

**PRACTICE THIRD MIDTERM EXAM, MATH 2243, FALL 2005**  
 INSTRUCTOR: MATTHIAS KURZKE

Name: \_\_\_\_\_ Discussion Session: \_\_\_\_\_

(031: Doyoon Kim TTh 2:30, 032: Javier Zuniga TTh 2:30, 033: Doyoon Kim TTh 3:35)

One-line scientific calculators are allowed. No books or notes. **Good Luck!**

Problem	1	2	3	4	5	6	Total
Maximum	10	15	20	20	20	15	100
Points							

1) (10 points). Are the following mappings linear transformations?

Yes    No

- ( )    ( )     $T : \mathcal{C}[0, 1] \rightarrow \mathbb{R}$  given by  $T(f) = \int_0^1 f(t)dt$
- ( )    ( )     $T : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$  given by  $T(A) = \det A$ . (Here  $\mathbb{R}^{2 \times 2}$  means the space of  $2 \times 2$  matrices).
- ( )    ( )     $T : \mathbb{R}^2 \rightarrow \mathbb{R}$  given by  $T(x, y) = xy$ .
- ( )    ( )     $T : \mathbb{R} \rightarrow \mathbb{R}^3$  given by  $T(x) = (x, x^2, x^3)$
- ( )    ( )     $T : \mathbb{R} \rightarrow \mathbb{R}$  given by  $T(x) = -\pi x$ .

2) (15 points). Consider the following matrix:

$$A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}.$$

Is  $A$  diagonalizable? If yes, find a matrix  $P$  such that  $B = P^{-1}AP$  is a diagonal matrix.

If not, why?

3) (20 points). Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

4) (20 points). Find the general solution to the system

$$\mathbf{x}' = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

5) (20 points). Find the general solution to the system  $\mathbf{x}' = A\mathbf{x}$ , where

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

- 6) (15 points). Find a basis for the kernel of the linear transformation given by the following matrix:

$$A = \begin{pmatrix} 3 & 1 & 4 & 2 \\ 2 & 3 & 5 & -1 \\ 4 & 1 & 5 & 3 \\ 0 & 7 & 7 & -7 \end{pmatrix}$$

What are the rank and the nullity of  $A$ ?