MATH 5652: Homework 5

(8.4, 8.16, 8.20 in Ch 4-Continuous-time Markov chains), and a, b, c below)

Let X_t be a continuous-time Markov chain on finite state space S with rate matrix Q.

a. Suppose that $S = \{1,2,3\}$ and that Q is symmetric (i.e. $q_{12} = q_{21}, q_{13} = q_{31}, q_{23} = q_{32}$). Find the stationary distribution of the process.

b. Suppose that $S = \{1,2\}$ with $q_{11} = -\lambda, q_{22} = -\mu$. Write down the system of differential equations for $p_t(i,j)$ using Kolmogorov's forward equations. Compute Q^n and use this to compute $e^{tQ} = \sum_{n=0}^{\infty} \frac{t^n}{n!} Q^n$. Using e^{tQ} write down the expressions for the probabilities $p_t(i,j)$, and find the limit of $p_t(1,1)$ and of $p_t(2,2)$ as $t \to \infty$. Find the stationary distribution of X_t and compare π_1 and π_2 with the two limits you computed above.

c. Suppose $S = \{0,1\}$ where the states denote two possible states for a service station: broken=0, and active=1. The time between failures is exponential μ and the time to repair a failure is exponential λ. Write down the rate matrix for this chain, and find its stationary distribution. Evaluate the limiting proportion of active time of the station.