

# High Order Reconstruction Methods for Piecewise Smooth Functions

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Reconstruction of piecewise smooth functions from their Fourier spectral coefficients is often studied. Applications arise in various scientific fields, in particular, the use of Fourier spectral methods are common in medical magnetic resonance imaging (MRI) because of their relationship to Radon transforms. Such images are not free from Gibbs phenomenon, as various tissue regions can be seen as piecewise smooth functions. Filtering is frequently used to alleviate the ringing in the images. However, abnormal developments often begin near the edges of tissues regions, and it is well known that filtering compromises the integrity of the image there. Hence we are motivated to use high order reconstruction techniques for purposes of earlier and better diagnosis.

Recently spectral reprojection methods, notably the Gegenbauer reconstruction method, have been developed to reconstruct piecewise smooth functions in their smooth sub-intervals and restore the exponential properties of spectral methods. Specifically, unlike standard filtering, the convergence rate does not deteriorate as the discontinuities are approached. This talk discusses these methods and demonstrate their capabilities in fields such as MRI reconstruction.

Another type of problem occurs when the given information is discrete (non-uniform) grid point data. Spectral reprojection methods can only be used if the data has a Gaussian type distribution. However, here we show that a similarly designed projection method, based on discrete variable orthogonal polynomials, can reconstruct piecewise smooth functions with spectral accuracy. The method is computationally efficient and robust.