A Survey of Max-Type Recursive Distributional Equations

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Abstract

In certain problems in a variety of applied probability settings (from probabilistic analysis of algorithms to statistical physics), the central requirement is to solve a recursive distributional equation of the form

\[ X = g(\xi_i, X_i), \quad i \geq 1. \]

Here \((\xi_i)\) and \(g(\cdot)\) are given and the \(X_i\) are independent copies of the unknown distribution \(X\). We survey this area, emphasizing examples where the function \(g(\cdot)\) is essentially a “maximum” or “minimum” function. We draw attention to the theoretical question of endogeneity in the associated recursive tree process \(X_i\), are the \(X_i\) measurable functions of the innovations process \((\xi_i)\)?

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