

MATH 5652: Homework 2

(9.2, 9.8, 9.22, 9.32, 9.46, 9.48 in Ch 1-Markov Chains, and a. below)

A branching process is a model for the growth of the population size, according to the following rules: at n th time step we have the population of the n th generation, each individual in the n th generation independently gives birth to k offspring with probability p_k and dies, the offspring of the individuals in the n th generation form the $(n+1)$ st generation. Let X_n denote the number of individuals (population size) in the n th generation.

a. In the case of binary branching each individual has 0 or 2 offspring with equal probability. Assume $X_0 = 2$. Show that $(X_n)_{n \geq 0}$ is a time-homogeneous Markov chain, determine its state space and its transition matrix, determine which states are transient and which recurrent, and compute its stationary distribution. (Can you tell whether it is unique?)