MATH 5652: Homework 3

(4.1, 5.13, 5.19, 5.20 in Ch 2-Martingales, and a. below)

A simple symmetric random walk on **Z** is the process $(S_n)_{n\geq 0}$ defined by
$S_n = S_0 + \sum_0^n \xi_i$, where $\xi_1, \xi_2, \ldots$ are independent random variables with distribution $P(\xi_i = +1) = P(\xi_i = -1) = \frac{1}{2}$.
Such a random walk is said to have absorbing barriers at 0 and N if $0 < S_0 < N$, and $P(S_{n+1} = 0 | S_n = 0) = 1$ and $P(S_{n+1} = N | S_n = N) = 1$, for any $n \geq 0$. The absorption time then is defined to be $\tau = \min\{n \geq 0 : S_n = 0 \text{ or } S_n = N\}$.

a. Show that the mean time until absorption is $E\tau = E(S_0(N - S_0))$. (you may do teh problem with $S_0 = x$ fixed for $0 < x < N$)