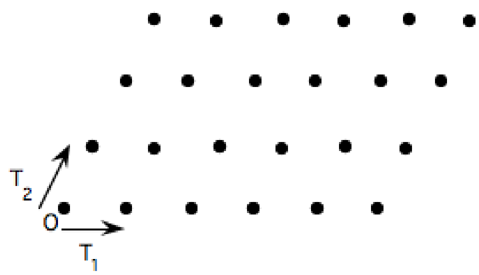
1 Wallpaper Patterns.

A wallpaper pattern consists of a motif, repeated at regular intervals in more than one direction. The framework on which any wallpaper pattern is built is called a net. A net is an array of points determined by an origin O and two translation vectors T_1, T_2, not in the same direction. All points in the net can be obtained by the translations $T_1^p \circ T_2^q$, where p and q are integers. Such translations form a group. Each parallelogram formed by the vectors T_1, T_2 is regarded as a **unit cell** of the **net.**



A unit cell of the net

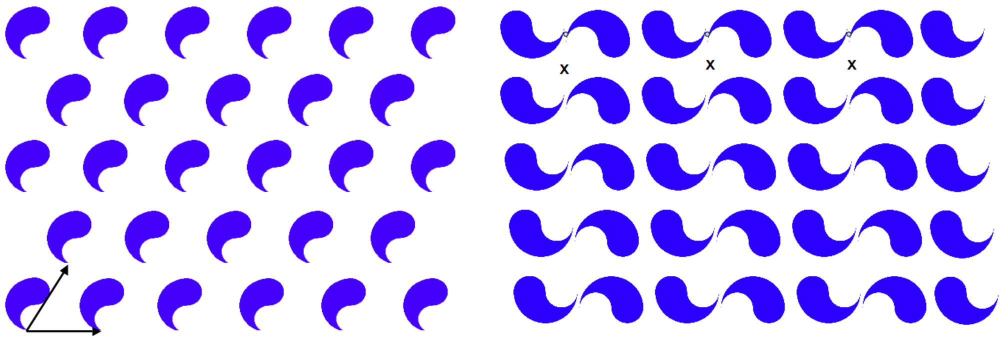
The net represents the mode of repetition. Any motif can be used, provided it is repeated in the way indicated by the net. Our illustrations are necessarily finite but the patterns are assumed to extend to infinity in all directions. There are only five types of nets possible for the wallpaper patterns. All nets have diad (180^o) rotation symmetry about the centers of the parallelograms, the vertices and the mid-points of the sides.

Parallelogram: The only symmetry of the parallelogram cell is the diad symmetry. A wallpaper based on this net may or may not have the diad symmetry as a whole. If the motifs used have the diad symmetry singly then the pattern as a whole has the diad symmetry and is referred to as $p2$, otherwise it does not have the diad symmetry and is referred to as $p1$.

The notation we use for describing the wallpaper group is given in the form $pijk$, where the prefix p is to indicate repetition in two directions.

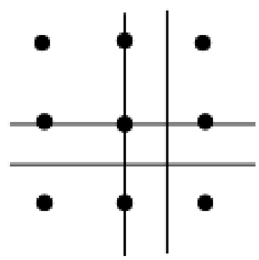
- i can take the value 1 to indicate there is no diad symmetry or 2 to indicate there is a diad (or 2-fold rotational) symmetry, 3 to indicate there is 3-fold, 4 to indicate there is 4-fold, and 6 to indicate there is 6-fold rotational symmetry. In addition m is used for the reflection symmetries.
- j can take the value m to indicate there is a vertical reflection symmetry otherwise we leave j out and k can take the value m to indicate there is a horizontal reflection symmetry otherwise we leave k out.

The following figures show examples of $p1$ and $p2$ for the parallelogram cell.



Left: p1 – Right: p2

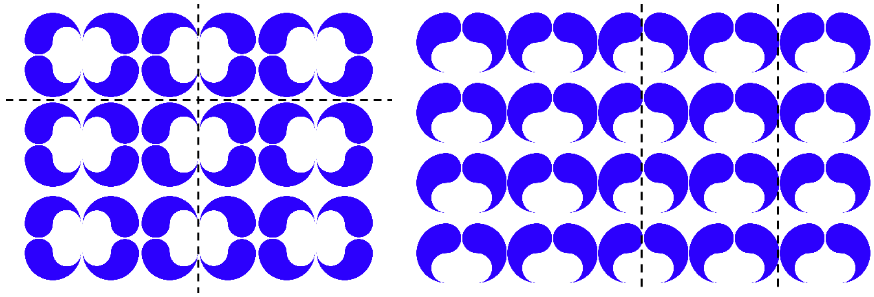
Rectangular: In addition to the diad symmetry the rectangular cell has both horizontal and vertical reflection symmetry about the centers of the rectangles and their vertices.



Rectangular cell

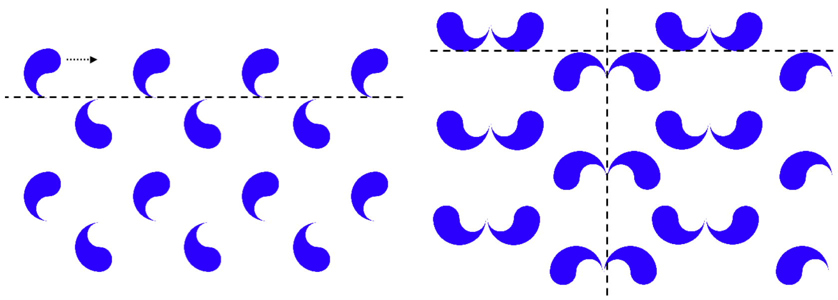
A motif applied to this net may have reflection symmetry in one or both directions. If the motifs have $D4$ as their symmetry groups then the pattern is referred to as $p2mm$ or pmm (2 is dropped because it is resulted from mm) . If the motif has reflection symmetry only with respect to the one-axis, it is referred to as pm.

Note that since we can interchange the x-axis and the y-axis in a wallpaper group, pm will represent both symmetries as they are considered to be congruent.

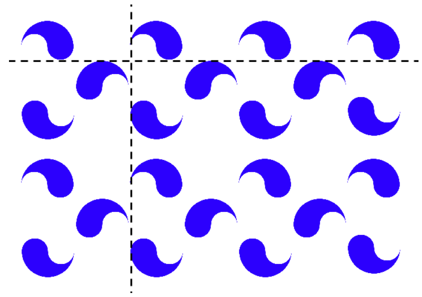


Left: pmm – Right: pm

As with frieze patterns, there are glide reflections associated with the rectangular cells. We can derive three new wallpaper patterns by using glide reflections in one or both directions. See pgg and pmg.

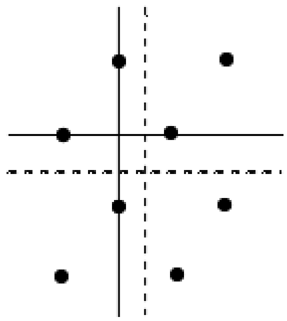


Left : pgg – Right: pmg



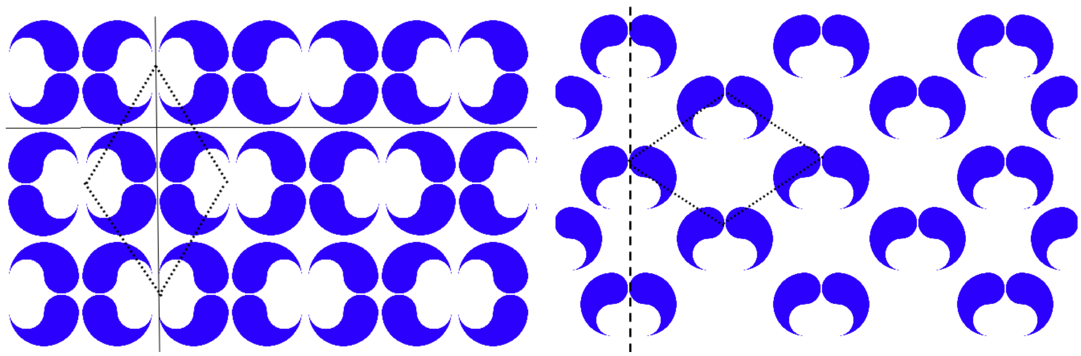
pgg

Rhombus : The Rhombus cell which is also referred to as centered rectangular has both horizontal and vertical reflection symmetry about the vertices and half way between the vertices and the center of the rhombus.



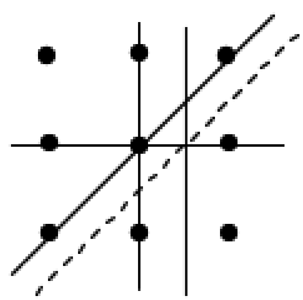
Rhombus cell

To distinguish the rhombus nets with the rectangular ones we use the letters c instead of p as a prefix to indicate repetition with a centered rectangular net. A motif applied to this net may have reflection symmetry in one or both directions. If the motifs have D4 as their symmetry groups then the pattern is referred to as *cmm*. If the motif has reflection symmetry only with respect to the x-axis or y-axis, it is referred to as *cm*. Since the glide reflection occurs naturally in these two types no other group is obtained by considering glide reflection.



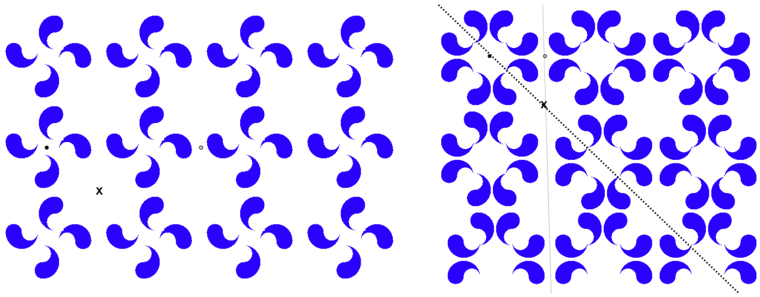
Left: cmm – Right: cm

Square: With a square net there is a wider range of possible symmetries. In addition to the diad symmetry about the mid-points of the sides, the net also has a tetrad symmetry (450rotation) about the vertices and centers of the squares. There are reflection lines not only along the squares, with intermediate lines through the centers, but also along the diagonals, with intermediate glide lines.

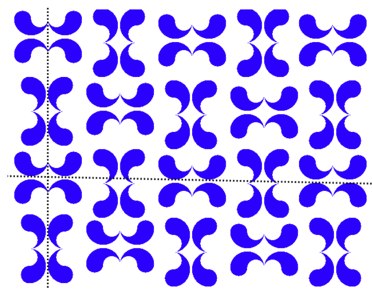


Square cell

There are three patterns based on a square net. The motifs may have a tetrad rotation *p*4, a tetrad rotation with reflections in both sides of the square, *p*4*m* or a tetrad rotation with a glide in the center of the square, *p*4*g* .



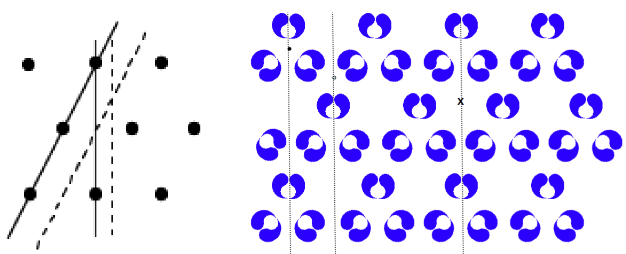
Left : p4 – Right: p4m



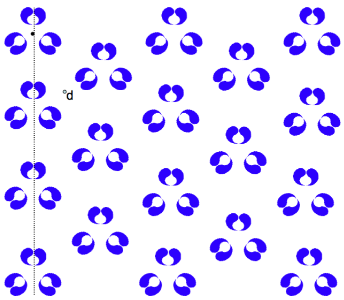
p4g

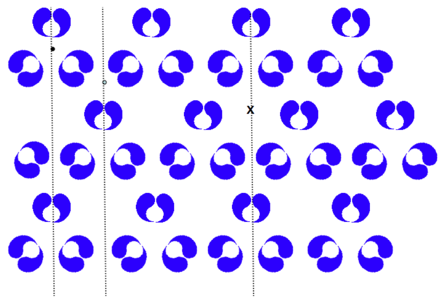
Hexagonal: $The 60^o$ rhombus, which divides into two equilateral triangles had hexad rotation(60^o) about the vertices of the triangles, triad rotation (120^0 rotation) about their centers, and diad about the mid-points of the sides, as well as reflection in the sides and altitudes of the triangles. We call this net hexagonal.

The motifs superimposed on a hexagonal net may have a triad rotation $p3$, a triad rotation with reflections in the altitudes of the triangles $p3m1$ or reflections in the sides $p31m$. They also may have a hexad rotation $p6$, or a hexad rotation with reflections in both the altitudes of the triangles and the sides $p6m$. Patterns of this type are shown below.

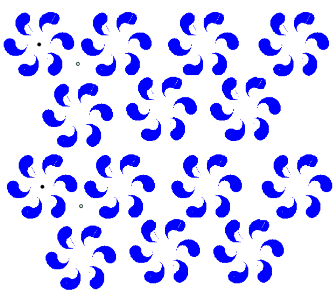


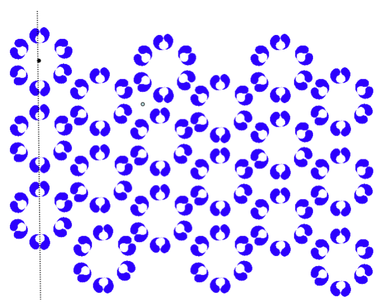
Left: Hexagonal cell – Right: p3



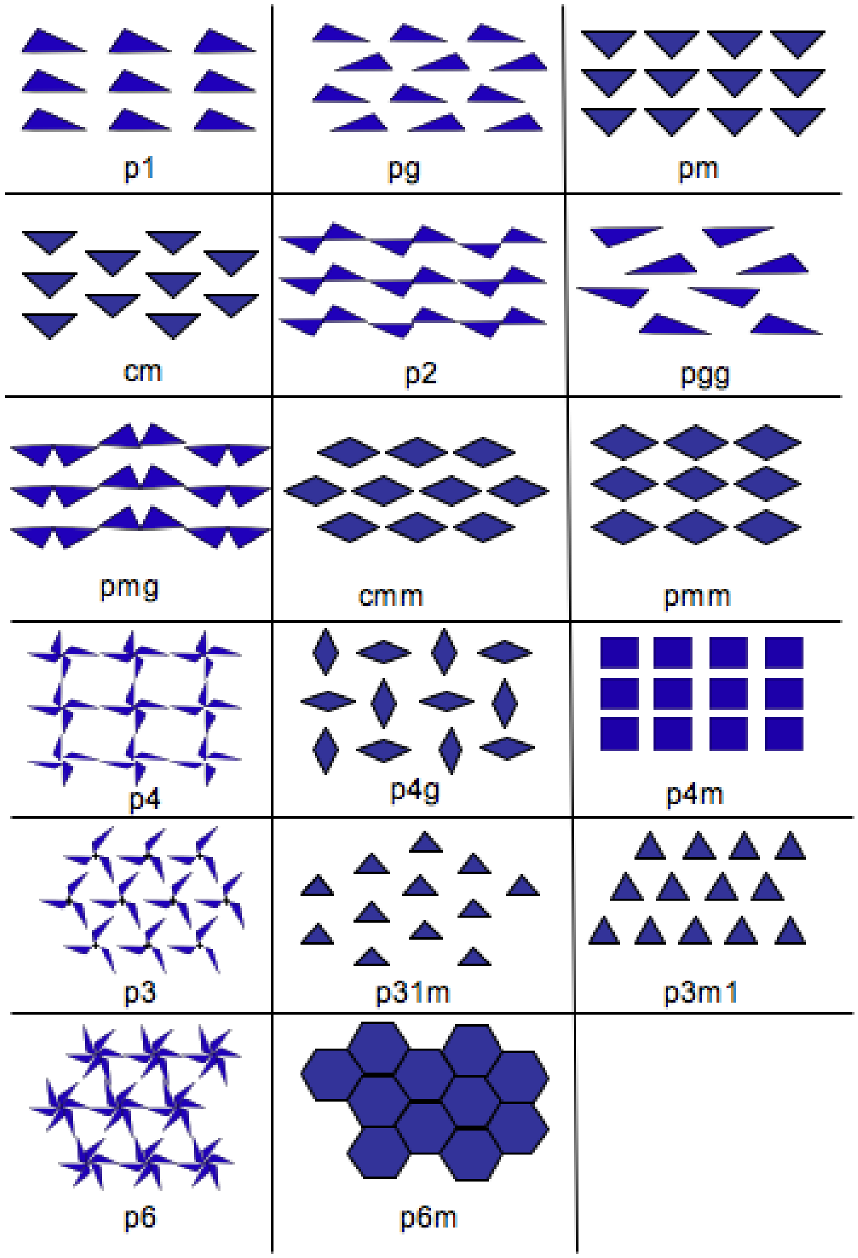


Left: p31m – Right: p3m1

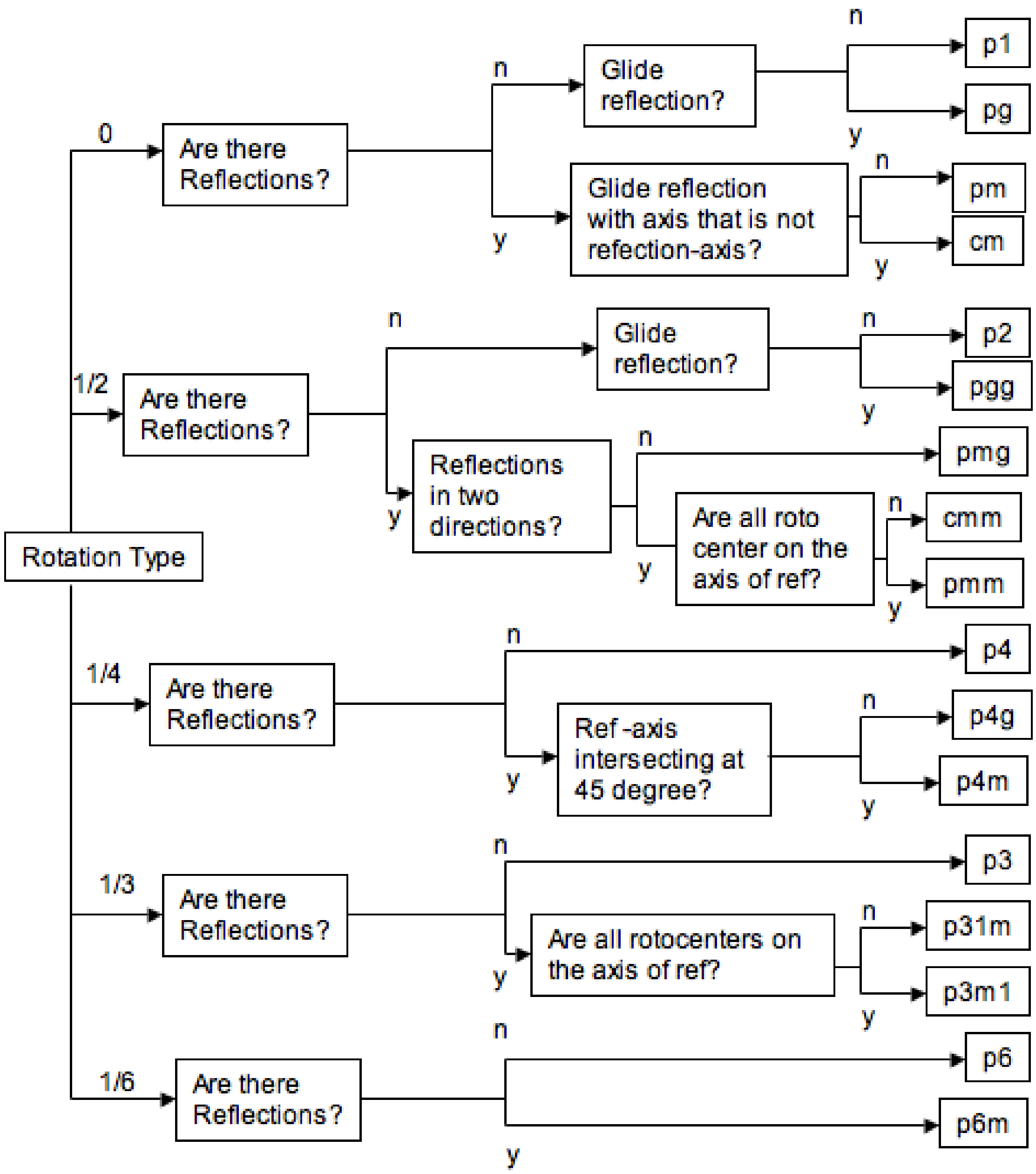




Left: p6 – Right: p6m

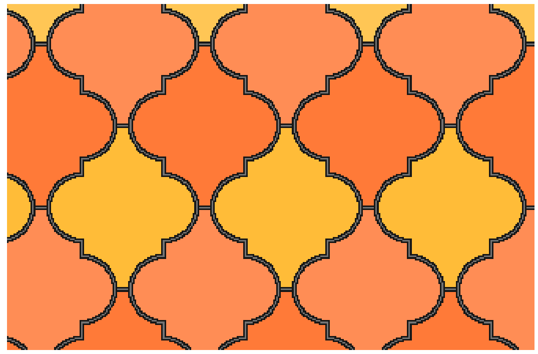


A table for classifying wallpaper patterns.

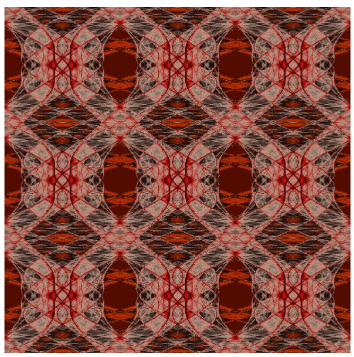
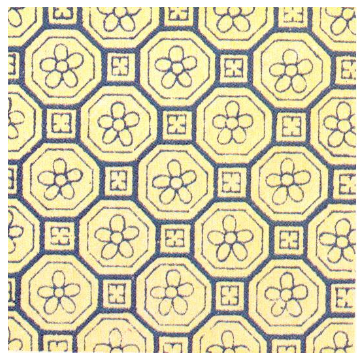


A Flow Chart for classifying wallpaper patterns.

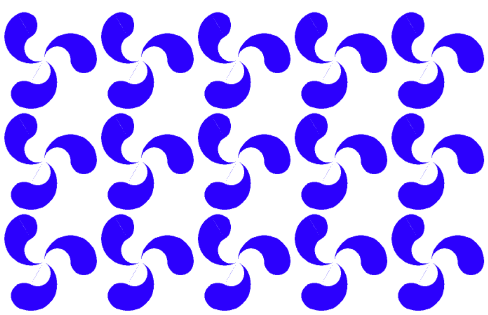
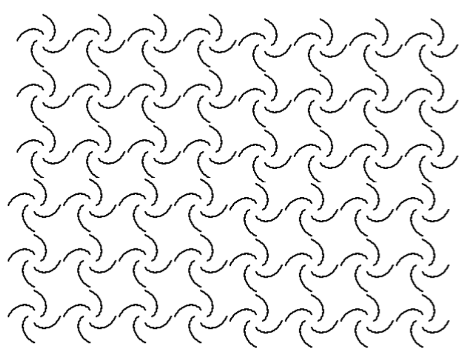
Examples of wall paper patterns:

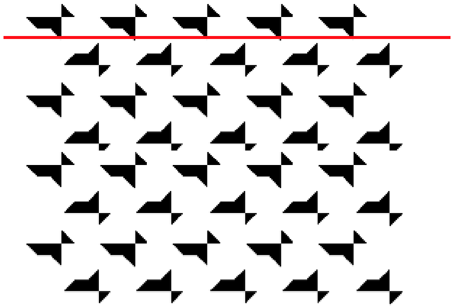
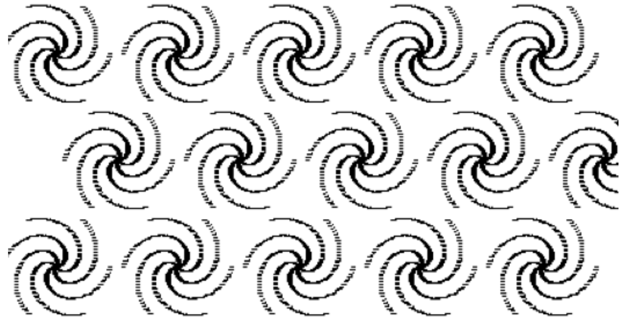
Left: cm – Right: cmm

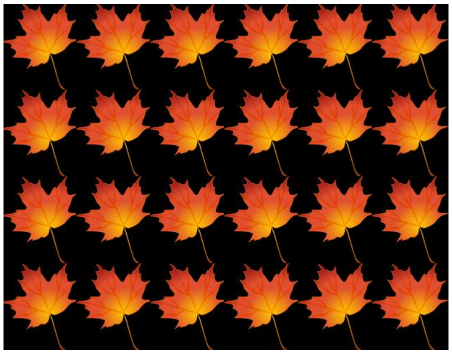
Left: pmm – Right: pm

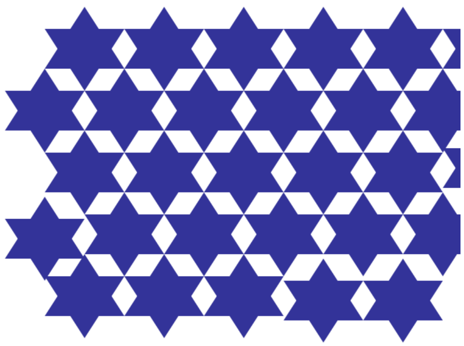
Left: p3 – Right: p4

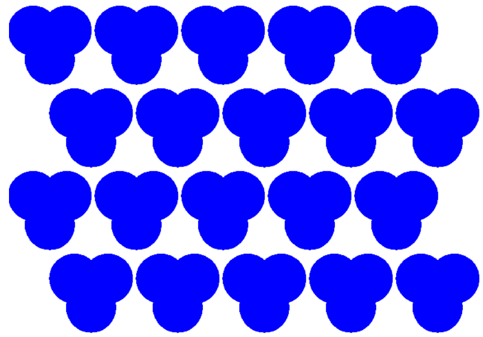
Left: pg – Right: p6



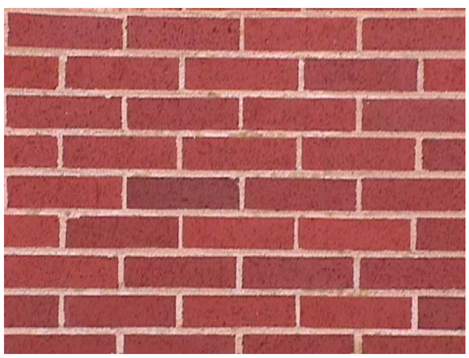
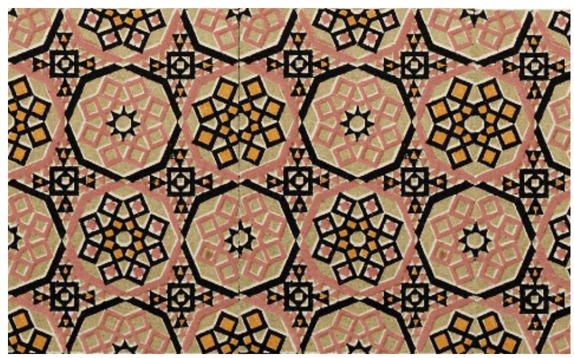

Left: p1 – Right: p2

Left: pgg – Right: p6m

Left: p31m – Right: p3m1

Left: pmg – Right: p4m