

PRACTICE SECOND MIDTERM EXAM, MATH 2243, FALL 2005
INSTRUCTOR: MATTHIAS KURZKE

Name: _____ Discussion Session: _____

(031: Doyoon Kim TTh 2:30, 032: Javier Zuniga TTh 2:30, 033: Doyoon Kim TTh 3:35)

One-line scientific calculators are allowed. No books or notes. **Good Luck!**

Problem	1	2	3	4	5	6	Total
Maximum	15	20	20	15	20	10	100
Points							

1) (15 points). Find a 2×2 matrix A with $A^2 = -I_2$.

Does there exist a 3×3 matrix B with $B^2 = -I_3$? Justify your answer.

2) (20 points) For any λ , find all solutions of the system

$$\lambda x_1 + x_2 = \lambda$$

$$x_1 + \lambda x_2 = -\lambda$$

3) (20 points). Consider the matrix A given by

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 8 \end{pmatrix}.$$

Compute $\det A$ and A^{-1} , if A is invertible.

4) (15 points). Solve the initial value problem

$$y'' + y' + y = \cos t, \quad y(0) = 0, \quad y'(0) = 1.$$

5) (20 points). Find a basis for the solution space of

$$y^{(4)} - 3y^{(3)} + 3y'' - y = 0.$$

6) (10 points). Which of the following sets is a vector space? (You don't need to justify your answer, "yes" or "no" is enough).

- (a) The set of all $(x, y) \in \mathbb{R}^2$ with $x^2 + y^2 = 1$.
- (b) The set of all $(x, y) \in \mathbb{R}^2$ with $x^2 + y^4 = 0$.
- (c) The set of all functions $f(t)$ with $\lim_{t \rightarrow \infty} f(t) = 0$.
- (d) The set of all functions $f(t)$ with $f'' + \cos(t)f = 0$.
- (e) The set of all polynomials whose degree is exactly three.