1. For A a square matrix, let $\kappa_p(A)$ denote the condition number with respect to the matrix l_p norm ($1 \le p \le \infty$). Show that
$$\kappa_1(A) \le n\kappa_2(A), \quad \kappa_2(A) \le n\kappa_1(A),$$
for all $A \in \mathbb{R}^{n\times n}$. Thus a matrix is well-conditioned or badly-conditioned with respect to the l_1 norm, if and only if it is with respect to the l_2 norm (up to a factor of the matrix size). State and prove a result of this type relating the condition numbers with respect to the l_1 norm and the l_∞ norm.

2. In computing a cubic spline interpolant using equally spaced interpolation points, with spacing h, we are led to solving a linear system with tridiagonal matrix
$$A = \begin{pmatrix} 2h/3 & h/6 & & & \\ h/6 & 2h/3 & h/6 & & \\ & h/6 & 2h/3 & \ddots & \\ & & \ddots & \ddots & h/6 \\ & & & h/6 & 2h/3 \end{pmatrix} \in \mathbb{R}^{n\times n}.$$
Prove that $\kappa_\infty(A) \le 3$ (independent of the spacing h or the number of points.

3. Write a column oriented algorithm to compute the Doolittle LU decomposition. That is, given a matrix A with nonsingular principal minors, your algorithm should overwrite the elements on or above the diagonal of A with the corresponding elements of an upper triangular matrix U and overwrite the below-diagonal elements of A with the corresponding elements of a unit lower triangular matrix L such that $LU = A$, and your algorithm should access the elements of A by columns. In addition to writing the algorithm, submit a direct Matlab translation along with a verification that it works using a random 4×4 matrix. Discuss the implementation of your algorithm using BLAS.