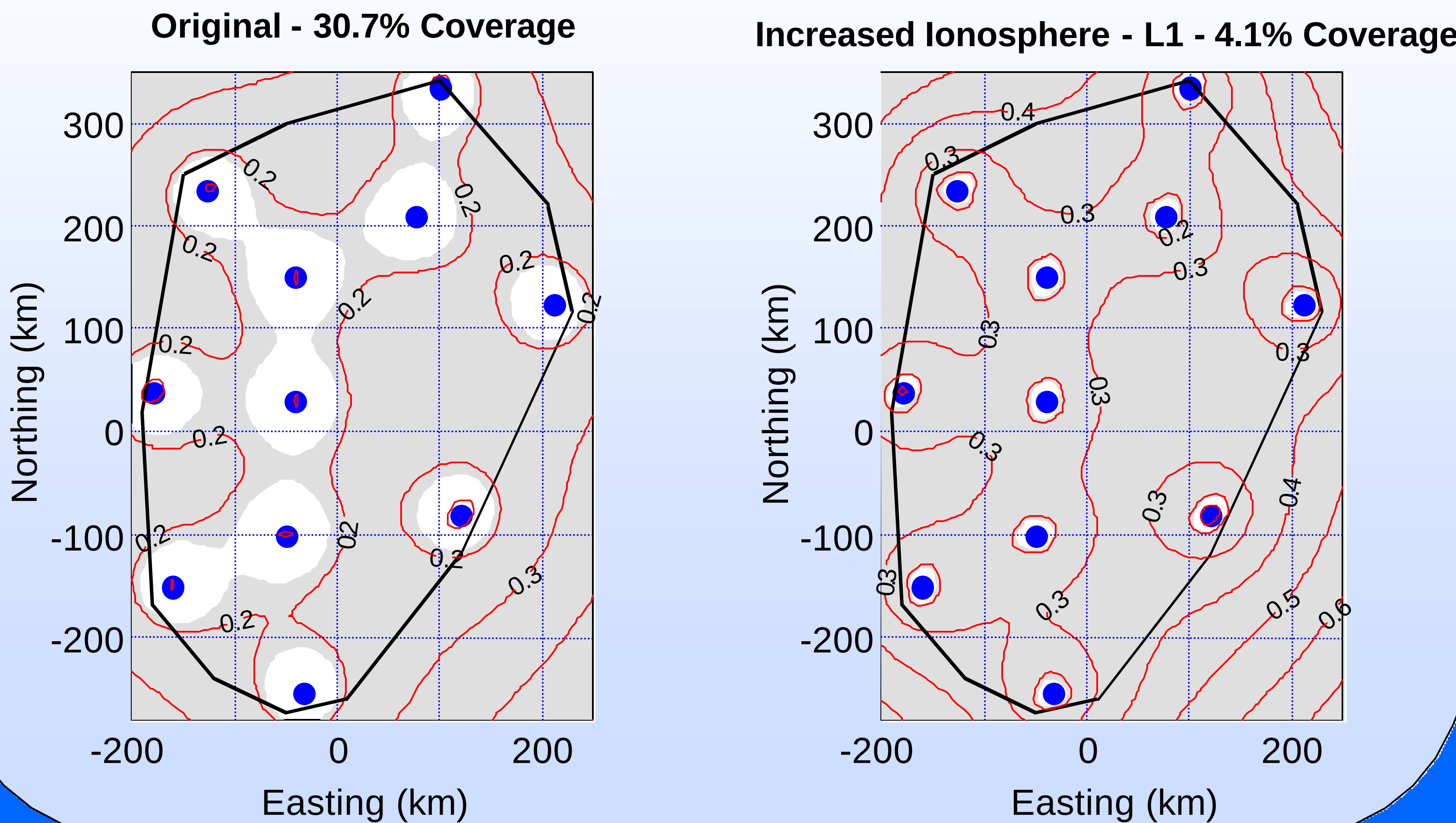
Analysis of Day Night Variation L1



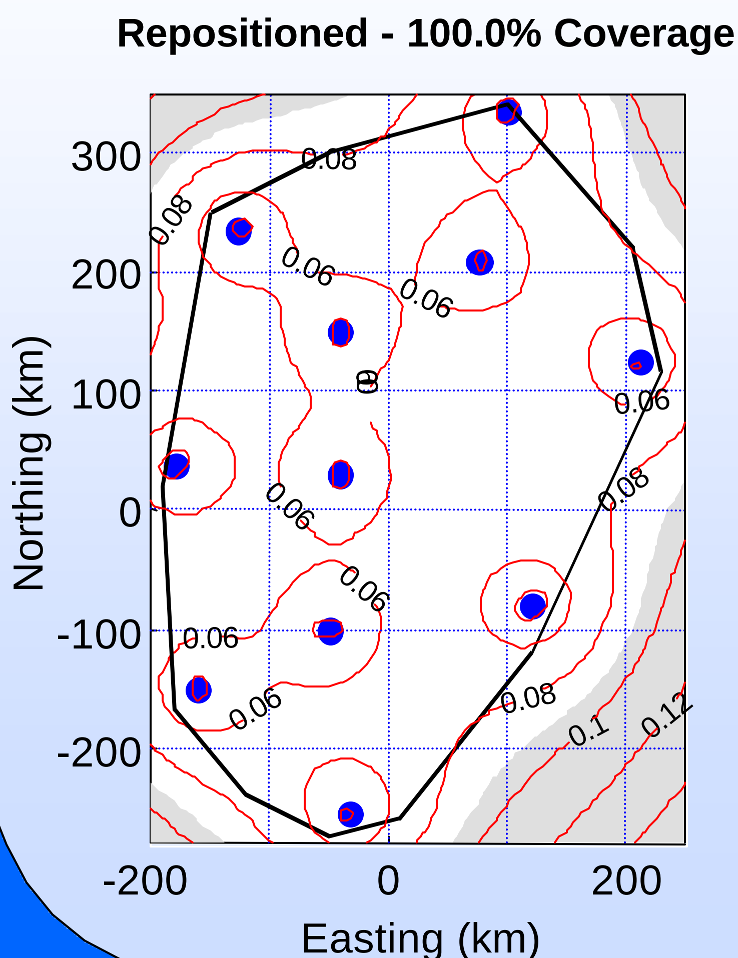
Prediction of Effect of Increased Ionospheric Activity

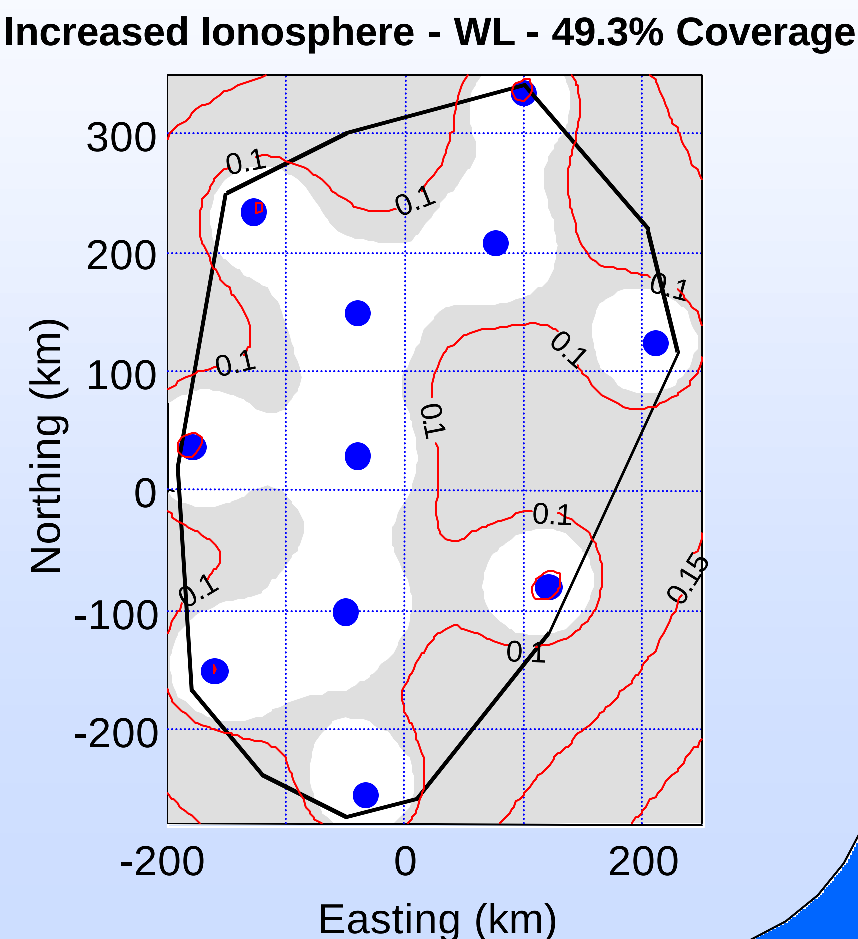
- Covariance analysis technique can be used to predict (simulate) high ionospheric activity
  - Covariance function represents combination of all errors (including ionosphere)
  - If ionosphere increases by some percentage, then total errors increase by a lesser percentage
    - Depends on the ratio of the ionospheric errors to all other error sources
    - Relatively easy to determine this ratio by using various L1/L2 combinations
- Measurement errors amplified by 1.5 (variance by 2.25) to simulate increased ionospheric activity

Effect of Increased Ionosphere (L1)



Effect of Increased Ionosphere (WL)





Overview

- Motivation
- Setting up the problem
- NetAdjust solution
- Implementation issues
- Putting this approach in context
- Covariance function description
- NetAdjust test results
- Covariance analysis technique
- Summary/Conclusion

Tests Accomplished

- Many post-processing tests have been performed
  - Holloman AFB, Aug 96
  - Norway, Sep 97
  - Norway, Sep 98
  - St. Lawrence Seaway, Nov 98
  - St. Lawrence Seaway, Aug 99
- Real-time implementation and testing underway
  - Norway
  - Japan

Conclusions

- Network approach shows promising results
  - Significantly educes both L1 and widelane errors
  - More effective with widelane ambiguity resolution, in tested networks
  - Not a cure-all
    - Depends on network spacing
    - Depends on error characteristics (especially ionosphere)
- Are many areas of ongoing work
  - Real-time network ambiguity resolution
  - Correction transmission schemes
  - Use of fixed and floating ambiguities

Additional Information

Dissertation and Related Papers:

http:/www.ensu.ucalgary.ca/GPSRes/multiref.html

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