MA2243 Exam 3 Fall 2005 Name:____________________________

(20 pts)1. Find the kernel, the nullity and the rank of the transformation T given by

$$T(x, y, z, w) = (2x - 12y + 5z + w,\ -x + 3y + 2z + w,\ x + 3y + 16z + 5w).$$

MA2243 **Exam 3** **Fall 2005** **Name:**____________________________

(20 pts) 2. Find eigenvalues and eigenvector(s) of $A = \begin{pmatrix} -1 & 0 & -4 \\ -1 & 3 & 0 \\ -1 & 0 & 2 \end{pmatrix}$.

(20 pts) 3. Determine whether the matrix $A = \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}$ is diagonalizable. If it is, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

(20 pts) 4. Find the solution of the initial value problem

$$\mathbf{x}' = \begin{pmatrix} -3 & 10 \\ -1 & -1 \end{pmatrix} = \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}.$$

MA2243 **Exam 3** **Fall 2005** **Name:**________________________

(20 pts) 5. Consider the linear systems $\frac{d\mathbf{y}}{dt} = A\mathbf{y}$. Let A have eigenvalues λ_1, λ_2 with corresponding eigenvectors $\mathbf{v}_1, \mathbf{v}_2$, respectively. Find the real general solution and determine the type and stability of the critical point. Sketch some trajectories in the phase plane (indicate directions of trajectories).

(1) $A = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}$, $\lambda_1 = 2, \lambda_2 = 7, \mathbf{v}_1 = [1 \ \ -1]^T, \mathbf{v}_2 = [2 \ \ 3]^T$.

type :

stability:

real general solution:

(2) $A = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$, $\lambda_1 = 2i, \lambda_2 = -2i, \mathbf{v}_1 = [1 \ \ i]^T, \mathbf{v}_2 = [1 \ \ -i]^T$.

type :

stability:

real general solution: