

Name: _____

Section: _____

Math 1571H. Final Exam December 15, 2005

There are a total of 235 points on this exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit.

Problem	Score
1.	_____
2.	_____
3.	_____
4.	_____
5.	_____
6.	_____
7.	_____
8.	_____
9.	_____
10.	_____
11.	_____
12.	_____
13.	_____
14.	_____
Total:	_____

Problem 1 Find the length of the curve $y = \ln \cos x$ between $x = 0$ and $x = \frac{\pi}{4}$.

Problem 2 A cylindrical tank of diameter 4 feet is lying on its side. If the tank is half full of rum having a density of 60 lbs./ft³, what is the force on one vertical end of the tank?

Problem 3 *Identify the graphs of the given curves expressed in polar coordinates. Be as specific as you can.*

1. $r = \frac{5}{\sin \theta}$

2. $r = 2/(1 - \frac{1}{2} \cos \theta)$

Problem 4 Find the arc length of the curve $\mathbf{r}(t) = t^3 \mathbf{i} + t^2 \mathbf{j}$ between $t = 1$ and $t = 2$.

Problem 5 Make the substitution $\sin \theta = 3x$ to evaluate the indefinite integral

$$\int \sqrt{1 - 9x^2} \, dx.$$

Show the details of your work.

Problem 6 Find the local maxima, the local minima, and the inflection points of the function $f(x) = e^x - 3e^{-x} - 4x$.

Problem 7 Solve the differential equation with initial condition

$$\frac{dy}{dx} = \frac{2y + 3}{x + 5}, \quad y(0) = 1.$$

Problem 8 Suppose the region under the graph of the curve

$$y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}, \quad 1 \leq x \leq 3$$

is rotated about the y -axis. Find the volume of the solid generated.

Problem 9 Find the equation of the plane containing the point $P(0, 1, -1)$ and the line with vector equation

$$\mathbf{R}(t) = (3 + t) \mathbf{i} + (1 + t) \mathbf{j} + (2 - t) \mathbf{k}.$$

Problem 10 Evaluate

$$\frac{d}{dx} \int_{2x}^{6x^2} \frac{t}{\cos t} dt.$$

Problem 11 Given that $\sum_{j=1}^n j^3 = \frac{n^2(n+1)^2}{4}$, use Riemann sums to compute the area of the region bounded by the curve $y = x^3$, the x axis and the line $x = 1$.

Problem 12 In a laboratory there are 10 grams of a radioactive substance with a half-life of 20 years. How many grams of the substance will remain after 25 years?

Problem 13 *The logistic equation is*

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where K and r are given positive constants and $0 \leq t$. Given some constant $P_0 > 0$, a solution of the initial value problem for this equation is a function $P(t)$, continuous on the infinite interval $0 \leq t$, twice differentiable on $0 < t$, satisfying the differential equation for all $t > 0$ and such that $P(0) = P_0$.

a. *Using the differential equation alone, show that*

$$\frac{d^2P}{dt^2} = r^2P\left(1 - \frac{P}{K}\right)\left(1 - \frac{2P}{K}\right)$$

b. *Argue that the graph of the solution $P(t)$ of the initial value problem has an inflection point at any time t_1 such that $P(t_1) = K/2$, i.e., is half the carrying capacity.*

Problem 14 *The front of a tank which is full of water has the shape of a regular trapezoid. In a suitable Cartesian coordinate system (x, y) the coordinates of the vertices of the top of the trapezoid are $(-4, 10)$ and $(4, 10)$, and the vertices of the bottom are $(-12, 0)$ and $(12, 0)$. Find the hydrostatic force exerted by the water on this tank front. The lengths are in feet and the water density is 62.5 lb/ft^3 .*

Brief solutions:

1. $\ln(1 + \sqrt{2})$

2. 320 lbs.

3. 1) vertical line $y = 5$, 2) ellipse in standard position $e = \frac{1}{2}$, $p = 4$.

4.

$$\frac{1}{27}(40\sqrt{40} - 13\sqrt{13})$$

5.

$$\frac{1}{6} \arcsin(3x) + \frac{x}{2} \sqrt{1 - 9x^2} + c$$

6. local max. at $x = 0$, local min. at $x = \ln 3$, inflection pt. at $x = \frac{1}{2} \ln 3$

7.

$$y = \frac{1}{10}(x + 5)^2 - \frac{3}{2}$$

8.

$$\frac{2\pi}{15}(11^{\frac{5}{2}} - 3^{\frac{5}{2}})$$

9.

$$x - 2y - z = -1$$

10.

$$\frac{72x^3}{\cos(6x^2)} - \frac{4x}{\cos(2x)}$$

11. $1/4$

12. $52^{-1/4}$ grams

13.

$$\frac{d}{dt} \frac{dP}{dt} = r \left(1 - \frac{2P}{K}\right) \frac{dP}{dt} = r^2 P \left(1 - \frac{P}{K}\right) \left(1 - \frac{2P}{K}\right)$$

14.

$$\frac{35}{6} 10^4 \text{ lbs.}$$