INTEGRATION BY PARTS

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1. Derivation of Formula for Integration by Parts

1.1. Product Rule for Differentiation.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

1.2. Indefinite Integral Notation.

$$\int [f(x)g'(x) + g(x)f'(x)]\,dx = f(x)g(x) + C$$

$$\int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx = f(x)g(x) + C$$

1.3. Formula for Integration by Parts.

1.3.1. *Indefinite Integral.*

$$\boxed{\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx}$$

Let $u = f(x)$ and $v = g(x)$. Then $du = f'(x)\,dx$ and $dv = g'(x)\,dx$.

$$\boxed{\int u\,dv = uv - \int v\,du}$$

1.3.2. *Definite Integral.*

$$\boxed{\int_a^b f(x)g'(x)\,dx = f(x)g(x)|_a^b - \int_a^b g(x)f'(x)\,dx}$$

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2. ILATE Rule for Choosing u in Integration by Parts

I: Inverse Trigonometric Function
L: Logarithmic Function
A: Algebraic Function
T: Trigonometric Function
E: Exponential Function

3. Examples

(1) Evaluate the following integrals:
   (a) $\displaystyle\int x \ln x \, dx$ **Answer:** $\displaystyle\frac{x^2}{2}\ln x - \frac{x^2}{4} + C$
   (b) $\displaystyle\int x \tan^{-1} x \, dx$ **Answer:** $\displaystyle\frac{x^2+1}{2}\tan^{-1} x - \frac{1}{2}x + C$
   (c) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x \csc^2 x \, dx$ **Answer:** $\displaystyle\frac{\pi}{4} - \ln\left(\frac{\sqrt{2}}{2}\right)$
   (d) $\displaystyle\int_0^t e^x \sin(t-x)\, dx$ **Answer:** $\displaystyle\frac{1}{2}\left(-\sin t + e^t - \cos t\right)$
   (e) $\displaystyle\int_0^1 \left(x^2+1\right) e^{-x}\, dx$ **Answer:** $\displaystyle -\frac{6}{e} + 3$
   (f) $\displaystyle\int_1^4 e^{\sqrt{x}}\, dx$ **Answer:** $2e^2$
   (g) $\displaystyle\int \sin^{-1} x \, dx$ **Answer:** $x \sin^{-1} x + \sqrt{1-x^2} + C$
   (h) $\displaystyle\int \sin(\ln x)\, dx$ **Answer:** $\displaystyle\frac{1}{2}\left[-x\cos(\ln x) + x\sin(\ln x)\right] + C$
(2) Find the area of the region bounded by the curves $y = 5\ln x$ and $y = x \ln x$. **Answer:** $\displaystyle\frac{25}{2}\ln 5 - 14$
(3) Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = e^x$, $y = e^{-x}$, and $x = 1$ about the y- axis. **Answer:** $\displaystyle\frac{4\pi}{e}$

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